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The Silk Roads: a Mathematical Model

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This paper concerns the mathematical modeling of historical processes; specifically, the temporal dynamics of the Silk Roads described by formal spatial equations. Historical data indicate that the location of the trade routes known as the Silk Roads varied dramatically from epoch to epoch. These changes arose from a number of causes—population oscillations, economic trends, diseases, and warfare—all of which affected the Silk Roads' geographical location in different eras, and also determined their rise and demise in each epoch. Mathematical simulation predicting the Silk Roads' location in each epoch could help to distinguish the most significant determinants of their fluctuations and to estimate where and when these factors were especially prominent. In this paper, we examine the hypothesis considered by Jeremy Bentley (1993), who suggested that one of the most important causes of the Silk Roads' prosperity was the development of large-scale empires across Eurasia. Empires stimulate the exchange of commodities for the rise of supply-and-demand of bulk and prestige goods, construct roads and related infrastructure that encourage trade, and bring stability to vast areas. The model takes these processes into account and demonstrates that oscillations of the Silk Roads' activities were induced by the rise and fall of large empires such as the Roman, Parthian, and Mongol empires, as well as the Han and Tang dynasties. Its ultimate demise might have been due to the rise of European maritime shipping, which increased ocean trade at the expense of overland, Eurasian routes.

Introduction

Computer simulation of historical events has great promise, given the interest in mathematical applications for social sciences. Formal laws, such as those discovered in physics and biology, that can describe the behavior (or, broadly, the evolution) of social structures do not yet exist; however, the history of the biological and physical sciences shows that formalization of ideas can lead to testable predictions, which a researcher can then reject or refine. Thus, some defined "social equations" may aid mathematical modelers of historical events

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in obtaining formal results and forecasts for social systems, just as hypotheses direct research on physical laws and systems.

Mathematical modeling of social processes is a discipline that has emerged primarily as a result of enhanced mathematical methods and computing machinery, but also due to a growing appreciation of the complexity of social interactions, risks, and threats. Great success in modeling complex physical processes spurred the application of mathematical models to the social sciences. Although the first steps of social modeling have shown that simple linear equations can sometimes give adequate predictions, these steps have also exposed some fundamental problems. One such problem is the "human factor." Mechanical systems have no free choice, so traditional physical-mathematical approaches do not permit the description of complex systems possessing free choice.

In order to construct a formal theory, it seems reasonable to consider macro-systems and to work with pre-modern, less-complicated societies. This way, we avoid both the problem of long-term prediction (we have information on the "future" dynamics of the pre-modern social systems) and the problem of the human factor (which does not dominate a large-scale system as it does a small one). Furthermore, the model builds on these historical, simple social systems to make predictions about modern, hierarchical, complex systems. Although the historical process is unique and unrepeatable, the same basic processes are observed in different societies in different ages. Thus, history gives us an extensive data set which is sufficient for both the construction and verification of the formal theory. This suggests that there may be some basic laws of social dynamics and these laws can be identified and formalized.

As I have outlined, the perspectives of historical modeling are promising. However, there exist few scientific works of this kind (but see Guseynova, Ustinov, Pavlovskii 1981; Nefedov 2001; Turchin 2002; Malkov 2002). In this paper, there are two goals: (1) to propose a formal framework and (2) to apply it to historical processes. As noted above, the attempts of macro-systems description seem to be the most creative approach under the current conditions. However, these studies have considered the internal relations of the society without including geographical properties as a major feature. This approach is acceptable for large, isolated agrarian societies, which were the foci of the above-mentioned works. However, there existed not only self-sufficient agrarian states but also a great number of societies that were essentially dependent on transit trade between these agrarian states. Societies of this type obviously depend on geography and location of agrarian neighbors. Consequently, the spatial factor is a major determinant of the dynamics of these societies.

To help with the construction of the model of spatial trade, it is helpful to take a look at previous studies of spatial economics, for example, the continuous model of transportation that was proposed and developed by Beckmann (1952). It concerns the process of commodity transportation in some geographical region (e.g., urban territory). However, it was constructed for trade flow optimization and does not pretend to describe the evolution of real flows and trade routes. So, the Beckmann model must be modified and generalized in order to describe spatial historical dynamics.

Beckmann suggests that some commodity sources (producers) and sinks (consumers) should be located with the geographical region under consideration. Every point of the region is also characterized by commodity flow **J** and transportation cost through the point. The problem considered by Beckmann was to find optimal flows, given a distribution of producers, consumers, and transportation costs. Beckmann proposed his model of the spatial market under the assumption that "traders must not suffer losses. This means that the gain from trade exactly equals transportation costs…" (Beckmann, Puu 1985: 16) and results in a stationary distribution of commodity flows. Beckmann proves that this distribution is optimal because the transportation costs are minimal along every flow line.

