Cliodynamics: the Journal of Theoretical and Mathematical History

Endogenous Population and Resource Cycles in Historical Hunter-Gatherer Economies

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This paper constructs a formal spatial model of a hunter-gatherer economy. By assuming that resource locations around a hunter gatherer camp can become congested I obtain the size of the area harvested as a function of population, resource density, gathering efficiency and time costs of commuting to locations. The model is then extended to include Malthusian and resource dynamics. The resulting dynamic properties are quite rich, with the possibility of stable steady states, as well as stable and unstable cycles. One result is that technological progress can actually cause such economies to collapse due to overharvesting of resources. Next, the model is extended to include the possibility of both group and individual migration. The former removes the possibility of collapse and exploding oscillations but introduces a new source of fluctuations in resources and population. Individual migration on the other hand, as long as there is no limit on new camp sites which can be settled by daughter colonies, will completely preclude the existence of oscillations.

Introduction

The purpose of this paper is to build a formal, tractable model of a historical, pre-Neolithic, hunter-gatherer economy. While the model borrows several building blocks and traditional methods from economics, the approach is interdisciplinary. In addition to economics, the analysis addresses issues which have been emphasized in ecology, anthropology and history.

As such, the work synthesizes insights from, and relates to, several different fields. It is related to economic analysis of the factors which led to the Neolithic revolution, as well as the evolutionary determinants of key economic and demographic characteristics. Connecting with anthropology, the paper examines territory size of hunter-gatherer groups, as well as parameters affecting migration patterns. By combining traditional Malthusian models from economic history with endogenous resource dynamics, the model is closely linked to predator-prey models in ecology.

In constructing the model I start with a spatial analysis which is reminiscent of Hotelling's famous model of firm location (Hotelling 1929). By analogy, a hunter gatherer camp corresponds to a location of a particular firm and the resource density around the camp plays the same role as the spatial

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Citation: Radek S. Szulga. 2012 Endogenous Population and Resource Cycles in Historical Hunter-Gatherer Economies. *Cliodynamics* 3: 234–270.

distribution of consumers in the Hotelling Model. While the setting is obviously very different, the formal apparatus is essentially the same because space plays a very similar role in both approaches. I then add a maximum resource-per-location constraint, which implies that locations can become congested. Aggregating the distribution of individuals across resource locations gives the production function for the entire economy, the size of the harvested area, the per capita resources gathered (income), and allows an analysis of how changes in various exogenous variables—population, resource density, gathering efficiency and time cost of commuting to a location—affect both the size of the harvested area and resources per person.

The model is then extended to a dynamic setting along two dimensions. First I add in population dynamics by allowing population size to change according to income. Hence the society studied here is a Malthusian hunter gatherer economy. The second dynamic dimension of the model allows the resource density to change over time as resources around the hunter gatherer camp are harvested. Here I borrow approaches, functional forms, and modeling techniques from the literature on resource and environmental Economics, as well as standard ecological models. Specifically I assume a hump-shaped natural growth rate of resources, often used in models of renewable resources such as the Gordon-Schaefer Fisheries Model (Gordon 1954 and Schaefer, 1954). This is then combined with a harvest rate which varies with the extent of the area harvested. This results in interplay between population and resources which creates the possibility of oscillations (in population, resources per capita and resource density) and even collapse. While these phenomenon have been noted previously, for example informally in Jared Diamond's *Collapse* (2004), or in a very specific setting in Brander and Taylor's (1998) model of the economy of Easter Island, the treatment here is more general and elucidates the connection between socially determined harvest rates and other factors. These include exogenous components of fertility and mortality and the time cost of commuting to locations and technology, as measured by harvesting efficiency.

One significant implication of the analysis is that the congestion of resource locations creates a bottleneck which prevents technological advances from increasing per capita income, and even creates the possibility of collapse. This means that the most technologically advanced hunter gathering societies were not necessarily the ones most likely to survive for long periods of time. In fact if advanced technology, as well as high population density, were prerequisites for a transition of a pre-Neolithic economy to agriculture, then the best candidates for this kind of transition would have been societies which balanced technological innovations with social developments; which successfully solved the well-known *Tragedy of the Commons* by limiting gathering per person.

The next dimension that I consider is the possibility of migration to new locations. I first consider only group migration, which has generally been the main focus of analysis in anthropology, and in fact has sometimes served as a defining characteristic of what constitutes a hunter gathering group (for example, according to Kelly (1995), Radcliffe-Brown defined a hunter-gatherer 'horde' by the fact no individual could leave it on his own). The possibility of migration serves to dampen the oscillations resulting from resource-population interaction, but it also introduces a new source of possible fluctuations. However, while static hunter gathering economies can oscillate in an exploding fashion and collapse, migrating hunter gathering economies can converge to a sort of 'quasi-limit-cycle.'[1] Linking the results to the Neolithic revolution, if relative sedentism was a prerequisite for the transition to agriculture then the model provides us with some idea of how other variables, both demographic and technological, determined migration patterns.

The final extension of the model involves individual migration. The main implication here is that as long as there is no 'frontier,' and virgin locations can be subject to settlement, then the outmigration of some households of an existing economy provides an escape valve which relieves population pressure on existing resources. As a consequence, individual migration will generally dampen oscillations and eliminate the possibility of collapse. It is only when the relevant geographic area becomes filled up that fluctuation in income and population reassert themselves. The filling up of the frontier may create the necessity of a meta-innovation such as the agricultural revolution as a means of avoiding collapse, in a manner reminiscent of Boserup's work on influence of population pressure on technological innovation (Boresup 1966 and 1981, also see Turchin and Nefedov, 2009 and Lee, 1986)

The next section of the paper develops the model. Subsequently its implications and the links with existing literature are discussed, as well as possible empirical tests.

The Basic Model

To develop the model, first consider a simple non-nomadic economy, where all consumption has to take place at a fixed single node, the main camp (the 'central business district' of the hunter-gatherer society, in the terminology of economic geography). Resources available for harvesting are spread out in front of the consumption node. For simplicity I assume in the model a unidirectional set up where the resources are placed on only one side of the village (with impenetrable mountains on the other), with the consumption node at 'location zero.' The model however easily generalizes to a bidirectional

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[1] This is not a limit cycle in the usual mathematical sense since it involves a discrete jump in one of the variables, which 'resets' the process, rather than a smooth oscillating movement in two variables.

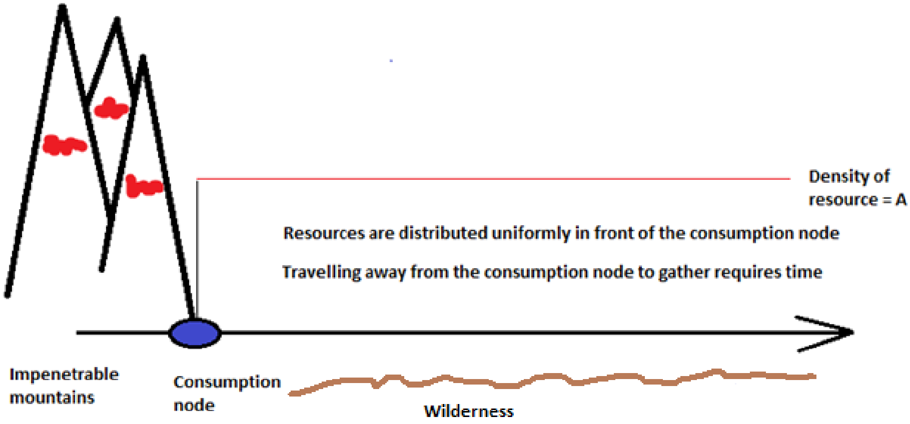

Figure 1. The basic setup of the model.

or even a two dimensional framework. The setup is presented in Figure 1. For mathematical details see the appendix.

Each location close to the camp is assumed to have the same density of resources, denoted by A. The model can be generalized to an arbitrary density distribution so this assumption is made here just for simplicity. A single person in the economy can harvest a maximum amount of resources per unit of time, denoted by β, which will be referred to as 'gathering efficiency.' Each individual has a total of one unit of time to devote to either harvesting resources or commuting to a location, and can harvest at most a single location. The total time cost of commuting increases with the distance of a location from camp.

A location is considered

- *congested* if total resources which would be harvested by all the individuals at a location exceed the total available resources
- *uncongested* otherwise

Hence *uncongested* locations are those where there are either very few workers, or each worker that has arrived has very little time left over for harvesting, or both. In *uncongested* locations each worker gatherers an amount equal to their gathering efficiency while in *congested* locations, the total available resources A get split up evenly among all the workers present.

If a particular location is congested then it will be worth it for a single individual to move to a location slightly further out if and only if the additional time cost of commuting to a new location is low enough. If a location is uncongested then, since time cost increases with distance, no worker from that location will wish to move farther out. This in turn means that all harvested locations are congested.[2] Otherwise an individual could move to a location closer to the consumption node and obtain higher output.

Assuming that resources are spread out on a continuum the workers who are harvesting at the furthest location, denoted by i_X, must be exactly indifferent between staying at i_X or moving just slightly further out. In this way the model resembles the Hotelling model of geographical firm location, except here resource density plays the role of consumer demand and the time travel costs are analogous to transportation costs. Hence i_X is the *maximum indifferent location*, and in the unidirectional setup, also measures total area harvested.

Of course individuals are also indifferent between the *maximum indifferent location* and any location closer to camp since all harvested locations are congested. The implication then is that given a uniform distribution of resources, workers will distribute themselves uniformly along all harvested locations as well. This allows us to determine the extent of the total area harvested as a function of population size, resource density, time cost of commuting and gathering efficiency.

