A Comparison of U.S. and Chinese Mathematics Textbooks and Teaching: Concept Definitions, Conceptual Information, and Classroom Instructions

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Since the 1990s, when international comparison of mathematics education took its popularity, projects using this approach have generated many insights. One of the main findings is great cross-cultural variability in student achievements on mathematical assessments. In particular, various international assessment projects such as TIMSS and PISA have revealed that China is among the top few countries with the highest student achievement in mathematics, and the United States is usually about the average (Mullis, Martin, Foy, & Arora, 2012; Tienken, 2013; Wu, 2009). Because of the constant report of student achievements in mathematics, the current study aims to explore the contributors to such a difference.

Many factors are believed to contribute to such differences in mathematical achievement and student learning. One of those factors, and possibly the most direct one, is how teachers teach the knowledge, i.e. the instructional patterns (Jonassen & Grabowski, 2012). A continuum of instructional patterns have been identified by international comparison research (Hiebert & Grouws, 2007; Rittle-Johnson & Alibali, 1999; Stevenson & Stigler, 1994; Stigler & Hiebert, 2009). On one end of the continuum is the conceptual instruction, which utilizes a top-down approach to address higher-level conceptual understanding first and then focus on developing specific procedures to solve problems later. This conceptual instructional pattern especially stresses the mental connections of concepts and ideas. On the other end of the continuum is the procedural instruction. Contrary to the conceptual instruction, the procedural instructional pattern is a bottom-up effort. That is, the general, high-level understanding is believed to be generated as students accumulate enough knowledge about certain procedures. As a result, an emphasis on practicing and working on exercise is a feature of this pattern.

Although previous research has extensively explored the connection between instructional patterns and learning outcomes, little is known about the origin of instructional patterns. Thus, this current research tries to trace the roots of different instructional patterns in mathematical textbooks. First, past research on how instructional patterns are associated with learning outcomes will be discussed. Then, a model will be suggested how textbooks can influence how teachers teach in class. The model will be translated into several hypotheses, and studies testing those hypotheses will be discussed.

Conceptual and Procedural Instructional Patterns

Past research has indicated that instructional patterns are predicted by the learning goals a teacher sets for the lesson (Hiebert & Grouws, 2007; Rittle-Johnson & Alibali, 1999). A procedural learning goal emphasizes the accurate, smooth, and rapid execution of action sequences to solve mathematical problems, i.e., the procedural knowledge. In contrast, a conceptual learning goal stresses the mental connections among facts, procedures, and ideas. Some researchers describe the latter learning goal as the implicit or explicit understanding of the principles that govern a domain and the set relationship between pieces of knowledge in the domain, i.e., the conceptual knowledge.

The bidirectional relations between conceptual and procedural knowledge (Rittle-Johnson & Alibali, 1999). In their study, children who received conceptual instruction increased both conceptual understanding and knowledge of problem-solving procedures, and children who received procedural instruction not only developed correct problem-solving procedures but also gained conceptual understanding of the material. However, the impact of conceptual and procedural knowledge on each other is asymmetric. The authors found in their study that the procedural learning group had significantly poorer performance on transfer tasks than those who received conceptual instruction, but the conceptual learning group did not experience more difficulty in learning a correct procedure than did the procedural learning group.

The findings in the previous study suggested that conceptual instruction enables better knowledge transfer, and this idea is further confirmed by Perry (1991). In Perry's study, students who received conceptual instructions performed better on knowledge transfer tests than those who received either procedural instruction or a combination of the two.

Because these findings have suggested the effectiveness of conceptual instruction with conceptual learning goals, many researchers have sought to identify features of such instructional pattern that promote conceptual understanding. Two key features have been identified. First, teachers and students attend explicitly to concepts (Hiebert & Grouws, 2007). For example, Hiebert and Wearne (1993) gave conceptual and procedural instructions to higher- and lower-achieving classes. Students receiving the conceptually based instruction spent more time on understanding the rationale of procedure and examining the legitimacy of invented procedures, whereas students receiving the procedural instruction spent more time on practicing the taught procedures. In addition, the lower-achieving class who received the conceptual instruction performed as well as the higher-achieving class who received the procedural instruction.

Similar studies have shown that conceptual instruction also facilitates the development of key mathematical reasoning skills (Good, Grouws, & Ebmeier, 1983) and the engagement with important mathematical ideas (Boaler, 1998).
The second feature of conceptual instruction that promotes conceptual understanding is that students need to struggle with important mathematical ideas (Hiebert & Grouws, 2007). Cognitive incongruity, a situation of perplexity, and the experience of confusion and doubt facilitate the development of mathematical reasoning skills (Hatano, 1988). For example, a study, which was conducted as part of the QUASAR Project, discovered that students whose teachers presented and confidently implemented more challenging problems exhibited higher level of conceptual understanding of the mathematical materials (Stein, Grover, & Henningsen, 1996; Stein & Lane, 1996). Inagaki, Hatano, and Morita (1998) recorded a class discussion of adding fractions with different denominators, which was filled with confusion, but led to improved understanding that was demonstrated through verbal and written evidence. Other case studies are also compelling evidence of greater conceptual understanding through struggling with challenging problems (Heaton, 2000; Lampert, 2001; Schoenfield, 1985).

As indicated by the studies mentioned above, the instruction method that a teacher uses significantly influenced the instructional patterns and the learning outcomes. Despite the richness of literature on the debate of conceptual and procedural instruction, little has focused on what causes the differences in learning goals.

Connecting Instructional Patterns with Textbooks

According to previous research, textbooks can shape teacher's knowledge on mathematical ideas and influence the learning goal that a teacher sets for a class (Ball & Cohen, 1996; Nathan, Long, & Alibali, 2002; Ding, Li, Li, & Gu, 2013). As discussed above, the learning goal in turn determines the instructional patterns. A visual elaboration can be found in Figure 1.

Specifically, this study examines two areas: First, it will investigate how concepts are defined and introduced in textbooks and whether there is other information in the text beyond the concept definitions that contributes to student's general understanding of mathematics. Specifically, the generalization of mathematical ideas, rules and principles (hereinafter referred to as Mathematical Generalizations, or MGs) and links between lessons are the two main focus besides definitions. The coding scheme will be developed based on previous work in mathematical taxonomy (Ginther & Henderson, 1966) and TIMSS video study (Jacobs et al., 2003). Second, another area of research interest is whether or not and how often teachers emphasize conceptual understanding in class and whether such patterns correspond to the characteristics identified in textbooks. Classroom videos from U.S. and Hong Kong collected as part of the TIMSS 1999 Video Study will be used for this comparison. Transcripts will be coded based on the past studies on conceptual and procedural instruction.

It is hypothesized that there are limited differences between the definitions of concepts in Chinese and U.S. textbooks. This hypothesis is partially supported by comparisons of Japanese and U.S. textbooks (Stevenson & Bartsch, 1991). They discovered that although several concepts are presented earlier in Japanese textbooks, around 90% of the concepts are the same and presented in a similar manner. Past studies have confirmed the similarity between Chinese and Japanese education system, so the same pattern is expected again in this study (Perry, 2000; Li, Chen, & An, 2009). The differences in concept definitions (types and strategies) will be addressed in Study 1.