Beckmann states that optimal flows $\mathbf{J}(x,y)$ will depend on price distribution

$$\frac{\mathbf{J}}{|\mathbf{J}|} = k\nabla p$$

at every point where $\mathbf{J} \neq 0$. Here $p(x,y)$ is price distribution and k is the coefficient.

This model is useful for the field of spatial economics because it can be used for effective trade flow control. Nevertheless, there are some limitations that preclude its application to the problems of historical dynamics. First, it is a stationary model and, as such, cannot describe dynamics. History is a non-stationary process; evolution is slow at times, fast at others, and any changes could be transient. To describe history, the model must be dynamical.

Second, the requirement of optimal route choice is unrealistic because we will inevitably lack enough historical information to make this determination. Although Beckmann assumes that the trader chooses the route with the absolute minimal transportation cost, it is clear that a real trader (especially an ancient one) can deal only with rough estimations of transportation costs. The model proposed by Beckmann gives an ambiguous solution for "neutral circuits" (Beckmann, Puu 1985: 38), when two distinct flow paths of equal cost exist between two points of a region. Under Beckmann's model, however, an infinitesimally small variation of the cost along one of these paths will destroy the neutral circuit and the trader will choose the best path. In other words, micro-level variations cause dramatic, macro-level changes. This situation is obviously unrealistic. A real trader is more likely to choose the route randomly (in a certain sense), but the probability of his choice essentially depends on roughly estimated transportation costs. Furthermore, risks, habits, prestige, and other factors will also affect path choice—and they become more significant factors as the costs of two paths approach equivalence.

Finally, the use of transportation costs as a territory characteristic works well only under the conditions of the modern society. Because it is difficult to reconstruct such data for historical periods, we need a method of estimating transportation costs. It is reasonable to introduce another meaningful parameter that could describe the trade conductivity of a territory that is less dependent on currency and prices, more inclusive of non-monetary factors that impact choice (risk, prestige, etc.) and is easily quantifiable. Thus, Beckmann's model requires modification and generalization in order to be applicable to the problems of spatial historical dynamics.

A Model for Trade Flows

Let us consider a closed region of spatial one-commodity market. Suppose $T(x,y)$ is the density of commodity, $q(x,y)$ is the excess density of production (i.e., the difference between the density of production and the density of consumption [q is positive if production exceeds consumption]), and t is time.

The divergent law for this process is

$$\frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{J} + q$$

where $\mathbf{J}$ is the commodity flow vector.

This well-known equation describes a continuous flow of any substance (e.g., heat, liquid, etc.) In the given case it can be verbalized as follows: "The increase of commodity density, $\partial T/\partial t$, is the sum of production q and the difference between the incoming and outgoing flows."

The price dynamics can be linearly considered as

$$\frac{\partial p}{\partial t} = \gamma(D - S)$$

where $p(x,y)$ is the distribution of commodity price, $D(x,y)$ is the demand density at the point, $S(x,y)$ is the supply density and γ is the constant of proportionality that implies the supply-demand imbalance sensitivity of market prices.

This equation can be verbalized as follows: “The prices grow if demand exceeds supply and fall if demand is less than supply.”

Under the assumption that there are no limitations on commodity selling,

$$\frac{\partial T}{\partial t} = S - D$$

That is, “overstocking results if supply is greater than demand and there are active sales if demand exceeds supply.”

Finally, the main assumption of the model is that

$$\mathbf{J} = k \nabla p$$

where the commodity conduction coefficient, $k(x,y)$, is a spatially uneven parameter. This equation means that the flow of commodity is proportional to the gradient of the commodity price. The verbalization is the following: “The flow of commodity transportation between adjacent points is proportional to the difference in prices at those points.”

This equation is the key difference from Beckmann’s model. Beckmann assumes that the flow must have the same direction as the price gradient (as it is known, the gradient vector is directed along the lines of the quickest ascent—in this case, the lines of the quickest price rise). Our assumption instead implies that the direction of the flow is the same as the gradient direction, but also that the absolute value of the flow is proportional to the amount of the gradient.

This refinement might appear to be a slight modification, but it is essential. Namely, this modification withdraws the problem of “neutral circuits” and decision-making under the information-poor conditions. That is, the traders can define the amount of the inter-local trade using the local properties of the market. They do not demand additional information about remote markets.

One of the advantages of the proposed model is that it does not fundamentally contradict Beckmann’s model. Moreover, Beckmann’s optimal stationary solution can be reached in this dynamical model as the time tends to infinity. That means that the system as a whole eventually comes to the stationary optimal solution where the cost of transportation is minimal. It is a more realistic behavior—suppose the system was initially stable, but suddenly, a considerable change of conditions occurs (e.g., new production startup, bankruptcy of an enterprise, armed conflict at the region of the trade route). Due to the lack of information, the flows do not come to the optimal configuration immediately after the change of conditions, but as time passes (if conditions do not change dramatically), the flows stabilize and become optimal under established conditions. Note that parameter γ corresponds to the speed of the information propagation and system response. The greater the γ value, the faster the optimal steady-state pattern establishes.