Mathematically i_X solves

$$\frac{A}{L} i_x = \beta(1 - d(i_x)) \qquad (1)$$

where $d(i_x)$ is the travel cost between the consumption node and i_x, and L is the total population of the hunter gatherer economy.

At this point we can say something about the production—and the aggregate production—of this economy. First note that as long as the time cost of commuting is independent of the number of individuals travelling to a location, [3] then changing resource density and population by the same percent will have no effect on the size of area harvested. In turn this means that if we increase both A and L by the same amount the total output of the economy will go up proportionally. This means that the *production function* of this economy exhibits *constant returns to scale* in resource density and population, a common and desirable property of production functions as used in economics.

Figure 2 illustrates total output, area harvested and per capita output of the economy as a function of population. Furthermore, if we assume that commuting costs are directly proportional to distance, $d(i) = \varphi i$, we can

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[2] The very last location could be uncongested. Since we are considering a continuum here, this has no effect on any of the results.

[3] An obvious exception would be if commuting to a location was itself subject to congestion, in that the costs of going to a particular location i increase with the number of individuals travelling to a particular location.

explicitly solve for area harvested, total resources harvested and resources harvested per capita. The solutions are given in the appendix.

It is also worth confirming that the comparative static effects of one variable on the *maximum indifferent location* are in line with intuition. Figure 2a illustrates the determination of i_x based on equation 1. The downward sloping line is $\beta L(1/i - \varphi)$ which represents total time input multiplied by the gathering efficiency, as a function of location. Total resources gathered are shown by the dark grey area. The other three panels show the effects of a change in a particular parameter. The change in total resources harvested is shown by the light dark grey area with a plus or a minus indicating an increase or a decrease.

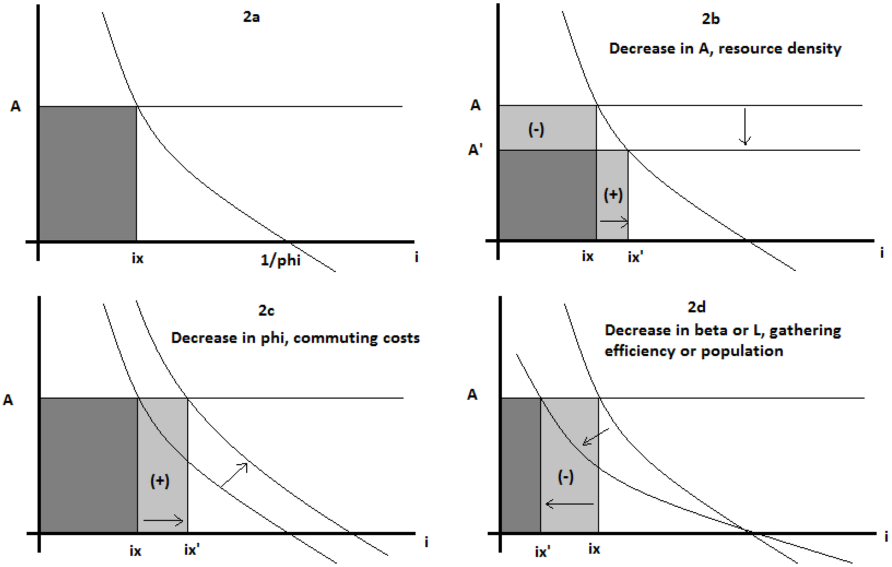

Figure 2. Comparative static effects on area and total resources harvested.

Figure 2b illustrates the effect of an decrease in resource density A. With a thinner distribution of resources, the hunter-gatherers are forced to spread themselves out further away from the consumption node because closer locations become congested faster. Figure 2c shows the effect of a decrease in commuting cost, φ. If φ is lower the workers will travel further out, which relieves congestion pressure on close-by locations, resulting in a larger area around the camp being harvested. Finally, the last panel, 2d, shows the effect of a decrease in gathering efficiency β, or a fall in the population level of the economy, L. With lower gathering efficiency, it takes more workers to congest a location since each one takes out fewer resources from that node. As a result the area harvested shrinks. Similarly, if the population decreases, that relieves congestion in closer locations. It should be noted that while a decrease in β lowers income per person, a decrease in L will increase it.

The Aggregate Production Function

One implication of the setup so far is that the total amount of resources gathered by the hunter gatherers is bounded above. This result is of interest and could be interpreted as one of the main differences between a pre-Neolithic hunter-gather economy and an agricultural one. In a hunter-gatherer economy, the maximum total output that can be 'produced' is constrained by the availability of resources, and the time that can be used to gather them. On the other hand, in an agricultural economy output can be cultivated and produced from previous output (circulating capital) and so, at least in practice is potentially unbounded provided enough inputs. Of course even in agricultural economies other factors—for example, Malthusian demographics or soil depletion - will limit the actual per capita output obtained.

The other implication of constant returns relates to the ownership of the common resource and competition. In this economy, despite the fact that there is congestion and no explicit individual property rights over the gathering of the resource there are no externalities, and no inefficiency. All output accrues to individual workers who are the only class in this society. A standard result in economics states that if a production function exhibits constant returns to scale and there is competition among employers, then workers are paid their marginal product, with the remainder of output accruing to other factors of production like land or capital. Since here we have neither of these, and workers are the only individuals in society, it immediately follows that in this economy the individual workers are 'paid' above their marginal product.

Hence if this economy were taken over by a group of 'entrepreneurs' (a warrior class?) who acquire the rights to gathering at various locations but who compete among themselves for workers to harvest these parcels, and pay them a 'competitive wage' equal to their marginal product, income per worker would fall. Thus, introducing property rights in this model creates income inequality, which could be both 'vertical' (between the owner class and the laborers) and 'horizontal' (among the owner class, according to how far the location they own is from the consumption node). In this way the situation would resemble that of a pre-industrial land-based economy, and the 'entrepreneurs' would be equivalent to Ricardo's class of rentiers.[4]

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[4] Note that there is a form of inequality present in the model, although it has to do with the distribution of leisure rather than resources; workers who have to travel further out

The case of a non-uniform resource distribution is briefly treated in the appendix.

Dynamics

In this section, dynamics are added to the static spatial model presented above. Ideally the model should incorporate both changes in overall population, according to Malthusian pressures, as well as the change in resources per particular location. The difficulty is that if resources at an individual location are allowed to change according to whether they are being harvested or not then the resource density will evolve over time and become non-uniform. This by itself substantially complicates the analysis. If a variable population is also added, this creates a feedback effect which makes the model more or less intractable.

As a result, the dynamic analysis actually carried out makes some sacrifices in terms of realism, for the sake of tractability. However, it generates interesting dynamics and sheds light on a couple pertinent issues. Specifically, I consider the case of ‘mobile resources.’ This set up can be thought of as more of the ‘hunter’ version of the hunter-gatherer economy or alternatively, a case where one unit of time represents a long enough period (say, a generation) so that even static resources have a chance to distribute themselves more or less evenly. In terms of the model, this leads to the assumption that the resources allocate themselves endogenously in such a way that their distribution remains uniform (though the overall availability in the relevant geographic area may decrease or increase).

As a motivating example, consider that killing a few deer in a forest (i.e. relevant geographic area) at a particular location, lowers the overall number of deer in the forest, but some of the remaining deer will move into the emptied location so that the distribution of deer in the forest remains more or less uniform. On the other hand, gatherers harvest resources which tend to be specific to a particular location—picking berries in one spot lowers the average number of berries in the forest, but lowers the number of berries available in that specific location even more. Hence, over time the distribution of berries evolves endogenously and will become non-uniform.

The assumption of a ‘mobile resources’ greatly simplifies the analysis in that it allows the assumption that the natural growth rate of resources is identical in all locations.

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enjoy less leisure. This is essentially a consequence of the assumption that each worker can harvest at most one location.

Resource and Population Dynamics

Resource Dynamics

Assume that at each location the density of resources evolves at the same rate. Harvested per capita resources are modified to depend not only on the resource density but also on the rate at which they are being gathered, denoted by r, which will be referred to as the *socially determined harvest rate*. The way to think of r is that it is determined through custom, taboo or other social mechanism and it is the fraction of resources which an individual hunter is allowed to harvest from each location (hence, each period $(1-r)A$ resources are left at a location).

The definition of *congestion* has to be modified appropriately and the same is true for both area harvested and per capita resources. The exact modifications are given in the appendix, but intuitively, area harvested will decrease with r since a higher harvest rate means locations close to camp become congested quicker, hence workers have to spread themselves further out. Likewise, per capita resources gathered at a point in time will increase with r.

To describe the evolution of resources over time I use a growth function borrowed from environmental and resource economics, specifically the Gordon fisheries model. The exact formula is given by

$$\frac{dA}{dt} = g\left(1-\frac{A}{T}\right)A - ri_xA \qquad (2)$$

Here A is the resource density at a particular point in time and T is the maximum possible resource density. The way to think of the above differential equation is to consider the part $g(1-A/T)$ as the natural growth rate of the resource in absence of any harvesting, and the ri_x part as the total harvest rate. The relationship is illustrated in Figure 3, for a constant harvested area.

The inclusion of i_x in the harvest rate can be justified by noting that each period a total of ri_xA resources is being removed from the relevant geographic area, while the remaining resources grow according to the first term of the above differential equation.

In Figure 3, if the straight line (the total harvest) lies above the parabolic curve (the natural growth) then resource density will decrease and vice versa. Keeping the area harvested constant, the system converges to a level of resource density, A^*. The complication is of course that the area harvested does not remain constant—the straight line in the figure pivots up or down as population and resource density itself changes.