Despite the similarity in concept definitions in textbooks from U.S. and China, differences in mathematical generalizations and links are expected. Specifically, MGs and links are expected to be more common in Chinese textbooks than in U.S. ones. For instance, the Chinese textbooks are expected to have references to prior or future lessons for the purpose of establishing a thorough concept "network." Moreover, the profound impact of specific ideas is expected to be discussed more frequently in Chinese textbooks than U.S. ones. In addition, evidence from classrooms is expected to support the notion that teachers in China remind students of concepts more often than those in the U.S. This hypothesis is supported by Perry's (2000) study, which compared how mathematical concepts are explained in classrooms in Japan, China, and the U.S.. In her study, Perry discovered that explanations of concepts occurred more frequently in the Japanese and Chinese classrooms than in U.S. classrooms, and typical explanations in these Asian classrooms were more substantive than those in the U.S.. The difference in MGs and links will be examined in Study 2.

The findings in this study are expected to illustrate the nuances in the textbooks that reflect different views towards procedural and conceptual instruction. As U.S. and Chinese textbooks are expected to differ by MG and link items, a similar pattern is anticipated in classroom instruction. That is, teachers are expected to spend a greater proportion of time on MGs and links in Hong Kong classes than in the U.S. ones. This association between textbooks and classroom instructional pattern will be investigated in Study 3.

Study 1

Methods

In this study, the difference among concept definitions from textbooks in China and the United States was explored. Specifically, this study explored two dimensions of difference in concept definitions: type and strategy. The type definition describes what relevant information is used as a means of elaborating the concept, and the strategy definition describes how a definition is introduced in the context and how it relates to previous and later content in the textbook. Both concepts will be introduced in greater details later.

Selection of Texts

The middle school mathematics textbooks that were currently used in the United States and China were selected for analysis. The American curriculum was represented by two textbooks widely used in California: Bennett et al. (2008), and Larson et al. (2008). There was less variety in the textbooks used in China, as all provinces except Shanghai and Zhejiang were required to adopt few textbooks that were developed in accordance with the national syllabus since the 1980s (Fan & Zhu, 2007). The most widely used textbook, Kecheng Jiaocai Yanjiusuo (2007), was selected in this study as a representative of Chinese curriculum.

The primary content of all selected textbooks was pre-algebra, including rational numbers, monomials and polynomials, and first-order equations with one unknown. The content was controlled because previous study suggested that the distribution of definition types and strategies vary on different topics (Ginther & Henderson, 1966). Thus, Sections 1.1 to 4.8 and Sections 12.1 to 12.6 in Bennett et al. (2008); Sections 1.1 to 6.2, Section 8.1 and Sections 12.1 to 12.5 in Larson et al. (2008); and Chapter 1 to 3 in Kecheng Jiaocai Yanjiusuo (2007) were selected for the coding purpose in this study, so that the coded content was the same across different textbooks.

Coding of Definition Types and Strategies

The coding scheme that was used in this study was modified based on the existing mathematical taxonomy research that was discussed earlier (Ginther & Henderson, 1966). In this study, a definition was defined as the sentence containing a bold or highlighted concept that includes (1) examples, (2) the necessary and sufficient conditions for an object to be classified as the concept being defined, and/or (3) synonyms for the concept. When a definition employed only examples to define a concept, then this coding scheme identified it as a denotative definition. When a definition used only necessary and sufficient conditions to define a concept, it was classified as a connotative definition. When a definition used synonyms to define a concept, it was classified as a synonymical definition. When a definition included both examples and necessary and sufficient conditions, the coding scheme classified it as denotative and connotative. Several cases of synonymical and connotative definitions were also found, which included both synonyms and necessary and sufficient conditions. Yet, due to the scarcity of such cases, they were combined with the synonymical definition category. A summary of these four types of definitions with examples from both American and Chinese textbooks was provided in Table 1.

Besides definitions, this study also explored the explanations of those definitions. An explanation of a concept definition must serve for at least one of the following purposes: (1) providing examples of the concept, (2) formally or informally defining a term that was a part of the concept definition body, or (3) specifying the scope to which the concept could apply. Based on the locations of the explanation of a definition, four strategies are identified in this coding scheme (Ginther & Henderson, 1966). An explanation-definition strategy has only an explanation proceeding a definition, a definition-explanation strategy has only an explanation following a definition, an explanation-definition-explanation strategy has explanations both before and after a definition, and a definition strategy does not have any explanation before or after a definition. A summary of these four definition strategies with examples from both American and Chinese textbooks was provided in Table 2.

Procedure

All definitions in the selected sections of each textbook were coded according to the above coding schemes. The first author, whose first language was Chinese and was fluent in English, coded both the type and strategy for each concept definition highlighted in the selected sections of the textbooks. Another bilingual coder independently coded 20% of concept definitions from each textbook, selected at random. After the initial coding, the results were compared. For definition types, the two coders displayed substantial agreement. Specifically, they agreed on 88% of their coding (). For the definition strategies, two coders displayed an agreement on 88% of their coding (). Each concept definition for which the coders did not agree was discussed until an agreement was reached on how the concept definition would be coded. Most of the disagreement with regard to definition types happened with synonymical definitions. The boundary between a synonymical and a connotative definition was sometimes unclear. The reason was that a concept could be defined by a synonym with a modifier, which included necessary and sufficient conditions. As mentioned before, due to the scarcity of these cases, they were not listed as a separate category in the coding scheme. In addition, all disagreement between the two coders regarding definition strategies were about whether there was an explanation after a concept definition (i.e., the distinction between definition and definition-explanation strategies). This problem might have happened because the specific definition of explanation in this coding scheme required coders to identify sentences following the concept definitions which served for one of the three particular purposes (giving example, explaining terms in the definition body, and specifying the scope of concepts). In reality, the sentence that followed a definition might serve for other purposes. In this study, coders were instructed to follow the definition of explanation in this coding scheme and ignored sentences that served for purposes other than the three listed above.

Results

There were 131 definitions collected from the selected sections of the textbooks, with 25 definitions from Kecheng Jiaocai Yanjiusuo (2007), 67 from Larson et al. (2008), and

39 from Bennett et al. (2008).

Because of low cell counts, the Fisher's exact tests were run, so that the statistical analysis no longer required a relatively larger sample size. First, the association between textbooks and definition types was tested, as shown in Table 3 and Figure 2. There was no significant association between the textbooks and the definition types (, Fisher's exact test). In other words, the distribution of definition types did not vary significantly across the three textbooks. In addition, the association between textbooks and definition strategies was analyzed, as shown in Table 4 and Figure 3. There was no significant association between the textbooks and the definition strategies (, Fisher's exact test). In other words, the distribution of definition strategies did not vary significantly across the three textbooks.