Assembling all previous equations, we can derive:

$$\frac{\partial p}{\partial t} = \gamma(k\Delta p - q)$$

This equation is well-known: it is a "heat conduction equation" that describes the evolution of a spatial system with heat sources and sinks and with an uneven heat-conduction coefficient. The heat conduction equation is well-studied (Tikhonov, Samarskii 1951) theoretically and practically, and has strong analytical treatments and computational methods.

The commodity-conduction coefficient k is, therefore, mathematically equal to the heat-conduction coefficient, so it will be discussed using this analogy. The commodity-conduction coefficient (CCC) is a very important factor for a spatial market. The higher this coefficient, the more profitable the conditions for both producers and consumers; a low CCC results in low prices for producers but high prices for consumers. The difference between these prices corresponds to transportation expenses. This situation is not favorable and can reduce both production and consumption. Although CCC is related to Beckmann's transportation costs coefficient, CCC is more general. It involves not only economical properties but also non-monetary aspects such as risks, prestige, and habits. Moreover, CCC is more measurable for historical processes. For example, if we have an estimation of the amount of the commodity flow between two isolated towns and an estimation of the price at each town, we can define the value of CCC along the route connecting these towns. Certainly it is not quite as easy as described—after all, the flow and prices are not constant through time—but the model is dynamical, so it is possible to distinguish the causes of changes (e.g., external conditions, evolution of the market itself).

Commodity Flow Equation

Commodity conduction k depends on properties of the territory. Let us assume that the price gradient is fixed. Thus, the price difference per one mile is $\Delta P = p$. How many caravans will move across the territory to satisfy existing demand? That depends on how fast they can move, because they will get paid

$$w = mv\Delta p$$

where w (money per hour) is hourly revenue of a caravan bringing m (tons of goods) with velocity v (miles per hour) following the gradient p vector (money per ton per mile).

We assume that the number of caravans is proportional to how much money they make from that journey (higher profits means more caravans). If there are no risks (no taxes, pirates, wars in the territory), caravans get w per hour. If there are risks, the probability of getting full revenue is lower: $(1 - r)w < w$, where r is a risk factor ($0 \le r \le 1$). When r equals 1, it means 100 percent probability of total profit loss; when r equals 0, it means that there is no risk and every dollar is secure.

Therefore, the number N of caravans bringing m of goods each is

$$N = a(1-r)w = a(1-r)mv\nabla p$$

where a is a coefficient of proportionality.

Finally, the flow **J** is the number of caravans N moving m of goods with velocity v:

$$\mathbf{J} = Nm\mathbf{v} = a(1-r)m^2v^2\nabla p = k\nabla p$$
$$k = a(1-r)m^2v^2$$

As we can see, commodity conductivity depends on two fundamental spatial parameters: r (risk of profit loss) and v (maximum velocity with which the caravan can move).

Even if the price difference is high, merchants will not conduct goods when territory is hostile, where risks are high (wars, pirates, high taxes, aggressive tribes, wild animals) and/or the caravan cannot move fast enough through physical obstacles (mountains, jungles, rivers, deserts). In order to make simulations of trade flows we need to estimate k, or, more specifically r and v.

Risks are hard to measure, and in our simulations we assume that risks are high outside the borders of large empires, and lower inside the borders because empires protect roads and support trade. $r(x,y) = 0$ inside the borders and $r(x,y) = 0.9$ outside. For movement velocity v we need another formal model, one that derives velocity from physical properties of the territory.

Evaluation of the Transport Friction

Many environmental factors can impact the velocity of transportation across a region. For example, one can travel much more rapidly over flat, sparsely vegetated terrain than mountainous or densely forested regions. Thus, we must define a transport friction coefficient that can be measured empirically.

The coefficient, μ, is comprised of two components: (1) the ruggedness of the terrain, and (2) the difficulty of travelling through a particular biome (an ecological zone). Rugged terrain, with frequent rises and falls in elevation, will incur a higher μ, as will biomes that are difficult to traverse, such as tropical forests.

In order to estimate this coefficient for the era of the Silk Road, we used data about the journey of Marco Polo, the famous Venetian merchant who traveled to China in the 13th century. Although his book contains a number of disputed and controversial issues, it is nevertheless an invaluable source of data about that ancient era. In particular, Marco Polo recorded the days required for a march from one point of his journey to the other. Such information is available for more than sixty marches (Appendix A), which is a good sample size for determining the coefficients on this journey.

I assume that the speed of movement through the terrain is related to the environment as follows:

$$v(x) = \frac{C}{\mu(x) + \left|\frac{dH(x)}{dx}\right|}$$

which relates movement speed v, the determination of which is made by historical data, elevation change H, for which we use the geographic information data (Figure 1), and the coefficients μ, which will be evaluated on the basis of this formula with the additional assumption that the coefficient $\mu(x,y)$ is a piecewise constant—although it will fluctuate across natural areas (Figure 2), within each region it is constant.