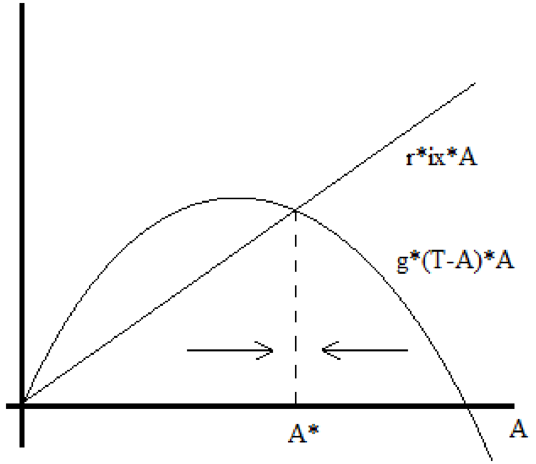

Figure 3. Resource dynamics. The ∩-shaped curve represents the natural growth of the resource, while the straight line is the harvest.

Population Dynamics

Population growth is the difference between the crude birth and death rate. To keep things simple—and relaxing this assumption does not alter the results qualitatively—I assume that there is only a *preventive check* in the economy in that while the birth rate depends on per capita income, the death rate is exogenously given.[5] Hence

$$\frac{dL}{dt} = fyL - mL \qquad (3)$$

where f is the exogenous component of the birth rate - the biological and cultural variables unrelated to income - while m is the exogenous death rate. y is once again resources gathered per person.

In steady state, where population is constant, $y^* = m/f$. The economy will have a long-run non-zero level of population as long as $\beta > m/f$, which is a natural assumption. It states that an economy where none of the harvesting

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[5] In interest of tractability it would be equally useful to assume that there is only a *positive check*—death rate is a function of income—and no *preventive check*. The situation where both of the Malthusian forces are present is for all intents and purposes the same, except that closed form solutions may not exist. For some discussion on the relative strengths of the positive and preventive checks in pre-industrial, but post-Neolithic, Malthusian economies see Crafts and Mills (2009) and Lee and Anderson (2002).

locations are congested (because population is low) has a higher per capita income than an economy at its Malthusian 'subsistence level.' Setting (3) equal to zero gives us a relationship between population and resources which must hold for population to be constant. Likewise, setting equation (2) equal to zero gives us the relationship at which the density of resources will be constant.

As it turns out these relationships can vary according to the level of the socially determined harvest rate r relative to other parameters. Several kinds of dynamics are possible. These are illustrated in Figures 4 and 5.

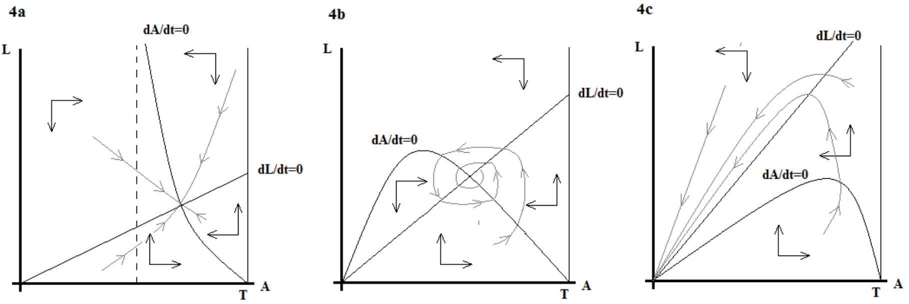

Figure 4. Possible dynamics of population and resources for different values of r.

The arrows in the figure denote the direction of change in population and resource density over time. The two lines indicate the combinations where population or resource density are constant. The intersection, if it exists, is the *steady state* where the system settles to a stasis, unchanged unless there is an exogenous shock which hits the economy.

Figures 5a and 5b illustrate an actual calibration of the model which shows the progression of the dynamics as r changes. In the upper panels the blue line shows possible paths of the economy, depending on initial conditions. The lower panels depict the time evolution of resource density and population.[6]

For low harvest rates the system converges to a unique long-run steady state. For very low r the adjustment is close to monotonic for both resources and population, but as the harvest rate increases 'swings' and eventually cycles occur.

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[6] The graphs were generated by the javascript program PPlane, by John Polking of Rice University, based on a MATLAB script. Images from PPlane are included with permission. The applet is available at http://math.rice.edu/~dfield/dfpp.html The parameters used in the simulation where $\{f = 1.5, m = 1, \beta = .8, g = .33, \varphi = .2\}$

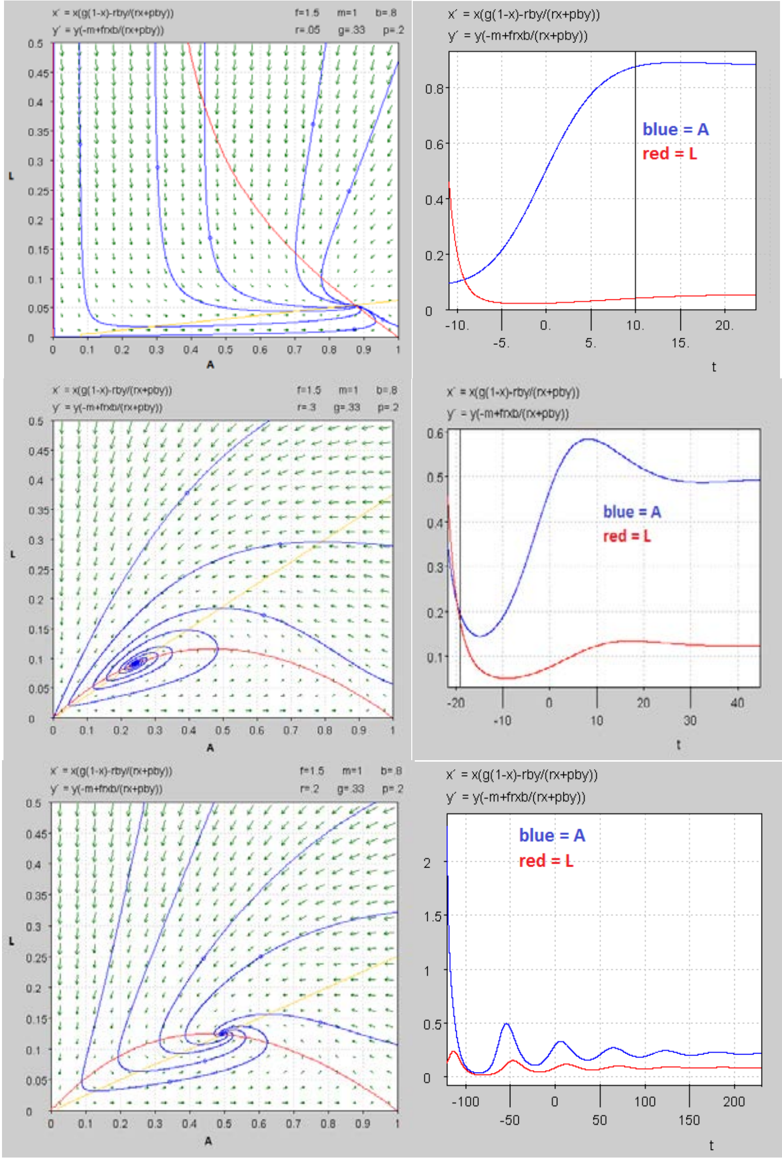

Figure 5. Dynamics of the model. (a) The model with stable dynamics.

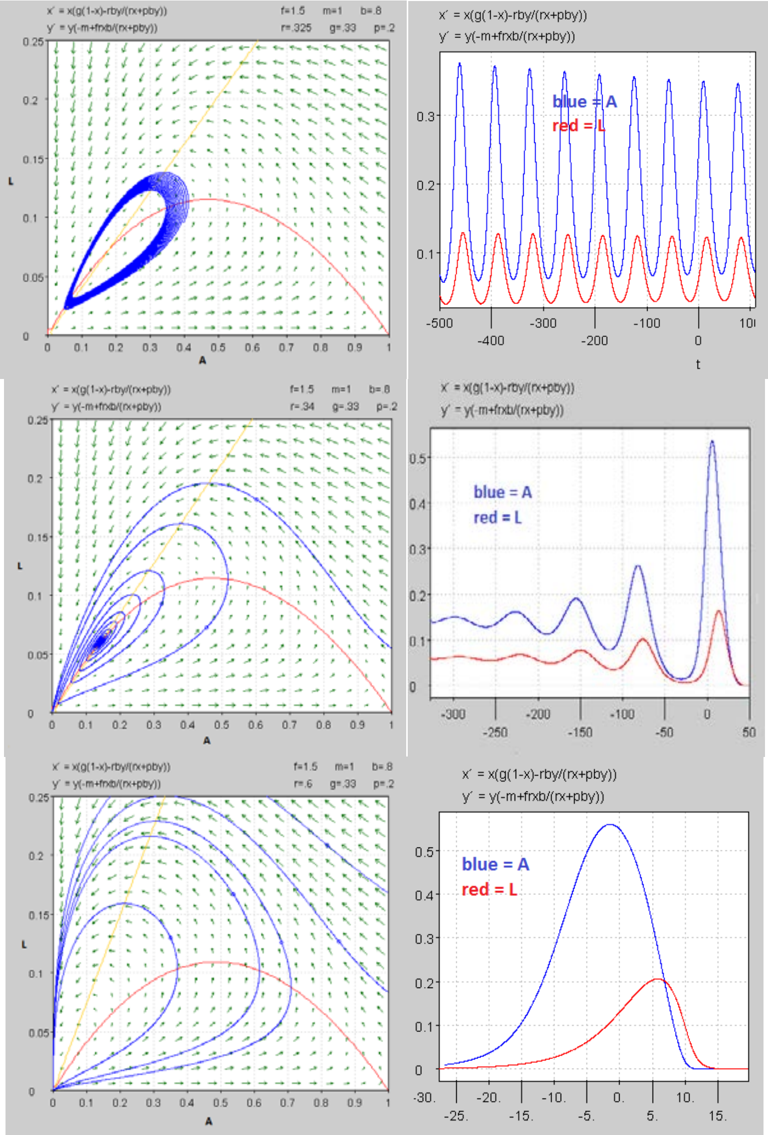

Figure 5, continued. (b) The model with cycles and unstable dynamics.