Study 2

Method

Besides concept definitions, there are other conceptual elements in textbooks. This study explored two other types of conceptual information in each textbook: conceptual mathematical generalization (describing the conceptual or structural nature of mathematics), and links to past or future learning. The conceptual mathematical generalization and links to past or future learning came from the Third International Mathematics and Science Study (TIMSS) in 1999 (Jacobs et al., 2003), and a modified version of their coding scheme was developed and applied in this study to textbook analysis.

Selection of Texts

This study used the same textbooks as in Study 1. Moreover, the sections selected for this study also remained the same as in the previous study.

Coding of Mathematical Generalizations (MGs) and Links

In the coding scheme for this study, Mathematical Generalization (MG) items were sentences that stated generic ideas, rules and principles and were independent of specific cases. Exercises and examples were not counted as MGs because they often were specific to certain cases. For instance, MGs included three types: conceptual mathematical generalizations, procedural mathematical generalizations, and definitional mathematical generalizations. Each of the three types is discussed in detail below, and examples selected from Chinese and American textbooks can be found in Table 5.
First, conceptual MGs contained two subcategories: implications and summary of ideas. An implication item is a sentence that explains how the introduction of certain concepts or principles relates to other concepts or principles and how it advances the understanding of mathematics. A summary of ideas is a sentence that summarizes key ideas and important philosophies that are demonstrated in examples, exercises, principles, or concepts. Moreover, procedural MGs were sentences that summarized rules, specific methods, and specific procedures. These items were often phrased in the following manner: "To do…, you can…." Lastly, definitional MGs also included two subcategories: principles and meanings. A principle item is a highlighted sentence that describes properties and is usually named as "Principles/ Properties of …" A meaning item is a sentence that specifies

the scope over which the concept definition or principle can or cannot be applied. Usually, special conditions and cases were mentioned.

In addition to MGs, how often the textbook referred to concepts or principles that were covered in the past or future lesson was used as a measure of how well-linked the lessons were in each textbook. A past concept is a concept or idea learned in past lessons that is brought up as a means of introducing a new concept, making an analogy, comparing similar ideas, and/or generalizing mathematical principles. Similarly, a future concept is a concept or idea that will be covered in future lessons and is brought up for the purpose of providing students with a full picture and/or stimulating students' interest. A summary of links with examples taken from Chinese and American textbooks can be found in Table 6.

Procedure

All selected sections in each textbook were coded according to the above coding scheme. The first author coded both Mathematical Generalizations (MGs) and link items for each selected section of the textbooks. Another bilingual coder independently coded 20% of the content randomly selected from each textbook. The amount of content was operationalized by page numbers. After the initial coding, the results were compared. For MG items, the two coders displayed substantial agreement. Specifically, they agreed on 91% of their coding. For the links items, two coders displayed a perfect agreement on 100% of their coding. Each MG item for which the coders did not agree was then discussed until an agreement was reached on how it would be coded.

Results

There were 196 Mathematical Generalization items collected from the selected sections of the textbooks, with 55 items from Kecheng Jiaocai Yanjiusuo (2007), 79 from Larson et al. (2008), and 62 from Bennett et al. (2008). Because of low cell counts, Principles and Meanings categories together were combined into the Definitional MG, and Implication and Summary of Ideas categories were combined into the Conceptual MG category. A chi-square test of independence was performed to examine the relation between MG types and region, as shown in Table 7 and Figure 4. The Chinese textbook contained more Conceptual MG items, and the two U.S. textbooks contained more Procedural MGs.

There were 27 Links items collected from the selected sections of the textbooks, with 9 items from Kecheng Jiaocai Yanjiusuo (2007), 6 from Larson et al. (2008), and 12 from Bennett et al. (2008). Because of low cell counts and a large proportion of zero counts, no statistical test was run to test the significance, as shown in Table 8 and Figure 5. It seemed that Bennett et al. (2008) contained the largest number of Links among the three textbooks, and Larson et al. (2008) the lowest. However, it was noticed that the selected proportion of both U.S. textbooks contained more sections than the Chinese textbooks, so it was reasonable to assume that there were more opportunities for U.S. textbooks to link across sections. Thus, a proper normalization method was needed. Although beyond the focus of the current study, the authors decided that page counts and section counts were not ideal methods, because sections in U.S. textbooks tended to be shorter and contained more exercises than the Chinese one. Word counts was not a good measure-ment either, because it failed to account for the linguistic difference between English and Chinese. Despite the lack of a proper normalization, however, it was still meaningful to discover that there was no link to future lessons discovered in either of the U.S. textbooks.

Study 3

Method

Now that the differences in mathematical textbooks across countries have been identified, it was thus reasonable to test whether such differences had an impact on how teachers teach mathematics in class. After all, instruction has the most direct impact on student's performance (Rosenshine & Stevens, 1986). This direction was promising because various international mathematical achievement programs, such as the Trends in International Mathematics and Science Study (TIMSS) and Program for International Student Assessment (PISA), have indicated difference in student mathematical achievement (Mullis, Martin, Foy & Arora, 2012; Turner, 2014). If differences in in-class instruction, which were similar to the differences in textbooks, could be identified across the countries, then it was promising to investigate the association between textbooks and the characteristics that teachers bring to class, such as learning goals. Thus, this study compared video transcripts from classrooms in the United States and Hong Kong SAR to further test this hypothesis.

Specifically, the total proportion of the lesson time in Chinese and U.S. classrooms spent on each type of coding items—concept definitions, Mathematical Generalizations, or Links to past or future lessons—was of particular interest in this study.

Selection of Video Transcripts

Transcripts of lessons from seven countries were collected by TIMSS Video Study in 1999, and they are available to the public at http://www.timssvideo.com/. Two lessons from the U.S. (US2 Writing Variable Expressions, n.d.; US3 Exponents, n.d.) and two lessons from Hong Kong (HK1 Square Roots, n.d.; HK4 Identity, n.d.) were used in this study, because these lessons were on topics addressed in Studies 1 and 2. The control of content enabled the association between textbooks and in-class teaching, without introducing other variables.

Coding of Total Proportion of Each Coding Type

The total proportion of lesson type spent each coding type was operationalized by summing up the time spent on each individual item of that coding type and dividing the sum by the total lesson type. The time spent on each individual item was calculated by taking the time difference between the in-points and out-points of that item.

A summary of the coding scheme used in this study with examples taken from Chinese and American textbooks can be found in Table 9.

Procedure

All selected video transcripts were coded according to the above coding scheme. The first author coded all transcripts for the four lessons, and a bilingual coder independently coded transcripts for one lesson from a U.S. class and one lesson from a Hong Kong class. The results were compared.

As with the original TIMSS coding, a window of 10 seconds was allowed for the difference between coders' codings of in-points and out-points (Jacobs et al., 2003). That is, an agreement was recognized as long as the coders did not differ more than 10 seconds in the judgment of each in-point and out point. The two coders displayed substantial agreement, agreeing on 86% of their coding. Each in-point or out-point for which the coders did not agree was then discussed until an agreement was reached on how it would be coded.

Results

There were 39 items identified in the selected videos, with 11 items from HK1 Square Roots (n.d.), 13 from HK4 Identity (n.d.), 6 from US2 Writing Variable Expressions (n.d.), and 9 from US3 Exponents (n.d.).