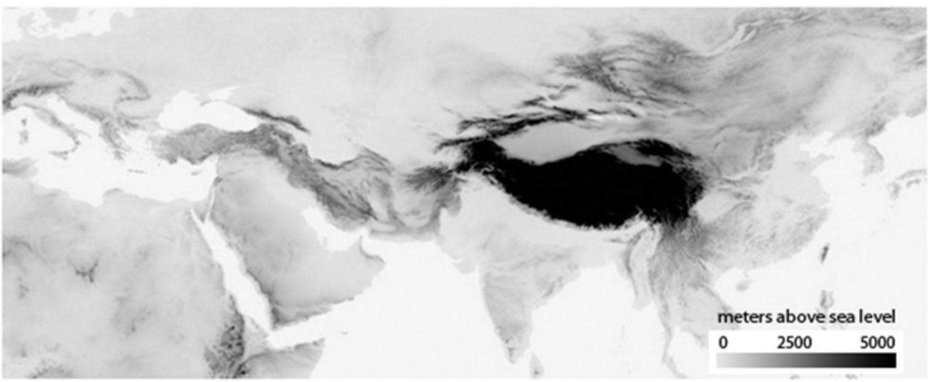

Figure 1. Elevation map. Darker regions indicate higher elevations.

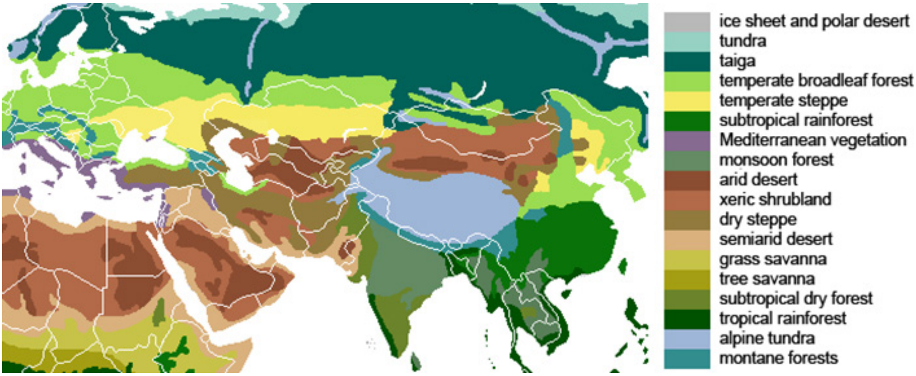

Figure 2. Map of ecological zones (biomes). Source: Wikipedia (http://en.wikipedia.org/wiki/Biome)

The idea of coefficient evaluation consists of solving the inverse problem. The march speeds, $v_{i+0.5,\,j}$ and $v_{i,j+0.5}$, are calculated by setting different values to the coefficients μ in accordance with the terrain an biome through which the route passed. Then, using the shortest path algorithm, the time τ necessary for a march between two points mentioned in Marco Polo's book is calculated (Figure 3). The calculated march time τ is compared with the corresponding time τ_{MP} mentioned by Marco Polo. As a result, we obtain a set of points (τ, τ_{MP}), which, if Marco Polo always moved with maximum speed and used the shortest path, should make a straight line passing through zero. Naturally, a perfect match is impossible to obtain; yet, by refining our estimates of μ to match the local terrain, we significantly improved the correlation between march time predicted by the model and the march time reported by Marco Polo.

In the simplest case, in which travel time was proportional to the distance between two cities, R^2 between τ and τ_{MP} was 0.44. Consideration of elevation changes and differences in the coefficient μ in different ecological zones increases R^2 to 0.69 (Figure 4). Consideration of height changes and differences in the coefficient μ allows us to produce model calculations more in line with the actual data. Quantifying elevation changes required the use of an ideal mesh size. For example, traversing a distance of 1 km with a smooth rise in elevation of 100 m is less difficult than traversing the same distance with many hills and valleys, but the same overall rise of 100 m. However, if one examines both of these hypothetical routes with a mesh size of 1 km, they will appear to be of equal difficulty.

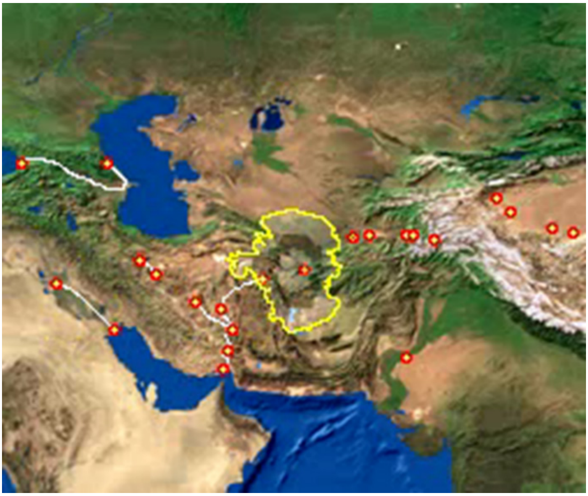

Figure 3. Several calculations of the shortest (in time) path between two points of Marco Polo's travel. The algorithm is run with different transportation friction coefficients for biomes to generate estimates of time, in days, it would take to travel from one city to another. The yellow line outlines how far caravans can travel starting from a city at the same time and moving in different directions under the transportation friction for that path.