As the harvest rate increases cycles emerge and both the population and resources orbit the steady state. For even higher harvest rates the dynamics become unstable, and the economy eventually collapses. The collapse can occur through ever increasing cycles or can happen as a boom and bust phenomenon.[7]

While the mathematics of the model are interesting in their own right, it is important to highlight the fact that these have a meaningful economic interpretation. The situation of 4a occurs when the harvest rate is low. In this case the resources have plenty of time to grow naturally and so the system winds up with a relatively high level of resources per location. At the same time, the low harvest rate implies that along the transition path income per person is low hence the final population is small.

In the case of moderate harvest rate the evolution of population dynamics is a race between the depletion of resources and the income which is derived from them. Starting at a medium resource density and low population, income will be very high, which means population will increase. At the same time, low population implies a small area harvested. This puts only minimal pressure on A, which has a chance to grow back naturally and also increases. The system moves north east.

At a certain point however, population—and hence the area harvested—will increase enough so that the pressure from harvesting is exactly equal to natural growth of the resource. Resources will begin to decline, while income is still high enough to keep on increasing the population. Finally, resources are depleted enough, and population is high enough so that income drops to a level beyond which population starts to decrease. While population growth is negative at this point the economy already has a large number of people and so the pressure on resources continues and A keeps on falling. This is the southwest movement of the model. If the harvest rate is only moderately high, then at a certain point population will drop to a level which implies higher income, hence it will begin increasing again. At this point, low population means that the resources get a chance to recover. For a higher r however, the turnaround will also occur, but at a lower level of A and L each time the system cycles. Hence the dynamics are unstable and eventually a point will be reached where the trajectory will take the economy to the collapse state of zero population and resources.

The results with respect to the harvest rate can be summarized as follows:

- For low harvest rates, the hunter-gatherer economy converges to its long-run steady state more or less smoothly. The final outcome is that of high resource density but low population.

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[7] Note that in the above graphs both time as well as A and L have been rescaled for the sake of presentation.

- For intermediate harvest rates, the hunter-gatherer economy will oscillate. If the rate is sufficiently low, it will converge to its long-run level—with higher population and lower resources—in cycles which dampen out over time. However if the harvest rate is sufficiently high, the adjustment will spiral out, resulting in population busts and booms which finally end in the collapse of the society as its resources are depleted.
- Finally, very high harvest rates will lead to a more or less smooth collapse of the society, without cycles. Initially income will be very high, but at the expense of a high depletion rate. Resources will not have time to recover and as a result they will eventually fall to a level which lowers per capita income significantly. At that point population itself will begin to decrease and the economy will eventually disappear.

It should be noted that the last result is a consequence of the interplay of demographic (exogenous birth rate and death rate, f and m), technological (gathering efficiency, β) and ecological (g and T) factors.

The role of other parameters of the model can be explained by analogy with the effects of the harvest rate. Increasing the gathering efficiency works the same as increasing r. This is intuitive since the former can be thought of as 'individual harvest rate' while the latter is the 'social harvest rate.' Note also that the role of both parameters implies an existence of a long-run/short-run trade-off. Per capita income, at any point in time is increasing in both β and r. However, if these parameters become too high the result will be unstable oscillations and collapse.

A change in the demographic variables, f and m, will not affect per capita income at an instant of time—they are growth rates—but it will change the steady state income that the system potentially converges to. A lower (higher) birth rate (death rate) leads to higher per capita income in steady state and will tend to stabilize the system as well. Hence reducing the birth rate—or in the counter-intuitive manner of all Malthusian models, increasing the death rate—can have both long-run and short run benefits, as long as income is the only measure of social well-being.

Unsurprisingly, a higher natural recovery rate, g, will also tend to stabilize the system and lead to outcomes with higher population and higher resource density. A change in maximum resource density, T, has no effect on the stability of the system because in steady state the ratios T/A and L/A are both independent of T.[8]

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[8] This means that in this model there is no 'Paradox of Enrichment' (see Rosenzweig 1971). This is because the predator (the hunter-gathers) population growth rate is Malthusian, in that it depends on both A and L and because the functional response, i_x , is a function of the ratio A/L.

Tables in the section below summarize the key binary relationships between variables and parameters.

'Social Welfare'

At this point it would be ideal to define some kind of 'social objective' that a particular hunter gatherer society tries to achieve and evaluate the various possible dynamics paths in relation to that objective. For modern economies this is usually done by specifying a lifetime utility (social welfare) function whose main argument is consumption per capita, possibly broadly defined to also include leisure. Here however, because this is a Malthusian model, income will always be pulled down to the 'subsistence level,' m/f, by demographic pressures. An alternative would be to consider income per capita along the transition path or the number of descendants of individuals in the model. Doing so in a precise manner however is no easy task.

Instead one way to proceed is to simply rule out extreme outcomes as socially bad. For example, the case of collapse, where the society converges to zero resources and zero population can simply be taken as undesirable by assumption. At the same time, it also makes sense that an economy which winds up with very few individuals and very low income per individual cannot be socially optimal either. What this means in light of previous discussion is that there is some intermediate level of the socially determined harvest rate, r, which is optimal from a social welfare point of view. While I don't define formally what this value may be, the intuition does suggest an interior solution for this rate. One way to think of it is that economies with too high of a harvest rate are suffering from the *Tragedy of the Commons* problem, while economies with too low of a harvest rate are not fully taking advantage of the opportunities present.[9]

Technology, Stability and Limits to Growth

Consider the case of an intermediate harvest rate and an intermediate gathering efficiency rate. As noted above, an increase in β can switch an economy from converging cycles to explosive cycles or even outright collapse. Hence, technological improvements can have deleterious results, if the socially determined harvest rate does not decrease in parallel. The story runs as follows: suppose initially the economy has a high socially determined harvest rate and a fairly low individual gathering efficiency—but that these parameters are such that the economy still converges to a stable steady state. Then some bright member of the tribe invents a better spear or fishing net, and this

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[9] The factors leading to a too low harvest rate occurring are not immediately obvious, especially if one assumes rational agents. However, this could occur for social or cultural reasons.

improvement can be easily copied by other members of the economy. Assuming that the socially determined harvest rate is unchanged, each individual can now gather more per unit of time which means that resource nodes get congested faster. This in turn implies the expansion of the harvested area. But while this results in an initial increase in per capita income, it also means a significant increase in pressure on existing resources—both because now the existing population will spread itself out further, and because population will begin to increase. The result is a boom and bust cycle and the economy eventually collapses. This example highlights the importance of the interaction between technological factors (the invention of better gathering technology) and social factors which determine the harvest rate (the ability of any society to overcome the tragedy of the commons).

For an economy to be able to continue to exist indefinitely three conditions need to be satisfied. First, it has to be the case that $\beta > m/f$, which just says that an uncongested, low population economy has higher income than an economy at its Malthusian steady state. If gathering efficiency is below this level then there will be a demographic collapse; the fertility rate is not high enough to sustain the continued existence of the society in question. Economies such as these will slowly fade out of existence due to low birth rates, but their members will enjoy a relatively high income during this process.

Second, the relative harvest rate and the gathering efficiency need to be such that an interior steady state actually exists. As mentioned above, this means that the hump-shaped curve has to be steeper at $A = 0$ than the straight line in Figure 4. For this to be true the socially determined harvest rate has to be low enough relative to other parameters. If the harvest rate is above this level, the economy will collapse in a non-oscillating fashion due to over harvesting of its resources.

Finally, given that a positive steady state exists, it needs to be stable in that all dynamic paths go towards it rather than run away from it. Otherwise even the smallest outside shock to the economy will send it on an oscillating path towards collapse. Mathematically, the stability of dynamics is analyzed by looking at the eigenvalues of a characteristic matrix of the system. Once again, the details and calculations are relegated to the appendix, but what we wind up with is another relationship between the harvest rate and other parameters.

Combining the $\beta > m/f$ condition, the existence condition, and the stability condition, the parameter values under which the system will result in either demographic collapse, converge 'monotonically,' converge in an oscillating fashion, collapse in an oscillating fashion or collapse 'monotonically' can be analyzed. Figure 6 below illustrates the resulting dynamics in terms of r/φ and β.