There was a significant variability among the four lessons, even between the two pairs of lessons collected from the same region, as shown in Table 10 and Figure 6. For example, HK4 spent a larger proportion of time on definitions than HK1, and HK4 and US3 spent more time on Definitional MGs than HK1 and US2. Moreover, HK1 and HK4 spent a larger proportion of lesson time on Procedural MGs, Conceptual MGs and Links than US2 and US3.

Discussion

According to the findings from the three studies, there was a significant difference in the Mathematical Generalizations across textbooks, but there was no significant difference in definitional types and strategies across different textbooks. However, the instructional patterns collected from Hong

Kong and U.S. classes only corresponded partially to the patterns identified in textbooks. Both textbooks and classroom transcript evidenced that Chinese mathematical teaching placed more emphasis and time on Conceptual Mathematical Generalizations and Links. Yet, classroom instructions also showed evidence that Hong Kong classes stressed Procedural MGs, contradictory to the findings from textbooks. Moreover, there was a significant variability in the proportion of lesson time spent on Definitions and Definitional MGs from classroom instruction, but such patterns were absent in the textbooks.

One explanation to such partially supporting evidence is rooted in a transactional model of instruction. That is, besides what was written in the textbooks, teachers also carried their own schemata (knowledge about mathematics and pedagogy, skills in mathematical reasoning and teaching, and attitudes and motivations towards teaching in general and teaching mathematics in specific) to a class (Adelman & Taylor, 2006; Stigler & Hiebert, 2009). As teachers studied textbooks, the textbook content interacted with teacher's own knowledge and understanding (Hill, Rowan & Ball, 2005). This transaction together helped teachers set the learning goals for the class, which determined the instructional patterns and performance sequentially. A visual illustration can be found in Figure 7.

An alternative explanation is found in the facts that the TIMSS videos were collected fifteen years ago, and that the teachers and students in those videos did not use the textbooks that were coded in this study. As a result, one could argue that the teaching pattern in TIMSS videos only corresponded to the textbooks they used at the time when those classes were recorded and that pattern might differ from the ones that were identified in the three textbooks that the authors coded in this study. To test this alternative hypothesis, future studies should consider increasing the time and monetary budget to systematically collect more recent video data from classrooms internationally. An example of a systematic approach to address videography procedures could be found in TIMSS-R data collection manual (LessonLab, n.d.).

Another limitation of the current study was that often the sample size was small. This effect was caused by the fact that only three textbooks were coded in the current studies, and that it was not likely to expand the selection of content because previous research indicated that the definition patterns might vary for different topics (Ginther & Henderson, 1966). For example, in U.S. textbooks, a larger proportion of algebraic definitions tended to be defined denotatively than geometric definitions, while a higher percentage of geometric definitions tended to be defined connotatively than ones in algebra. The different patterns due to varying topics limited the coding content. Future studies should thus consider selecting more textbooks and using the developed coding scheme to study a controlled, limited content, rather than carelessly expand the content in each textbook.

In addition, it should be noticed that this study was designed to only study the correlation between textbook patterns and content and the teaching strategies employed in real classroom. The causality, however, was not directly addressed in the current study. This in fact would not eliminate the possibility of a third variable, which was caused by textbook patterns and further lead to the different instructional strategies. Moreover, teachers' collective experience with engaging students in mathematical classes would reciprocally determine how the textbooks were written. After all, most textbooks were written either by professors or personnel who had direct teaching experience or had expertise supervising teaching on a large scale, as was the case with the three textbooks selected for the current study. Further research exploring this direction should consider designing cognitive experiments that allow learners to engage with novel mathematical content with exposure to a variety of teaching strategies and learning goals. The final learning results should be compared to determine the effectiveness of each teaching method. What was crucial to such a design was how novel content was selected or constructed in order to control for the learners' previous knowledge, skills, attitudes, and motivations (collectively referred to as the schemata; Adelman & Taylor, 2006). Other important factors to consider to design the stimuli should be the construct validity and the exposure time to the new content.

Despite the limitations of the design and data of the current study, the findings still provided some valuable insights that could help explain the differences in student achievement levels and in instructional patterns identified in China and the United States. Such understanding could help both countries to realize alternative approaches to teach mathematics, so that future research could compare the results in an experimental setting. Once causal relationship between textbooks, learning goals, and instructional patterns could be proven in each country in the near future, programs and policies that facilitate changes in teaching and textbooks editing systems to improve student learning based on scientific research should be implemented. In addition, international comparison studies should try to include more countries and regions so that more unique methods of teaching could be discovered and valued by the international community of scholars.

References

Adelman, H.S., & Taylor, L. (2006). The implementation guide to student learning supports in the classroom and schoolwide: New directions for addressing barriers to learning. Thousand Oaks, CA: Corwin Press.

Ball, D. L., & Cohen, D. K. (1996). Reform by books: What is—Or might be—The role of curriculum materials in teaching learning and instructional reform? Educational Researcher, 25(9), 6-8, 14.

Bennett, J. M., Burger, E. B., Chard, D. J., Jackson, A. L., Kennedy, P. A., Renfro, F. L., … & Waits, B K.. (2008). Holt California Mathematics: Course 2: Pre-algebra. Austin, Texas: Holt, Rinehart and Winston.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. Journal for Research in Mathematics Education, 29, 41-62.

Ding, M., Li, Y., Li, X., & Gu, J. (2013). Knowing and understanding instructional mathematics content through intensive studies of textbooks. In Y. Li & R. Huang (Eds.), How Chinese teach mathematics and improve teaching (pp. 66-82). New York, NY: Routledge.

Fan, L., & Zhu, Y. (2007). Representation of problem-solving procedures: A comparative look at China, Singapore, and U.S. mathematics textbooks. Educational Studies in Mathematics, 66, 61-75.

Ginther, J. L., & Henderson, K. B. (1966). Strategies for Teaching Concepts by Using Definitions. The Mathematics Teacher, 59(5), 455-457.

Good, T. L., Grouws, D. A., & Ebmeier, H. (1983). Active mathematics teaching. New York: Longman.
Hatano, G. (1988). Social and motivational bases for mathematical understanding. In G. B. Saxe & M. Gearhart (Eds.), Children's mathematics (pp. 55-70). San Francisco: Jossey-Bass.

Heaton, R. M. (2000). Teaching mathematics to the new standards: Relearning the dance. New York: Teacher College Press.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. Second handbook of research on mathematics teaching and learning, 1, 371-404.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. American educational research journal, 30(2), 393-425.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. American educational research journal, 42(2), 371-406.

HK1 Square Roots [Video file]. Retrieved from http://www.timssvideo.com/43

HK4 Identity [Video file]. Retrieved from http://www.timssvideo.com/65

Inagaki, K., Hatano, G., & Morita, E. (1998). Construction of mathematical knowledge through whole-class discussion. Learning and Instruction, 8, 503-526.