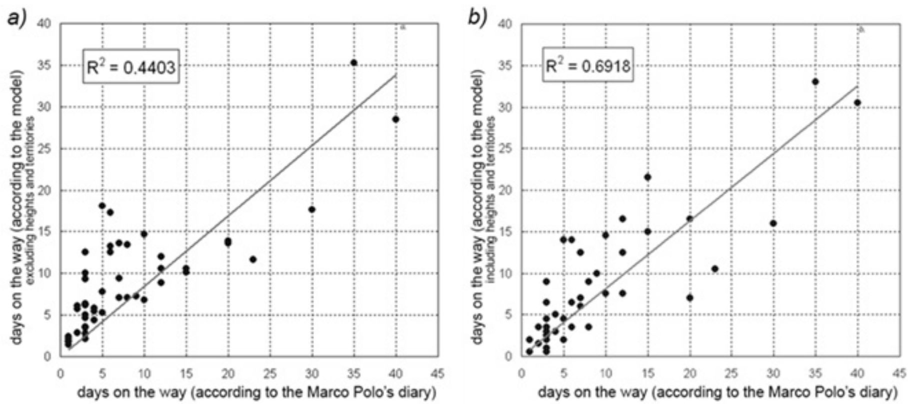

Figure 4. Relation between the calculated march times and time spent by Marco Polo (a) excluding heights and territories (μ = const, H = const); (b) including heights and territories (biomes).

The problem was solved as follows. Mesh $H[i,j]$ of the heights of the Earth in polar coordinates with a spacing of about 0.1° was used. Meshes $H_K[i,j]$, which grew larger along the *y*-axis, were consistently created for meridians. The closest values in the original mesh were taken as the values in the nodes of the new mesh:

$$H_K[i,j] = H\left[i, \left\lfloor j\frac{\lambda_K}{\lambda_J} \right\rfloor\right]$$

where λ_J, λ_K—mesh spacing along the meridian in the original and, respectively, calculated mesh. The sign denotes rounding to the nearest floor. After the calculation of $H_K[i,j]$, the sum is calculated for the entire mesh,

$$S_K = \sum_i \sum_j \left| H_K[i,j+1] - H_K[i,j] \right|$$

indicating the dependence of the S_K height change sum on mesh size λ_K. The results (Figure 5), suggest that the change in mesh spacing leads to a change in the height change sum.

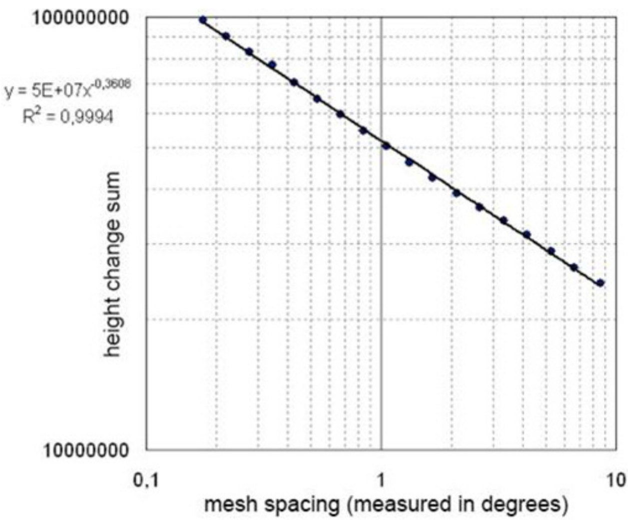

Figure 5. Dependence of the height change sum along the meridians on mesh spacing.

Given that μ can vary considerably across different regions, accurate measurement of μ was crucial for model prediction accuracy. Indeed, the best correlations between τ and τ_{MP} resulted from μ values differing by an order of magnitude in different terrain (Figure 6).

Testing the Model

The next step is to test the model by applying it to historical data. If it makes accurate predictions, then we can believe the model is correct. We need an example of a spatial market system that is sufficiently large (spatially and temporally) and well-described and analyzed. There is a system that adequately satisfies all these conditions: the famous trade-route system known as the Silk Roads.

The Silk Roads system is a unique phenomenon. It is the most long-lived, large-scale trade route system in the world. It was not only a merchant route but also a basic factor of the unification of Afroeurasia (Chase-Dunn, Hall 1997). The Silk Roads had complex historical dynamics, with three main epochs: the ancient Silk Road (II B.C.E.–III C.E.), Islamic conquests (VI–IX C.E.), and the Mongol Empire (XI–XIV C.E.). Each epoch saw intensification of trade on the Silk Roads, and trade diminished at the end of each epoch. However, the pattern of the main trade routes was different in each epoch, and these changes will be the object of our further attention.