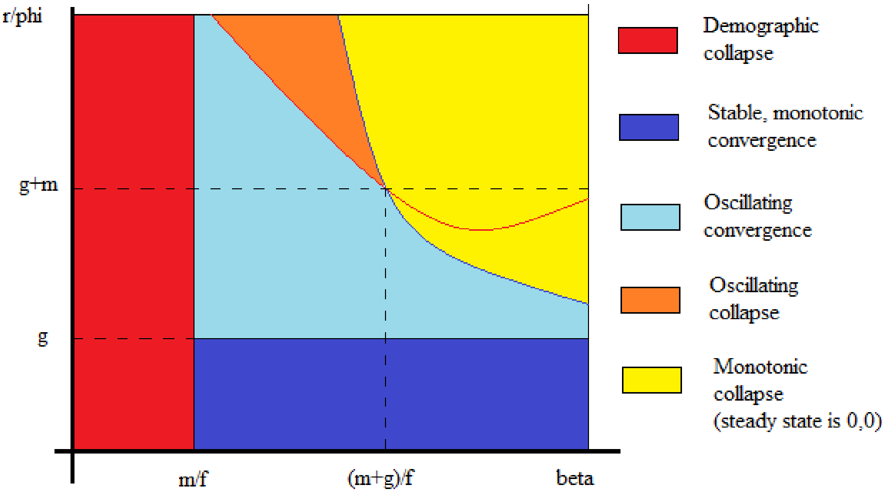

Figure 6. The dynamics of the system as a function of harvest rates and gathering efficiency.

In the above graph the blue line represents the steady state existence boundary; parameter values above this line imply that no interior steady state exists. The red line is the stability boundary. If harvest rates and gathering efficiency are above this line then, if a steady state exists, it is unstable.

Considering hunter gathering economies in evolutionary terms the ones which survive (avoid collapse) will be ones which balance the three factors highlighted so far: demographics, technology and social organization. Specifically a surviving economy will have

- Both exogenous fertility and mortality which are neither too high nor too low
- A harvest rate which is intermediate or low
- A gathering efficiency which is not too high

The selection then is going to favor ‘mediocre’ economies. Additionally, the kind of societies which make for good candidates for a transition to agriculture should satisfy some further plausible criteria:

- Achieve a high enough population density (which may be necessary in the presence of fixed costs of agriculture)
- Are technologically advanced

The discussion so far should make it clear that to a significant extent the two sets of goals can be in conflict with one another. A hunter-gatherer economy with high fertility or low mortality could achieve a high population density, but it is also likely to collapse before it can transition to agriculture. Likewise, an economy with a high rate of innovation would generally be more likely to transition to agriculture, but the very ability to harvest resources quickly will make it unstable.

The kind of economy which works best as a viable candidate for the Neolithic transition would then be one where the commons problem is adequately solved in a dynamic fashion. In other words, it is an economy where the harvest rate itself reacts to other developments, such as technological improvements. This in turn implies not just a society which achieves a particular harvest rate but one which can adapt as circumstances themselves change. To put it in the language of dynamic optimization, the society which does best is one which doesn't just choose an optimal harvest rate, but one which has enough social sophistication to be able to choose an optimal *policy rule*.

Migration

As the section above shows the interaction between resource density changes and population growth can generate oscillations, cycles, and even collapse. However, another source of fluctuations (aside from random shocks) is the possibility of nomadism. Hunter-gatherer groups can be more or less stationary and it is of interest to consider what kinds of factors determine the degree of mobility.

Some anthropological literature has emphasized hunter gatherer mobility as an explanation for supposed low cultural 'materialism' of hunter gatherer groups (Kelly, 1995). Since it is costly to carry many physical possessions in a society which moves frequently the value of these possessions will be low. However, this kind of explanation takes migration itself as given. It could very well be that the causality runs the other way. Groups which have run down their resources and have arrived at a low level of per capita income (and hence few physical possessions) are exactly the ones with the greatest incentive to move in search of 'virgin pastures.'

In a historical context, if sedentism is taken as one of the preconditions of adoption of agriculture, then it is of interest what kind of factors determine mobility patterns, since these will be the variables which will be correlated with early adoption of agriculture and the timing of the Neolithic revolution.

There are two types of migration that can be considered; group migration, where the whole economy changes location together, and individual migration, where the decision to move is undertaken at the household level. The latter kind of migration can lead to the formation of 'daughter groups' and is a likely candidate explanation for emergence of units at the sub-tribal level, and how a given geographic area becomes populated over time. Intuitively, by relieving population (and hence resource) pressure at origin, the individual migration will serve to dampen out population-resource cycles. On the other hand, group migration can generate cycles all on its own, by resetting the level of resources to their initial level. Hence, the two forms of migration can have quite different effects.

Group Migration

An implicit assumption behind group migration is that there are substantial benefits to keeping the whole hunter gatherer society together. Migration has to be coordinated by the society as a whole. An obvious example of such costs/benefits is security and safety during the period of travel, as well as risk sharing in the face of uncertainty. While it may be too dangerous for a single individual or family to undertake the migration decision, if the economy as a whole decides to move the dangers of migration are mitigated sufficiently and the risks are spread across many individuals. These considerations are not modeled explicitly here; rather they serve as a justification for assuming in this section that only group migration is possible.

To keep things simple consider an economy which is characterized by relatively low harvest rates so that the collapse scenario can be ruled out. The choice of group migration is modeled in as simple terms as possible. Specifically, assume that if society moves its consumption node there is a fixed cost, F, which each person in the economy has to pay. This could represent both a time cost (harvesting foregone) as well as a physical cost in terms of resources.[10] If it is worth it for a single individual to migrate, then it is worth it for the whole economy to migrate. Notice that this assumption means that the total cost of moving the camp is proportional to population.

The assumption of fixed costs of migration is actually weaker than it may appear at first glance. While I do not go into detail here, it is possible to set up the model where the costs of migration are proportional to the distance by which the economy migrates. As it turns out, given a choice between staying in an area where resources have been depleted and a virgin territory, the optimal choice involves a corner solution; the economy either does not migrate or it moves far enough from the initial location. This makes the problem equivalent to assuming a fixed cost of migration.

I also assume that if the economy does migrate, resources at the new location are at the maximum, $A = T$. This is of course unrealistic; potential migration locations available might be more or less fertile than where the economy is currently located. What it means in practical terms is that

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[10] Since this is a one good economy the choice of units in which resources are measured is arbitrary and income can be indexed either in terms of time or in terms of physical resources.

eventually the society reaches a place where the next best available location has low enough resource density so that all further migration stops. Alternatively it could be that the economy switches between two or more locations, but the dynamic processes involved are fast enough such that the resources have a chance to recover in other locations in the meantime. The discussion above leads to a model which is illustrated in Figure 7, which is analogous to the first figure of this paper.

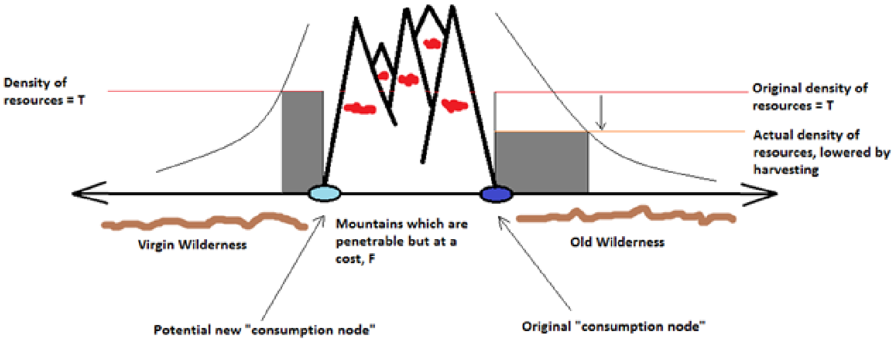

Figure 7. The hunter-gatherer economy with possibility of group migration.

Mathematically, the above discussion can be summarized by a threshold equation for group migration:

$$y_{stay}(A) = y_{move}(T) - F \qquad (4)$$

The difference in per capita resource between 'stay' and 'move' will in turn depend on how far below maximum resources have been depleted at the old site. If the left hand side is greater, the economy will not wish to migrate and vice versa. Hence, at the combination of A and L where (4) holds with exact equality, the economy is indifferent between migrating and staying.

This boundary is illustrated in Figure 8. If by some chance the economy finds itself to the left of this boundary it will immediately migrate, moving horizontally to $= T$. Otherwise the dynamics are the same as in the previous section. Hence this is a system with the possibility of a discrete jump in one of the variables.

Notice that conditions which determine the steady state of the economy are independent of the fixed cost of moving. At the same time, the steady state

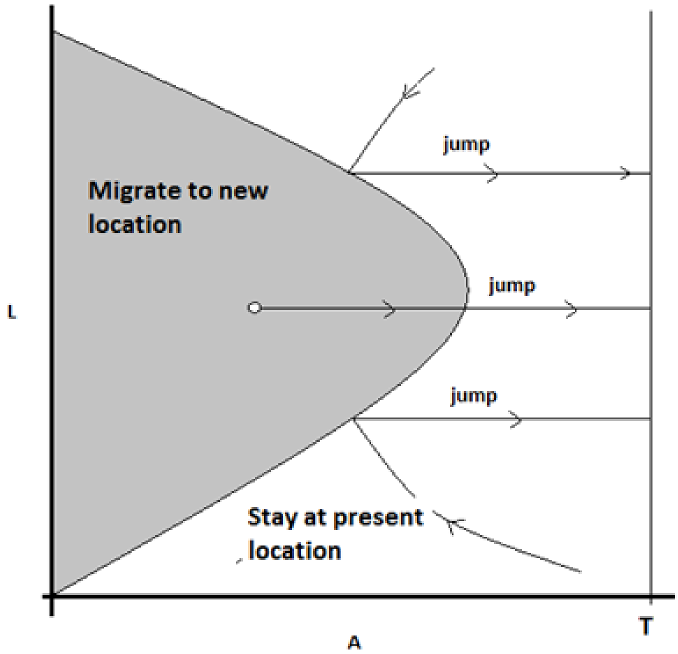

Figure 8. Dynamics with a migration threshold.

depends on natural growth of resources and the birth and death rates which do not directly affect the level of per capita income. This means that the steady state to which the society would converge in absence of migration can lie either to the left or to the right of the threshold condition. If it is to the right of the migration threshold, then eventually the economy will converge to it, albeit with the possibility of several jumps beforehand. However, if the steady state lays to the left of the migration threshold then the economy will never achieve it, as whenever it gets close to it, it will choose to migrate.