Jacobs, J., Garnier, H., Gallimore, R., Hollingsworth, H., Givvin, K. B., Rust, K., Kawanaka, T., Smith, M., Wearne, D., Manaster, A., Etterbeek, W., Hiebert, J., & Stigler, J. (2003). TIMSS 1999 Video Study Technical Report: Volume 1: Mathematics Study, NCES (2003-012),U.S. Department of Education. Washington, DC: National Center for Education Statistics

Jonassen, D. H., & Grabowski, B. L. (2012). Handbook of individual differences, learning, and instruction. Routledge.
Kecheng Jiaocai Yanjiusuo. (2007). Shuxue, Qinianji [Mathematics, grade 7] (Vol. 1). Beijing: Renmin Jiaoyu Chubanshe.

Lampert, M. (2001). Teaching problems and the problems of teaching. New Haven, CT: Yale University Press.
Larson, R., Boswell, L., Kanold, T. D., & Stiff, L. (2008). Pre-algebra (California ed.). Evanston, Illinois: McDougal Littell.

LessonLab (n.d.). TIMSS-R data collection manual. Retrieved June 13, 2015, from http://www.timssvideo.com/sites/default/files/Data Collection Manual.pdf

Li, Y., Chen, X., & An, S. (2009). Conceptualizing and organizing content for teaching and learning in selected Chinese, Japanese and US mathematics textbooks: The case of fraction division. ZDM, 41(6), 809-826

Mullis, I. V., Martin, M. O., Foy, P., & Arora, A. (2012). TIMSS 2011 international results in mathematics. International Association for the Evaluation of Educational Achievement. Herengracht 487, Amsterdam, 1017 BT, The Netherlands.

Nathan, M. J., Long, S. D., & Alibali, M.W. (2002). The symbol precedence view of mathematical development: A corpus analysis of the rhetorical structure of textbooks, Discourse Processes, 33, 1-21.

Perry, M. (1991). Learning and transfer: Instructional conditions and conceptual change. Cognitive Development, 6(4), 449-468.

Perry, M. (2000). Explanations of mathematical concepts in Japanese, Chinese, and US first-and fifth-grade classrooms. Cognition and Instruction, 18(2), 181-207.

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other?. Journal of educational psychology, 91(1), 175.

Rosenshine, B., & Stevens, R. (1986). Teaching functions. In M. Wittrock (Ed.), Handbook of research on teaching (3rd ed.) (376-391). New York: Macmillan.

Schoenfield, A. (1985). Mathematical problem solving. Orlando, FL: Academic Press.

Starnes, D. S., Yates, D., & Moore, D. S. (2010). The practice of statistics (4th ed.). New York, NY: W.H. Freeman.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. American Educational Research Journal, 33, 455-488.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. Education Research and Evaluation, 2(1), 50-80.

Stevenson, H. W., & Stigler, J. W. (1994). The learning gap: Why our schools are failing and what we can learn from Japanese and Chinese education. New York: Simon & Schuster.
Stevenson, H. W., & Bartsch, K. (1992). An analysis of Japanese and American textbooks in mathematics. In R. Lectsma & H. Walberg (Eds.), Japanese educational productivity (pp. 103–133). Ann Arbor: Center for Japanese Studies.

Stigler, J. W., & Hiebert, J. (2009). The teaching gap: Best ideas from the world's teachers for improving education in the classroom. Simon and Schuster.

Tienken, C. H. (2013). Conclusions from PISA and TIMSS testing. Kappa Delta Pi Record, 49(2), 56-58.

Turner, R. (Ed.). (2014). PISA 2012 technical report. Washington, D.C.: Organization for Economic Cooperation & Development.

US2 Writing Variable Expressions [Video file]. Retrieved from http://www.timssvideo.com/80

US3 Exponents [Video file]. Retrieved from http://www.timssvideo.com/69

Wu, M. (2009). A comparison of PISA and TIMSS 2003 achievement results in mathematics. Prospects, 39(1), 33-46.

Appendix A

Table 1

Definition type coding scheme for Study 1

	Definition	Example from Chinese Textbook	Example from American textbooks
Denotative	A definition that only uses examples and set membership to define a concept	五百这个数只是接近实际人数，但与实际人数还有差别，它是一个**近似数**。 [Five hundred is just a number that approximates the actual number, but there is still a difference. It is an **approximate number.**]	Adding and subtracting by the same number are **inverse operations.** Inverse operations “undo” each other.
Connotative	A definition that only uses necessary and sufficient conditions to define a concept	上面个方程都只含有一个未知数（元），未知数的次数都是1，这样的方程叫做一**元一次方程**。 [The equations above all contain one unknown, and the order of the unknown is 1. Such an equation are called **linear equation with one unknown**.]	**Integers** are the set of whole numbers and their opposites.

	Definition	Example from Chinese Textbook	Example from American textbooks
Synonymical	A definition that uses synonyms to define a concept	一般地，数轴上表示数a的点与原点的距离叫做数a的**绝对值，记作|a|**。 [Generally, the distance on the number line between point a and the origin is the **absolute value** of a, denoted as |a|.]	The **absolute value** of a number is its distance from 0 on a number line.
Connotative & Denotative	A definition that uses both examples and necessary and sufficient conditions to define a concept	像3, 2, 1.8%这样大于0的数叫做**正数**。 [Numbers like 3, 2, 1.8% that are greater than 0 are called **positive numbers**.]	In Example 1, the expressions 2(90+60) and 2(90)+2(60) are called **equivalent numerical expressions** because they have the same value.

Note. All the examples in this coding scheme are from the three textbooks selected for this study. The two coders coded these examples independently, and they coded these examples identically. The term being defined is bolded in each example. Moreover, examples from the Chinese textbook are provided with an English translation.

Table 2

Definition strategy coding scheme for Study 1

	Definition	Example from Chinese Textbook	Example from American textbooks
Definition-Explanation	An explanation follows a definition.	像3, 2, 1.8%这样大于0的数叫做**正数**。 [Numbers like 3, 2, 1.8% that are greater than 0 are called **positive numbers**.]	An **expression** is a mathematical phrase that contains operations, numbers, and/or variables. A **variable** is a letter that represents a value that can change or vary.
Explanation-Definition	An explanation proceeds a definition.	分析上上面列出的式子，，，它们都可以看做几个单项式的和…像这样，几个单项式的和叫做**多项式**。 [In the above expressions ,, and , they can be regarded as sums of monomials… Such an expression is called a **polynomial**.]	When you combine like terms, you change the way an expression looks but not the value of the expression. **Equivalent expressions** have the same value for all values of the variables.

	Definition	Example from Chinese Textbook	Example from American textbooks
Explanation-Definition-Explanation	Explanations are presented both before and after a definition.	像与，与,与这样，所含字母相同，并且相同字母的指数也相同的想叫做同类项。及格常数项也是同类项。 [Like and and , and and , terms that share the same letters whose powers are also the same are called **like terms**. Several constant terms are like terms.]	You can solve a real-world problem by creating a verbal model and using it to write a variable expression. A **verbal model** describes a problem using words as labels and using math symbols to relate the words. The table shows common words and phrases that indicate mathematical operations.