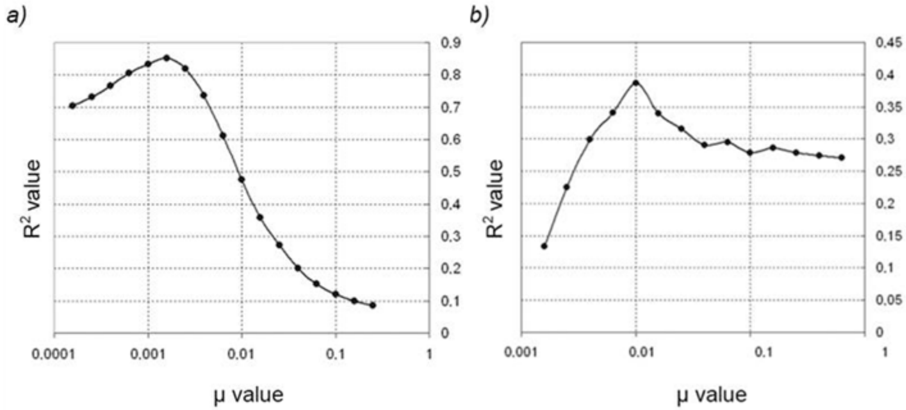

Figure 6. R^2 for correlation between travel times as predicted by the model and as reported by Marco Polo, at different μ. The value corresponding to the maximum R^2 is taken as the coefficient for zones: (a) zone of broad-leaved forests; (b) mountain zone.

What were the main factors affecting the intensity and location of the routes? Although a large number of factors could be considered, we can reduce the system and select one or several prominent, governing factors.

Our calculations show that the main factor that predetermined the location of the Silk Roads was the spatial layout of large-scale empires. This point of view is similar to that of Jerry Bentley (1993), who examined cross-cultural links, such as the Silk Roads, and implied that the large-scale empires were presumably the main factor behind the Silk Roads' existence and dynamics:

> "The era of the ancient silk roads—roughly 200 B.C.E. to 400 C.E.—thus figures as the first major period of cross-cultural encounter. The consolidation of large, imperial states pacified enough of Eurasia that trading networks could safely link the extreme ends of the landmass....Beginning about the sixth century, however, a revival of long-distance trade underwrote a second round of intense cross-cultural encounters. The revival of cross-cultural dealings depend again on the foundation of large imperial states....The second period did not so much come to end as it blended into a new era—roughly 1000 to 1350 ... The distinctive feature of this era ... had to do with the remarkable military and political expansion of nomadic peoples, principally Turks and Mongols, who established vast transregional empires and sponsored regular interactions between peoples..." (Bentley 1993: 26–27).

Unquestionably, the empires themselves are not independent phenomena. There are many other factors that induce the rise and fall of empires. However, we do not take these factors into account when testing the model. We only derive that the existence of empires predetermines the pattern of commodity-flows. Large empires demand and supply more goods for prestige-goods trade networks, support roads and other infrastructure, and bring stability to the areas of commerce. There are many other components of the beneficial influence of large empires, but fortunately, most of them can be easily described in terms of the spatial trade model, which was proposed above: *Large-scale empire increases the commodity conductivity of the territory inside its bounds.*

This means that the commodity-conduction coefficient of a geographical region increases when this region belongs to an empire (e.g., after being conquered) and decreases when the empire loses control in this region (e.g., after imperial collapse). An increase of conductivity results in a reduction of transportation costs (as was discussed above), which in turn reduces the expenses of traders. Imperial interlinks become faster and safer, and the roads inside the imperial bounds become more attractive for traders—even if a shorter path outside the empire exists.

However, many spatial and temporal factors can change. The original location of main commodity-flows can become less profitable and, therefore, unstable after the formation of an empire nearby. The general flow pattern can change considerably from epoch to epoch as the empires rise and fall, even if the locations of the main commodity producer and consumer remain constant.

As a result, the simulation of the Silk Roads involves the following assumptions:

1. The model of spatial trade is used as a mathematical basis for the dynamics of the Silk Roads.
2. Initial conductivity of each geographical point is estimated using the conditions of the respective territory.
3. Three historical epochs are considered: the epoch of the ancient Silk Road, the epoch of Islam, and the epoch of Mongols.
4. For each epoch, two points are assigned: the point of the main production of commodity (silk) and the point of its consumption.
5. For each epoch, the layout of main empires is assigned, and the commodity conductivity coefficient increases inside the empire during the respective epoch.

Mathematically, the model corresponds to the parabolic equation with a point source, point sink, spatially variable coefficients, and the boundary condition of the flux equaling zero. The simulation was counted out using the finite-difference methods.

The locations of the Silk Roads for each epoch are given in Figs 7–9. In each, the known geographic boundaries of an epoch's main empires are illustrated, followed by the numerical results of the model for that epoch. Finally, the third figure in each panel corresponds to the real historical data (World History, 1956-1958) on the location of major trade routes.