The case where the steady state occurs to the right of the migration threshold is illustrated in Figure 9. An example path where migration will occur is shown in blue, initiating at point a. The dashed portion of the line is the path that the economy would have taken if migration was not an option. However, once the economy arrives at the threshold (point b) it will migrate, and its resource density will reset itself to *T* (point c). Since the steady state is in the no-migration portion of the graph, from c the economy will converge 'normally' to point d.

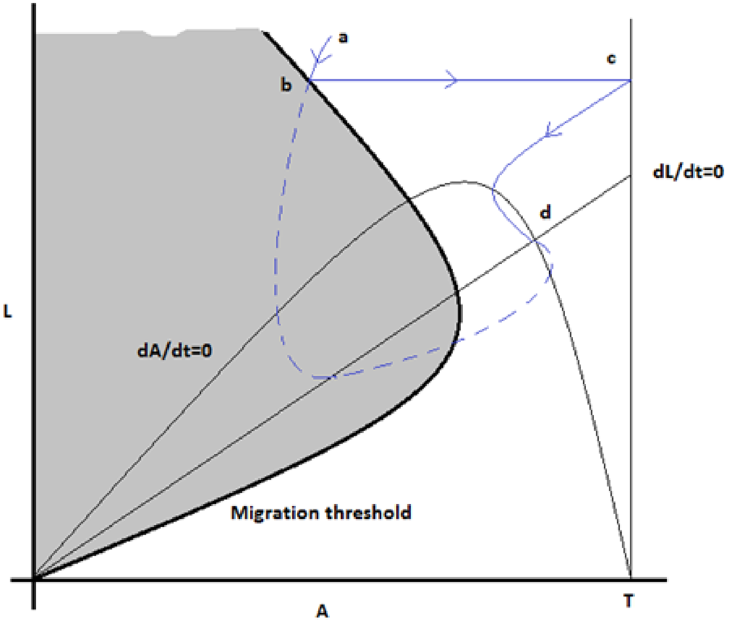

Figure 9. Migration when steady state is to the right of the threshold boundary.

In this case, while there may be one or two instances of migration, eventually the economy will get to a point from which it will proceed to converge smoothly to the actual steady state. Note that this is true even if the underlying dynamic path is cyclical as in panel c of figure 4. In this sense the possibility of migration eliminates the endogenous cyclicality created by the resource-population growth interaction.

What happens if the steady state lies to the left of the migration threshold? In that case the steady state will never be achieved since at that point of resources and population, the hunter gatherer economy will always prefer to move to new territory. Figure (9) shows the case where the society starts at maximum resources and then proceeds to gradually deplete them over time. With a high enough initial population income will be low, both population and resources will decline until eventually the economy reaches the migration threshold. It will then migrate and resources will reset themselves. This takes place at a lower population level. The process resumes until the dynamics

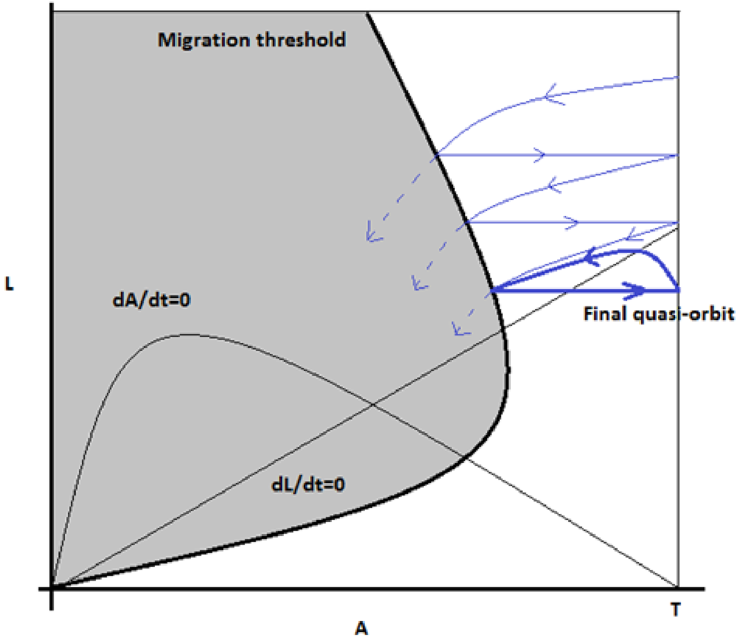

Figure 10. Migration when steady state is to the left of the threshold boundary.

converge to a 'quasi limit cycle.' This isn't a limit cycle in the standard use of the term—where the parameters are such that a regular orbit results—but rather a closed loop which is due to the jump in the resource base. In this case both population and the resources available will cycle but the reason is different from that of the previous section. There the cyclicality relied on the model parameters aligning themselves 'just right' for cycles to result—and hence the chance that this would occur was pretty low. Here, however, the cyclicality is a natural and not unexpected outcome of the possibility of migration.

As can be easily checked, in the case where the initial conditions involve high resources but low population, the oscillating path will 'climb upwards' until they reach the same orbit as depicted in Figure 10.

Individual Migration

This section considers the possibility of individuals, or individual households, migrating and splitting off from the original economy on their own.

In equilibrium individual migration has to be either zero, or there has to be just enough of it so that each household is indifferent between migrating or not. In other words the condition for individuals analogous to (4) is:

$$y_{stay}(A, L_{stay}) = y_{move}(T, L_{move}) - F \qquad (5)$$

where $L_{stay} + L_{move} = L$ is total population, and L_{move} is the number of individuals who migrate to a new location. The difference between (4) and (5) is that now the migration condition depends not just on the difference in resources between the old and new locations, but also on the number of individuals who decide to move. This set up resembles the Specific Factors model used in various applications in economics, with the specific factors here being location-particular resources. The first figure (11) below illustrates the case where resource density at the original location is close to maximum, hence there is no incentive for even a single individual to move.

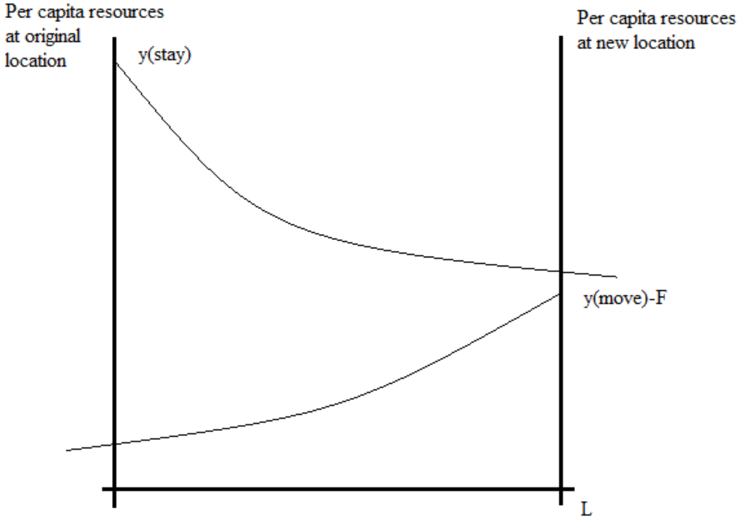

Figure 11. No individual migration; not enough incentive to move given cost of moving.

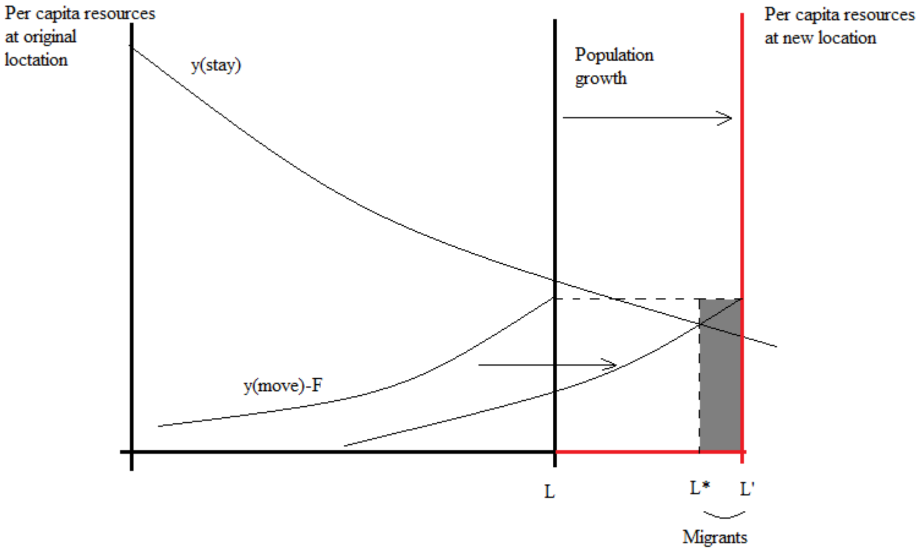

Figure 12. Population growth at original location induces migration to a new daughter colony.