	Definition	Example from Chinese Textbook	Example from American textbooks
Definition	A definition that uses both examples and necessary and sufficient conditions to define a concept	上面个方程都只含有一个未知数（元），未知数的次数都是1，这样的方程叫做一元一次方程。 [The equations above all contain one unknown, and the order of the unknown is 1. Such an equation are called **linear equation with one unknown**.]	A **variable** is a letter used to represent one or more numbers.

Table 3
Number of instances of each definition type in Study 1

Concept definition type	Chinese Textbook	U.S. Textbook 1	U.S. Texbook 2
Connotative	14	45	31
Denotative	2	4	3
Synonymical	5	16	4
Connotative & Denotative	4	2	1
Total	25	67	39

Note. In the table above, "Chinese Textbook" denotes Kecheng Jiaocai Yanjiusuo (2007), "U.S. Textbook 1" denotes Larson et al. (2008), and "U.S. Textbook 2" denotes Bennett et al. (2008).

Table 4
Number of instances of each definition strategy in Study 1

Concept definition strategy	Chinese Textbook	U.S. Textbook 1	U.S. Texbook 2
Definition	5	32	19
Definition-Explanation	12	24	14
Explanation-Defintion	4	7	3
Explanation-Definition-Explanation	4	4	3
Total	25	67	39

Table 5

Coding scheme of mathematical generalizations (MG) in Study 2

	Definition	Example from Chinese Textbook	Example from American textbooks
Procedural mathematical generalizations (PMG)	A sentence that summarizes of rules, specific methods and specific procedures. It often takes the form "*To do…, you can…*"	多个有理数相乘，可以把他们按顺序依次相乘。 [To multiply several rational numbers, you can multiply them by order.]	To solve a system of two equations, you can reduce the system to one equation that has only one variable.

	Definition	Example from Chinese Textbook	Example from American textbooks
Definitional mathematical generalizations (DMG)	A highlighted sentence that describes properties and is usually named as "Principles/ Properties of ___."	分配率 [Associative property]	Properties of addition and multiplication (including commutative property and associative property)
Definitional mathematical generalizations (DMG)	A sentence that specifies the scope over which the concept definition or principle can or cannot be applied. Usually, special conditions and cases are mentioned.	显然，正数的任何次幂都是正数，0的任何正整数次幂都是0. [Obviously, any power of a positive number is still positive, and a power of 0 with any positive exponent is 0.]	Whole numbers and integers are part of the set of rational numbers, as shown in the Venn diagram.

	Definition	Example from Chinese Textbook	Example from American textbooks
Conceptual mathematical generalizations (CMG)	A sentence that explains how the introduction of this concept or principle relates to other concepts or principles and how it advances the understanding of mathematics.	可以看出，方程是分析和解决问题的一种很有用的数学工具. [It is easy to see that equations is a great mathematical tool to analyze and solve problem.]	You can use the least common denominator (LCD) to compare and order fractions.

Note. All the examples in this coding scheme are from the three textbooks selected for this study. The two coders coded these examples independently, and they coded these examples identically. Moreover, examples from the Chinese textbook are provided with an English translation.

Table 6
Coding scheme of links in Study 2

	Definition	Example from Chinese Textbook	Example from American textbooks
Reference to past concept	When the textbook refers to past learning, a concept or idea learned in past lessons is brought up as a means of introducing a new concept, making an analogy, comparing similar ideas, and/or generalizing mathematical principles.	前面我们结合实际问题，讨论了如何分析数量关系、利用相等关系列方程以及如何解方程. [Previously we have discussed with real-world problems how to analyze quantitative relationships and use equality to write equations and how to solve them.]	To solve a multi-step inequality like , you should use the properties of inequality from Lessons 3.4 and 3.5 to get the variable terms on one side of the inequality and the constant terms on the other side.

	Definition	Example from Chinese Textbook	Example from American textbooks
Reference to future concept	When the textbook refers to future learning, a concept or idea that will be covered in future lessons is brought up for the purpose of providing students with a full picture and/or stimulating students' interest.	在本章还会看到，我们不仅可以用字母或含有字母的式子表示数和数量关系，而且还可以将这样的式子进行加减运算. 这些内容将为下一章一元一次方程的学习打下基础. [In this chapter we will also see that we can not only express quantitative relationships by letters or expressions containing letters, but also add or subtract these expressions. This content will become a foundation for the next chapter on first order equations with one unknown.]	

Table 7

Number of instances of each type of mathematical generalizations (MG) in Study 2

Mathematical Generalizations (MG)		Chinese Textbook	U.S. Textbook 1	U.S. Texbook 2
Conceptual MG		5	32	31
Definitional MG	Principles	18	27	19
Conceptual MG	Meanings	9	8	3
	Implication	14	6	3
	Summary of Ideas	9	6	6
Total		55	79	62

Table 8

Number of instances of each type of links in Study 2

Links	Chinese Textbook	U.S. Textbook 1	U.S. Texbook 2
Reference to past concept	6	6	12
Reference to future concepts	3	0	0
Total	9	6	12

Note. In the tables above, “Chinese Textbook” denotes Kecheng Jiaocai Yanjiusuo (2007), “U.S. Textbook 1” denotes Larson et al. (2008), and “U.S. Textbook 2” denotes Bennett et al. (2008).

Table 9
Coding scheme of in-points and out-points in Study 3

	Definition	Example from Hong Kong classes	Example from U.S. classes
In-point	The *in-point* denotes the time when the mention of definitions, MGs or links begins, and it is defined as the first moment when the teacher mentions some new information since the last out-point of the same or different concept.	What will be the expanded form for the right-hand side? Two X. Plus 10. Constant terms, both are the same, 10. X terms, the same, two X. Therefore, will they be always the same? Yes. In fact, on both sides, the expressions are exactly the same. ‖ Or we say that they are *identically* the same. Therefore, no matter what's the value of X, it is- you substitute for X, the changes will be the same	Okay, that's a variable. ‖ What I wrote on the board, seven H, is called a variable expression because it contains a variable. You'll see it up there as one of the examples and the other two are also examples of variable expressions. [" ‖ "marks the beginning of a definition item]

	Definition	Example from Hong Kong classes	Example from U.S. classes
Out-point	The out-point represents the moment when the mention of definitions, MGs or links ends, and it is defined as the time at which the instructor stops explaining the information and transitions to other tasks. Possible follow-up tasks include an example exercise on the blackboard, an in-class exercise, and a question from a specific example in which this exmplanation of concept is applied.	And for identity, in between the two sides, you can use a new symbol with three lines. And we read it as, is identical to. Or you can say that they are identically equal. Okay? ‖You have some class practices here. Page one-four-seven. ["‖" marks the ending of a definitional Mathematical Generalization item]	So now you've got the forth rule. We've only got one more rule to learn. Okay. ‖ Now on this one. On the rule we need to take these and expand them out this way so that you see the rule develop. Okay. ["‖" marks the ending of a procedural Mathematical Generalization item]

Note. All the examples in this coding scheme are from the four TIMSS videos of classroom instruction in Hong Kong and the U.S. selected for this study. The two coders coded these examples independently, and they coded these examples identically. All classes were taught in English. The in-points and out-points in the examples above are marked by "‖".