From II B.C.E.–III C.E. (Figure 7), during the epoch of the ancient Silk Roads, the main empires were the Roman, Parthian, Kushan, and Han Empires. It was the first time in history that Eurasia became an integrated system. However, only prestige-goods networks were integrated. Military networks and bulk goods networks of these world-systems were much smaller than those for prestige-goods (Chase-Dunn, Hall 2003).

From VI–IX C.E. (Figure 8), during the epoch of Islam propagation, we treat the whole Islamic region as one empire with no borders. Other empires of the period are Byzantine Empire and Tan Dynasty in China. The Silk Roads flourished during this era because Islam is an auspicious religion for merchants.

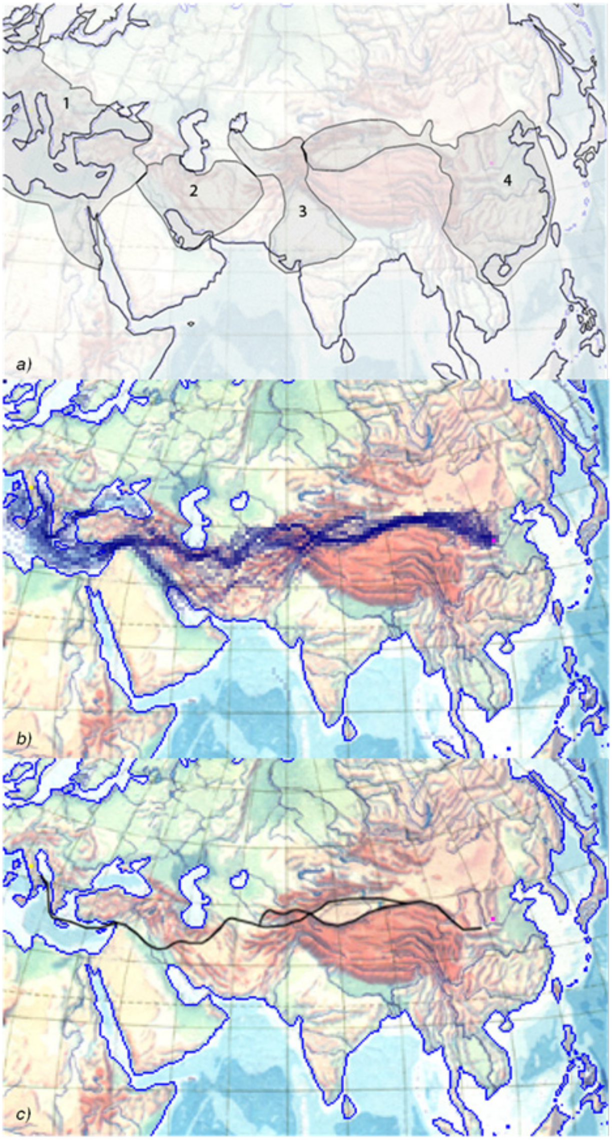

Figure 7. II B.C.E.–III C.E. The epoch of the ancient Silk Roads. (a) Main empires of the epoch: 1.Roman, 2.Parthian, 3.Kushan, and 4.Han Empires. (b) Calculations for commodity flow; the darker the point is the higher the commodity transportation through the point is. (c) Historical data on the Silk Roads location during this epoch.