Population is measured from left to right for the original location, and from right to left for the potential new location. Resources per capita are a decreasing function of population at each consumption node. Since, initially, both nodes have roughly similar density of resources the existence of migration costs implies that it is not worth it to migrate even for a single household. This situation is an equilibrium if F is high enough and this is a Malthusian steady state. The interesting question is what happens if income at this point is high enough so that population keeps increasing. This is illustrated in the next figure.

Here population at the original location increases from L to L'. In this case, for some portion of individuals it will be worthwhile to move to a new location and initiate a 'daughter colony.' The per capita income of the old location and the net—of migration costs—income of the new colony have to be equalized. As a result $L^* - L'$ individuals move from the original location while L^* individuals remain at the old location.

After this point both locations are governed by Malthusian as well as resource dynamics. In the new location, since the migrant population is initially low, the pressure on resources will be mild and incomes will be high, which will attract further migrants from the parent colony. At some point the new location will become sufficiently populated so that the migration costs are once again too high to justify any further movement. Additionally, the growing population in the new colony will increase faster than the mother colony because both population is low and resources are still high. What happens next depends on whether there are further locations which can serve as 'granddaughter colonies.'

The resulting dynamics are more complicated as now there are four dynamic equations, two for each location. If further 'granddaughter colonies' are considered, then with each one two more differential equations are added. The intuition however is fairly straight forward. First, if the old colony is at its long-run Malthusian income level, and this (determined by exogenous fertility and mortality) is higher than the uncongested income an individual would receive by migrating, then there is no migration and the model remains in stasis. However, if the old colony has not yet reached Malthusian subsistence, or the level of income is below gathering efficiency, then migration will occur. The ability for individuals to migrate to a new colony will dampen the Malthusian pressures at the original site and amplify them, at least for a while, at the new site. In turn, resource pressure will be relieved at the old site and, at least initially, exacerbated at the new site.

This means that unlike group migration, individual migration is more likely to eliminate cycles or even the possibility of collapse—as long as there remain potential new nodes that the new migrants can move to.

The question whether endogenous resource or migration cycles will be present then depends on the factors which determine to what extent group and individual migration is possible. If there are tribe-level fixed costs in the form of insecurity, uncertainty or logistics, then the result will be either absence of migration or group migration. In this case it is possible to get resource based cycles. Alternatively, if it is relatively costless for individual households to split off from their mother group on their own, then cycles will be eliminated.

Possible Empirical Applications

Like other formal models of hunting and gathering, the framework presented above can be difficult to test empirically. It does however yield some very specific predictions. Tables 1 and 2 present the implied binary relationships between exogenous variables and endogenous outcomes, for non-migratory and migratory economies respectively. These relationships constitute the main implications and testable predictions of the model.

A full set of tests however, is simply not possible. First, because the model is stylized, it leaves out important characteristics of real world hunter gatherer economies. Second, it is not always straight forward to map model variables into recorded data. For example, for economies which rely primarily on

Table 1. Binary relationships for non-migratory economies, non-collapsing hunter gatherer economies.

Effect of an increase in:	*On population*	*On population density*	*On resource density*	*On total resources gathered*	*On resources per capita*	*Notation*
Maximum resource density	Up	Up	Up	Up	None	T
Natural resource recovery rate	Up	Up	Up	Up	None	g
Commuting cost	Down for low r, ∩-shaped for high r	Up	Up	Down for low r, ∩-shaped for high r	None	φ
Gathering efficiency	Up for low r, ∩-shaped for high r	Down	Down		None	β
Social harvest rate	∩-shaped	∩-shaped	Down	∩-shaped	None	r
Exogenous death/birth ratio (steady state income in absence of migration)	Down for low r, ∩-shaped for high r	Down for low r, Up for high r	Up	Down for low r, ∩-shaped for high r	Up	m/f
Notation	L	L/i_x	A	$Y = ri_xA$	$y = Y/L$	

Note that the effect is on steady state variables.

∩-shaped*: If the independent variable is initially low and then increases, the effect is to increase the dependent variable. If the independent variable is initially high and then increases, the effect is to decrease the dependent variable.*

aquatic resources, the notion of 'area harvested' would naturally correspond to the fishing range, but the available relevant data usually considers only land. Finally, we run into the more general problem of data availability. Most of the information we have concerns present day hunter gatherer groups (while the model is more historical in nature), the data is almost exclusively cross-section rather than time-series (and this is a dynamic model) and most of it comes from shortly after 'first contact' of hunter gatherer groups with modern civilization (which makes the assumption of long-run equilibrium problematic).

A further complicating factor concerns the possibility of collapse. By their very nature such economies are not going to be observed over long periods and at any point in time there is unlikely to be many of them present. As such, the available data is essentially a non-random sample of mostly those economies whose parameters are such that they are not on the path to collapse.

Table 2. Relationships for migratory economies, effect in the quasi-limit cycle.

Effect of an increase in:	*On frequency of migration*	*On variance in resources per capita (income)*	*On variance in population*	*On variance in existing resources*	*On variance in population density*	*Notation*
Maximum resource density	Down	Down	Up	∩-shaped	∩-shaped	T
Natural resource recovery rate	Down	Down	Down	Down	Down	g
Commuting cost	Down	Down	Down	Down	Down	φ
Gathering efficiency	Up	Up	Down	Up	∩ shaped	β
Social harvest rate	Up	Up	∪-shaped	Up	Up	r
Exogenous death/birth ratio (steady state income in absence of migration)	∩-shaped	∩-shaped	Down	∩-shaped	∩-shaped	m/f

The effects on average values are not included since these are the same as in the non-migratory case, even if the average values themselves are be different.

Nonetheless it is worth considering some limited tests. The most comprehensive single data source for hunter gatherer economies is the Standard Cross-Cultural Sample (SCCS), which includes almost two thousand variables on economic, sociological and cultural characteristics of more than a thousand human societies (Murdock and White, 1969). Further data on ecological and environmental factors is also available from numerous other sources. Finally, there is some direct data on demographic factors of at least some hunter gatherer societies. Unfortunately the coverage of the SCCS is uneven and it is not immediately clear if the variables there can be easily mapped into the parameters of the model. In particular, the social harvest rate, gathering efficiency and time commuting cost present considerable difficulties. On the outcome side, there is very little comprehensive data on variables such as ‘resources per capita’ or ‘total resources.’ Even something as seemingly straightforward as ‘population density’ may present problems for migratory economies whose area harvested oscillates over time. Time series data which would permit the computation of sample variances do not exist on a sufficient scale. As a result, although future extensions of this work may use SCCS variables as indices or proxies more extensively, here I limit myself to a couple variables for which a there is decent coverage, specifically those used in Kelly (1995). I use population density and residential moves per year as dependent variables, and primary production and effective temperature as explanatory variables which are taken as proxies for maximum resource density (T), and to some extent resource recovery rate (g). I also use a set of regional dummies to capture region specific characteristics.

The results from straightforward linear regressions are presented in Tables 3 and 4.

Table 3. Linear regressions for population density and resources. Dependent variable: population density (persons per 100 km^2).

Independent variables	*I*	*II*	*III*	*IV*	*V*	*VI*
Constant	−40.26	−37.43	−155.46	−175.78	−153.89	−166.05
Log of primary production	11.44	15.54	25.07	26.78	26.84	27.11
	(p = .105)	(p=.052)	(p=.001)	(p=.005)	(p=.003)	(p=.006)
Effective temperature		−2.02			−0.89	−0.934
		(p=.262)			(p=.68)	(p=.771)
Regional dummies	No	No	Partial	Full	Partial	Full
R^2	0.067	0.1	0.39	0.4	0.39	0.4
N	40	40	40	40	40	40

Partial regional controls include dummy variables for Africa, Alaska/Arctic, American Northwest/West Canada, American Southwest/Mexico and Australia/Tasmania. Full regional controls add East and South Asia, and North American Plains, though neither of these is strongly associated with population density.
p-values are included in parentheses.

The estimated relationship between log of primary production and population density is positive and fairly strong; the point estimates indicate that a 10% increase in primary production increases the number of hunter gatherers per 100 km^2 by between 11.4 and 27.1 people. The relationship becomes stronger if we also add effective temperature as evidenced by the much higher p-values.[11] Notice also that including controls for regions both greatly improves the overall fit of the model (higher R-squared) and makes the

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[11] Primary production is defined as net above-ground plant production, calculated from evapotranspiration E, $PP = .0219\, E^{1.66}$. Effective temperature is given by

$$ET = \frac{18\,W - 10\,C}{W - C + 8}$$

where W and C are the warmest and coldest temperatures. This metric captures both the intensity of solar radiation as well as its seasonal distribution (Kelly 1994).

conditional correlation of primary production even stronger. The two strongest regional effects are for Alaska/Arctic and the American Southwest/Mexico (not shown here). Hence, these results are in line with a basic prediction of the model.

Table 4. Linear regressions for residential moves and resources. Dependent variable: residential moves per year.

Independent variables	*I*	*II*	*III*	*IV*	*V*	*VI*
Constant	21.32	27.62	69.69	70.15	69.02	75.75
Log of primary production	−0.89	−3.08	−6.31	−4.25	−6.06	−6.84
	(p=.763)	(p=.552)	(p=.046)	(p=.25)	(p=.28)	(p=.203)
Effective temperature		0.54			−0.065	0.84
		(p=.605)			(p=.96)	(p=.496)
Regional dummies			Partial	Full	Partial	Full
R^2	0.003	0.01	0.38	0.48	0.38	0.49
N	32	32	32	32	32	32

Partial regional controls include dummy variables for Africa, Alaska/Arctic, American Northwest/West Canada, American Southwest/Mexico and Australia/Tasmania.
Strong regional effects on number of residential moves are present for Alaska/Arctic, American Northwest/West Canada, American Southwest/Mexico and Australia/Tasmania, all of which are negatively associated with migration relative to the excluded group. Africa variable also has a negative coefficient but the relationship is weak.
p-values are included in parentheses.