Table 10
Total proportion of lesson time spent on Definitions, MGs and Links in U.S. and Hong Kong classes

Definition	HK1	HK4	US2	US3
Mathematical Generalization	2.76%	8.77%	5.33%	0.00%
Procedural MG	4.89%	2.48%	0.00%	0.00%
Definitional MG	1.02%	7.74%	1.93%	6.81%
Conceptual MG	6.35%	9.28%	3.59%	1.37%
Links	2.91%	2.73%	0.00%	0.00%
Total	17.93%	31.00%	10.85%	8.17%

Appendix B

Figure 1
The hypothesized model of textbooks and instruction

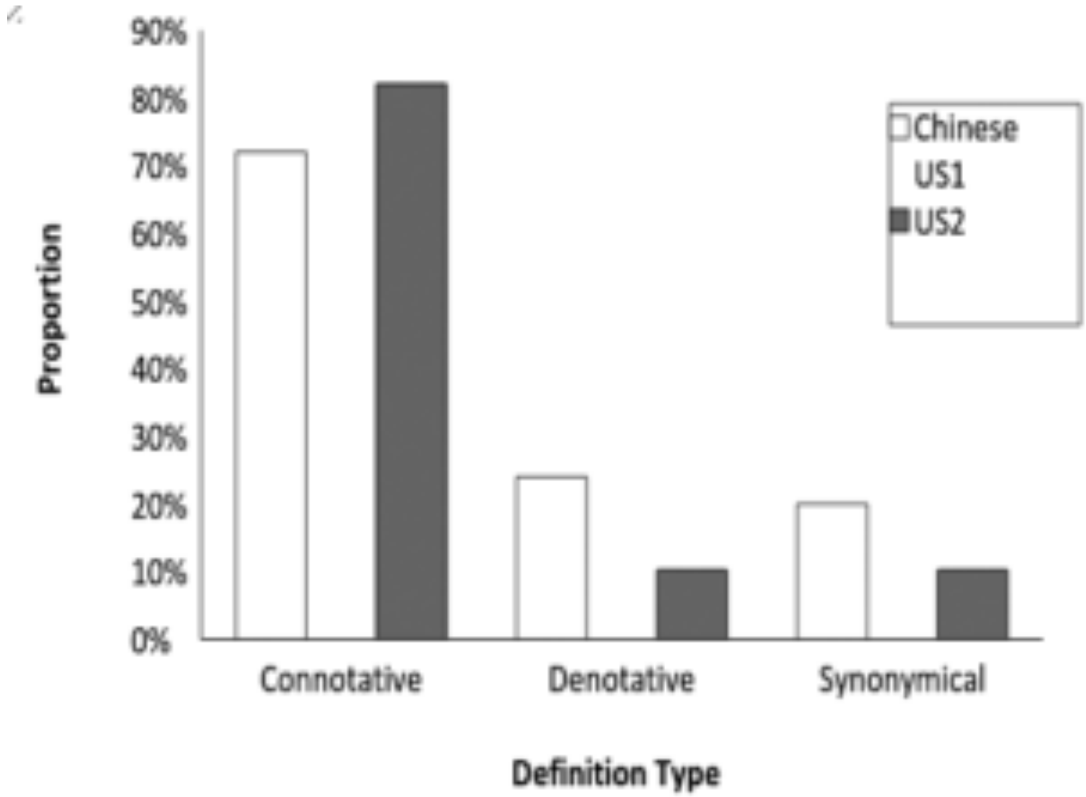

Figure 2
Definition types in Chinese and U.S. textbooks