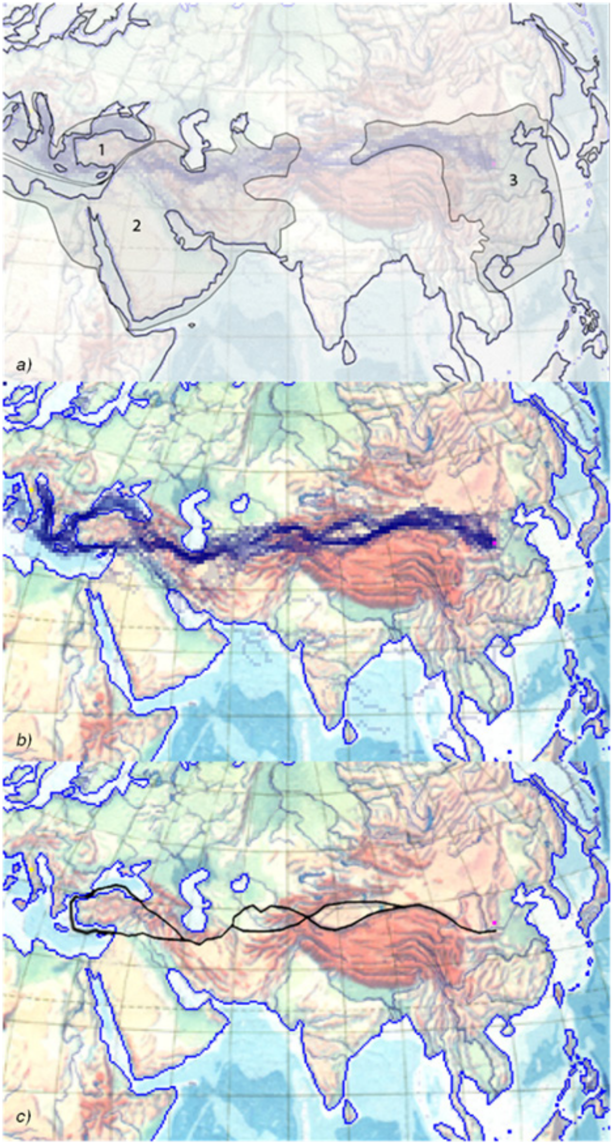

Figure 8. VI–IX C.E. The epoch of Islam propagation. (a) Main empires of the epoch: 1.the Byzantine Empire, 2.Islam states, 3.Tang Empire. (b) Calculations for commodity flow; the darker the point is the higher the commodity transportation through the point is. (c) Historical data on the Silk Roads location during this epoch.

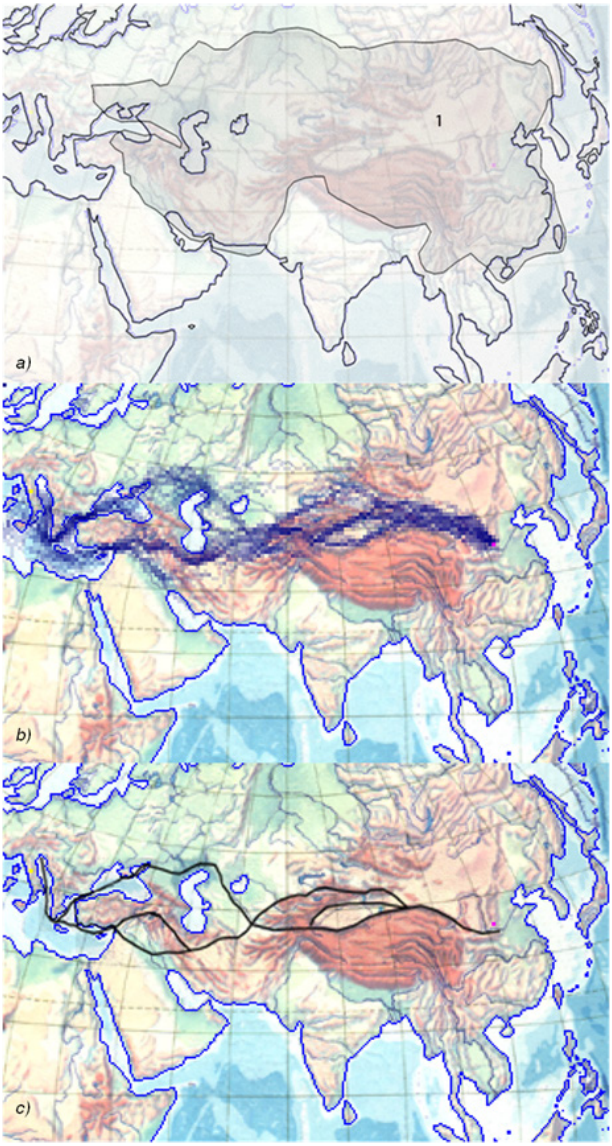

Figure 9. XI–XIV A.D. The epoch of the Mongol Empire. (a) The huge Mongol empire was the main power of the epoch. (b) Calculations for commodity flow; the darker the point is, the higher the commodity transportation through the point is. (c) Historical data on the Silk Roads location during this epoch.

Figure 10. Changes in commodity conductivity after 15th century. As European naval empires grow, Indian Ocean conductivity increases, whereas overland conductivity decreases.

From XI–XIV A.D. (Figure 9), the huge Mongol empire was the main power of the epoch. It was the first time that military networks reached such a scale. Due to the Mongols' activity in the region, a stable, intensive Silk Roads route to the north of the Caspian Sea appears for the first time. The system of main routes converges upon Venice as the destination point in Europe.

After the fall of the Mongol Empire, the system of the Silk Roads degrades and never resumes its former importance. The model gives a curious explanation of this fact. According to the model, European maritime trade was the cause of this process. If we increase the commodity-conduction of the Indian Ocean as European empires' navies expand, overland trade decreases while transoceanic trade increases (Figure 10).

Conclusions

Let us say a few words in conclusion. The obtained results are intriguing, but they are still rough. We used a rather simple model that involves only one factor: large empires. It is impossible to expect precise prediction at every point as models do for physics. To adjust the results, we must take into account other factors, add new equations, and expand the model. However, new equations must be proposed only after successful approbation and testing; it is not reasonable to hurry model expansion. The results presented here show that the model is valid in principle, so our further work is to apply it to more precise historical data, propose more effective methods of conductivity evaluation, and try to obtain actual predictions.

Acknowledgment

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