Table 4 indicates that the prediction of the model with respect to the effect of resource abundance on migration is likewise not rejected by the data. If the amount of primary production increases, then on average the hunter gatherer group moves fewer times during a year. This particular data includes moves which are both seasonal as well as those which occur to due resource depletion. The model only considers the latter. This would not be a problem if the seasonal component of residential moves was either exactly the same across all regions and groups or uncorrelated with the resource-depletion motive for migration. In practice neither of these assumptions is likely to hold. As a consequence, what we see in the results is that if we do not control for regional characteristics (which can pick up at least some of the regional differences in seasonal migration) the relationship between residential moves and primary production is extremely weak. However, once regional controls are included (columns III through VI), the relationship becomes much stronger. In fact, the best specification is the one with 'Partial' controls. Along the same lines, effective temperature does not seem to affect migration much and its inclusion weakens the link between migration and primary production. This is most likely due to the fact that this variable is an amalgam of several factors, as well as due to its multicollinearity with primary production (the correlation coefficient between the two variables is 0.795).

Hence, it appears that some of the most basic predictions of the model are supported by the available data, scant though it may be. However, due to the limitations enumerated above it is probably the case that the most fruitful ways to proceed in regard to future empirical applications of the model lie in looking at case studies rather than cross sectional statistical tests. Specifically, the model can be useful as a concept organizing framework when applied to the historical experience of particular hunter gatherer economies. It can shed light on the reasons why some of these saw periods of expansion, stasis or collapse, by providing a rigorous and formal framework for the explanation which is often missing from historical narratives.

This is particularly true in regard to cases of historical collapse, such as those discussed in Jared Diamond's *Collapse*, where a unique historical dynamic development does not lend itself well to statistical testing.

Discussion

Since the analysis of the paper is linked to several different strands of inquiry, from economics to anthropology and ecology, this section places the results in the context of the wider literatures on hunter-gatherers.

Economics

The economic literature on hunter-gatherers has generally followed two strands. The first has looked at how various parameters which are of relevance to modern economies may have evolved during the pre-Neolithic phase of human existence. The aim here is to 'endogenize' important aspects of human behavior, which are often otherwise taken as given by economists. The second aspect has focused on the Neolithic transition from hunter gathering to agriculture.

Pre-Neolithic Economies and Preferences

Robson and Kalpan (2003) consider the co-evolution of greater intelligence and longer lifespans in humans by modeling the brain as a form of capital, and increases in intelligence over time as a form of investment. This work points to the hunter-gatherer period of human history as an important determinant of what has traditionally been taken as an 'exogenous' component of human demographic behavior. In a separate paper the authors make an argument for a general consideration of historical hunter-gatherer societies within economics (Robson and Kaplan, 2006).

In a similar manner, the argument in Greg Clark's *Farewell of Alms* (2003) concerns the biological or social evolution of the rate of time preference (impatience). This trait would develop differently in an economy where a sufficiently efficient storage technology is unavailable, such as a pre-Neolithic hunter gatherer society, and a settled agricultural one. Clark's hypothesis is at least partly based on previous work about the evolution of time preference, most notably that of Rogers (1994). Additional papers on the evolution of time preference in historical societies include Hansson and Stuart (1990), Robson and Samuelson (2009) and Chu, Chien and Lee (2010).

While in this paper I do not consider the connection between impatience or life expectancy and hunter gatherer production, it is pretty straightforward that several parameters of the model—such as gathering efficiency and the social harvest rate—are closely linked to these considerations. This is particularly true since the model suggests that societies with high social impatience will collapse or, due to the possibility of exploding cycles, be especially vulnerable to environmental shocks.

Transition to Agriculture

The second strand in the economic literature on hunter gatherers has attempted to explain and conceptualize the Neolithic Revolution. However, in many, but not all, of the relevant papers the story of the hunter gatherers takes a backseat to the transition itself.

The importance of the timing of the transition to agriculture has been widely acknowledged. Ashraf and Galor (2012) point out that an implication of the Malthusian framework, combined with the assumption that post-Neolithic economies were characterized by positive technological growth is that the timing of the transition would show up in population densities across regions but not in incomes per capita. Hence, the observed relationship between the timing of the Neolithic revolution and these two variables many centuries later (in Ashraf and Galor, in 1000 AD and 1500 AD) can serve as an empirical test of the Malthusian hypothesis itself.

One aspect which is left unexplained however, is why continuous technological progress was ushered in only with the introduction of agriculture. The present paper provides at least a partial answer: pre-Neolithic economies which were too productive risked collapse as technological innovations could lead to over extraction of resources. In turn, this most likely had a negative effect on the rate of technology adoption (if not innovation). Interestingly, in what I believe is a fairly novel extension, the paper considers to what extent migration, whether by groups or individual households, could ameliorate these disincentives.

A more in depth consideration of the nature of pre-Neolithic hunter gatherer economies is provided by Matthew Baker (2003, 2008). The author considers a model where both property rights and adoption of agriculture emerge simultaneously. While here I do not deal with the issue of property rights—in the model there are none—this is most certainly an issue which should be included in future extensions of the model.

Additional key works which consider hunter gathering economies and the transition to agriculture include Smith (1975), as well as more recent papers by Weisdorf (2004, 2005).

Anthropology and Ecology

A good bit of work in anthropology and ecology has considered the issue of territoriality, which can be roughly related to 'area harvested' of the present model. The main work horse of this literature is the Economic Defensibility Model (EDM), originally due to Dyson-Hudson and Smith (1978). Hunter-gatherer groups devote resources to defending a geographic area, with the costs and benefits determined by resource density and resource predictability. Accordingly, geographic areas with high resource density and predictability result in stable, well-defined territory, while hunter gatherer groups in areas with low resource predictability and density will be characterized by frequent migration and high dispersion of economic activity. While the EDM model has generally proven useful in classifying hunter-gathering economies, there have been instances where the data appear to contradict its main conclusions (see Baker 2003 and Kelly 1995 for a perspective from an economist and an anthropologist, respectively).

For instance Cashdan (1983) notes an inverse relationship between territoriality and density/predictability of resources among the Kalahari Bushmen. Interestingly, Cashdan also states that 'there is empirical support for the effects of resource density on territoriality. Many studies have shown an inverse relationship between territory size and resource density.' As it turns out, a similar conclusion—if 'territory' is replaced by 'area harvested'—is one of the implications of the present model, and the connection between migration and resources is one of the main points of focus of this paper. Here however, the relationships arise not due to any consideration of the costs and benefits of defending a territory but simply from the time constraint imposed on the hunter gatherers; higher resource density implies that locations close to the hearth of the hunter-gatherer society can be harvested more intensively before they become congested.

Another important point made by Cashdan (though one should keep in mind the publication date) is that the ecological literature has not formally considered population density in the territoriality models, whereas given the Malthusian framework I use, here it is one of the key elements. On the other hand, the issue of predictability of resources, which plays a key role in territory defensibility, is ignored here since the focus is different.

However, a substantive difference between the literature on the EDM and this paper is that it is my intention to analyze historical hunter gatherer populations rather than modern ones (though of course some of the insights might very well generalize to post-Neolithic hunter gatherer economies). As a result the background assumption in place here is that the world is relatively sparsely populated and that the factor of production 'land'—though not 'resources'—is not scarce. In other words, the hunter gatherers of this model operate in relative isolation from other groups and there is no frontier at which further land can become unavailable. Rather, the limiting factors here are the density of resource per location and the time needed to both travel to a particular location and to actually harvest the resource itself. Since land is not a scarce factor there is no need for defensibility. Hence, while one can make a rough analogy between 'territory' of the EDM literature and 'harvested area' of the present model, the focus and intent of the analysis is different.

The present paper is very much similar in spirit to Bruce Winterhalder's (1997) mathematical simulations of hunter gatherer dynamics. The key difference is that the present work builds an explicit theoretical framework and obtains analytic solutions. As a result I can more precisely characterize the interaction between various variables and parameters. In that sense the present work can shed light not just on 'what' will happen if a particular variable changes, but 'why' it happens and what the underlying process is.

Conclusion

This paper has analyzed a hunter gatherer economy in a formal manner. According to the model, the area harvested by the group depends positively on population and gathering efficiency, and negatively on resource density and time costs of travelling to resource locations. When population growth and possibility of resource depletion are included, the resulting dynamic paths may be smooth and converge to a stable steady state, or the economy can oscillate, explode and collapse. Higher socially determined harvest rate, as well as higher fertility and gathering efficiency make collapse more likely. One implication then is that as technology increases, the hunter gathering economy needs to decrease the socially determined harvest rate in order to avoid collapse.

The model is then extended to include migration. Group migration can serve as a way of avoiding collapse and dampens the possible oscillations, if these exist, resulting from the population-resource interaction. However it creates a new source of fluctuations, as each time the hunter gatherer group migrates, the resource stock around its camp 'resets' itself. Finally, individual migration, if it is possible, can serve as escape valve for high populations and in that way prevents oscillations or collapse from occurring.

Acknowledgment
I wish to thank Faress Bhuiyan and Mark Kanazawa for their extremely helpful comments and advice. The development of this work certainly encountered some technological and methodological bottlenecks along the way and I would not have been able to overcome these without their help.

Mathematical Appendix: to access this file, go to the article view and click on the 'Supporting Material' tab in the left-hand column.

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