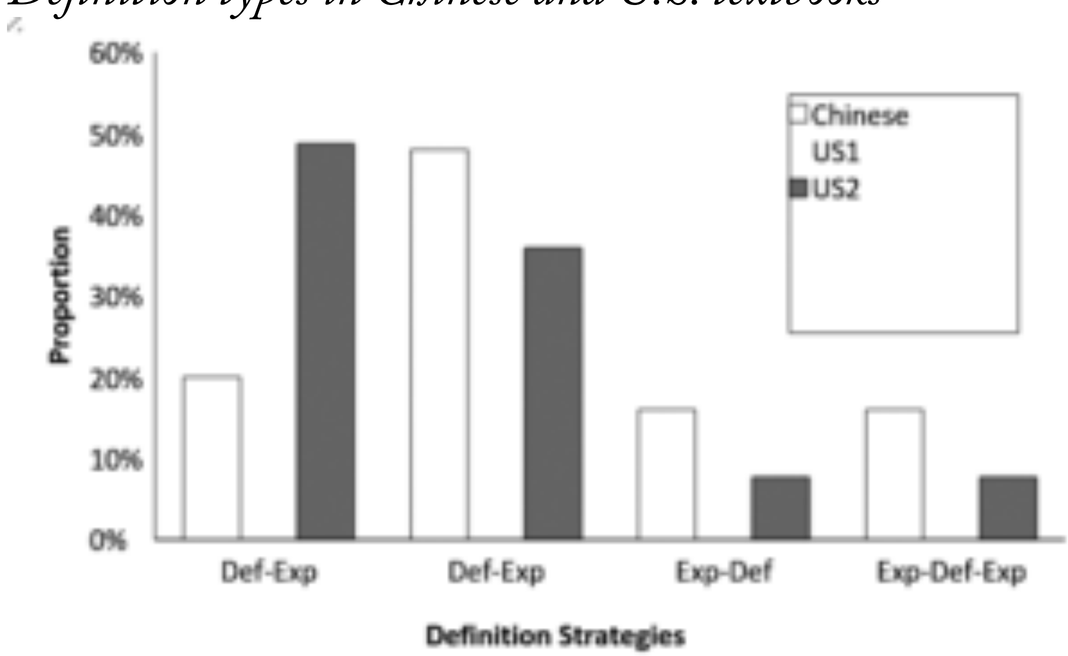

Figure 3
Definition strategies in Chinese and U.S. textbooks

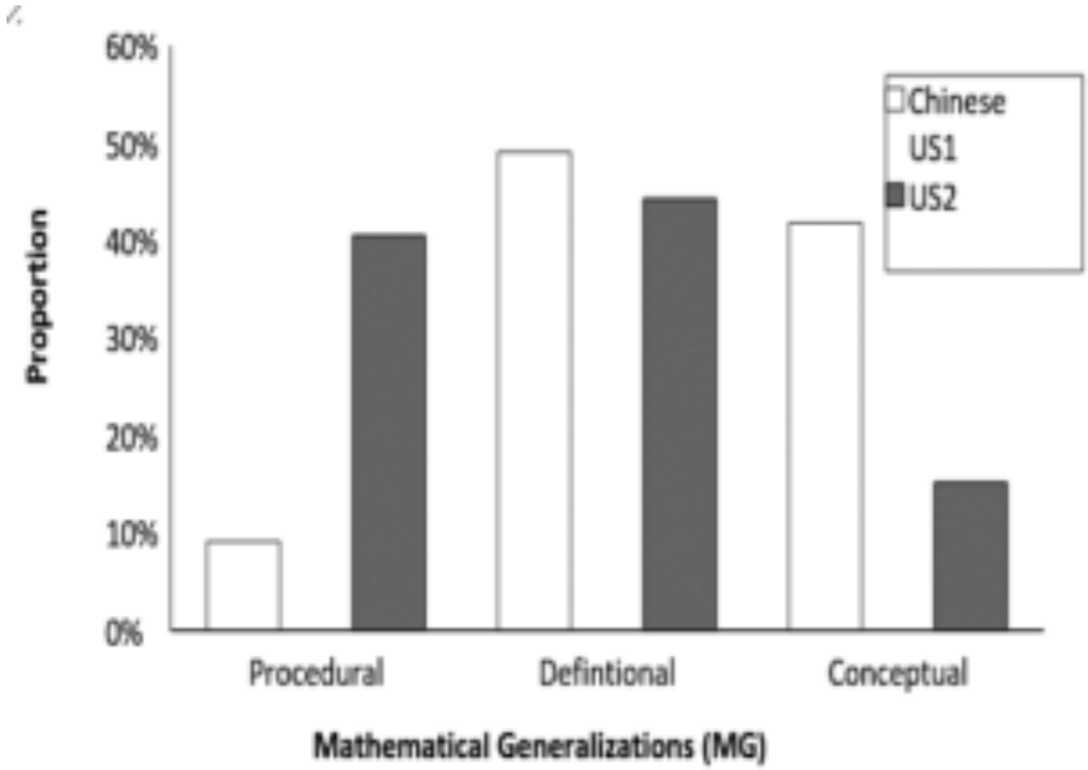

Figure 4
MGs in Chinese and U.S. textbooks

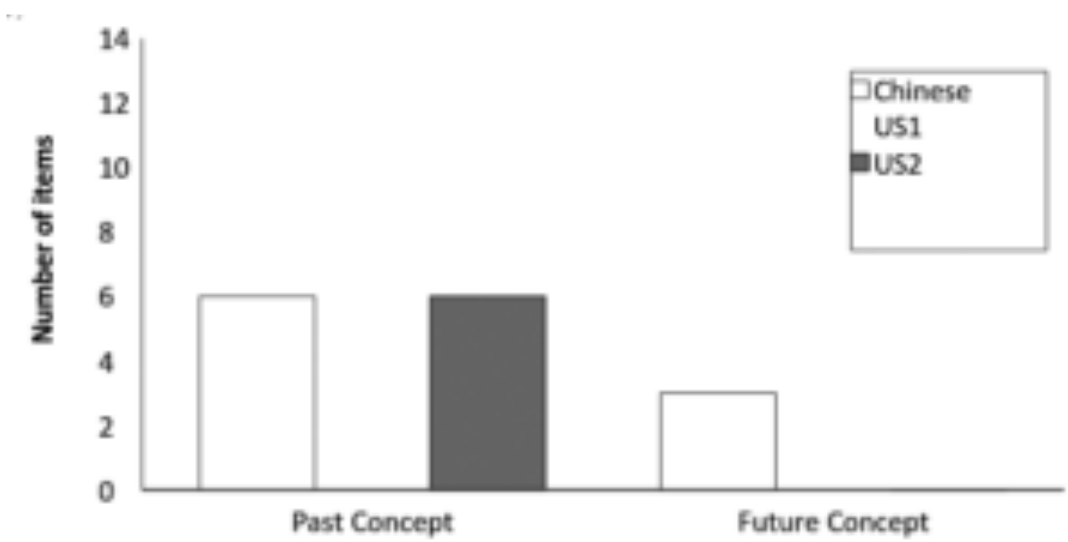

Figure 5
Links in Chinese and U.S. textbooks

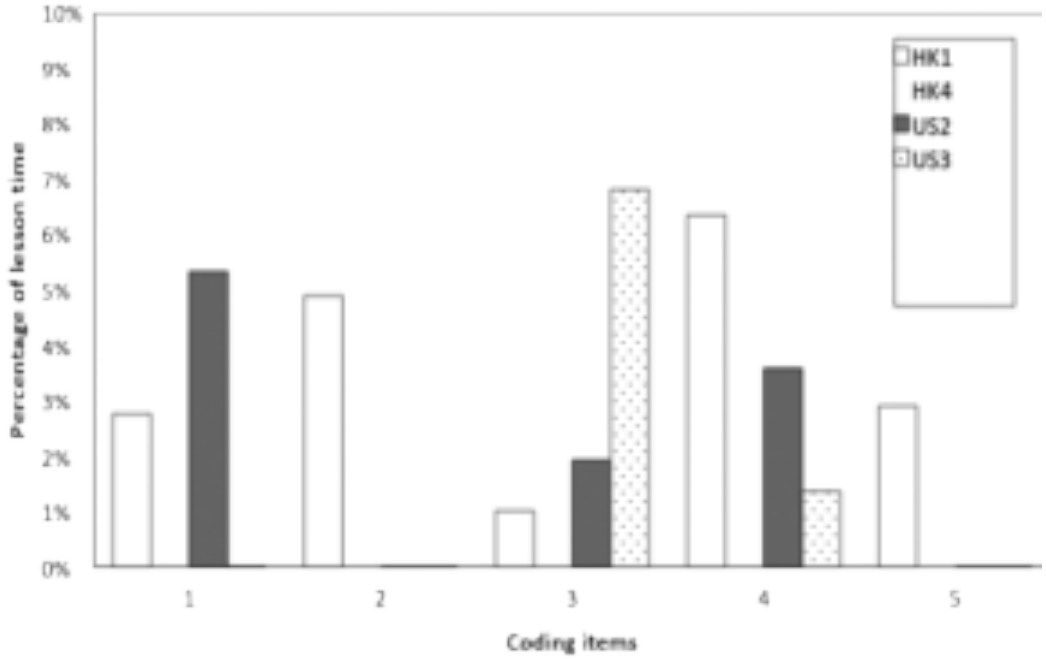

Figure 6
Total proportion of lesson time spent on coding items in Chinese and U.S. classrooms

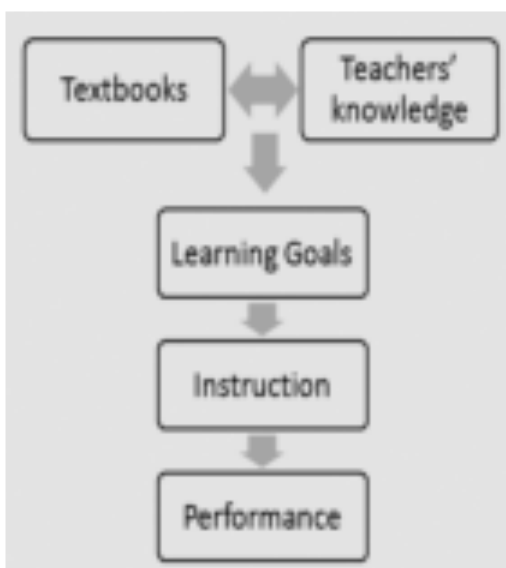

Figure 7
The transactional model of textbooks and instruction