

# **Application of Mathematical Models to English Secular Cycles**

Michael Alexander

*Pfizer Corporation*

## **Abstract**

Secular cycles are 2–3 century oscillations in population associated with periodic state breakdown. Turchin and Nefedov (2009) find two secular cycles in England: the Plantagenet (1150–1485) and Tudor-Stuart (1485–1730). This paper proposes modified dating for these cycles (1070–1485 and 1485–1690) and two adjacent cycles: Anglo-Saxon (ca. 880–1070) and mercantile (1690–undetermined). Several mathematical models for secular cycles were investigated for their ability to model trends in population, state strength, elite number, and internal instability during the Plantagenet and Tudor-Stuart cycles with the modified dating. The demographic-fiscal model (Turchin 2003) uses just six adjustable parameters and gave as good a fit to the population data as a polynomial model with ten parameters. Sociopolitical instability has been proposed as the primary factor in delayed population recovery following secular decline. This did not seem to be the case for England, at least when instability was measured in terms of large-scale events. For such events the dominant pattern was the fathers and sons cycle, not the much longer secular cycle.

## **Introduction**

Since the middle of the last century it has become clear that Europe experienced a series of long-term fluctuations in prices and population during the second millennium (Postan 1950, Braudel 1973; Le Roy Ladurie 1974; Abel 1980, Fischer 1996). Such oscillations, which include similar cycles in political and social variables, are known as secular cycles (Turchin and Nefedov 2009, 5) and are the subject of this paper. Secular cycle theory has been used to detect “more than 40 demographic cycles in the history of various ancient and medieval societies of Eurasia and North Africa, thus demonstrating that the demographic cycles...should be regarded as a general feature of complex agrarian system

*Corresponding author's e-mail:* [mikebert@yahoo.com](mailto:mikebert@yahoo.com)

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dynamics” (Korotayev 2006, 4). Adjacent secular cycles in England, France, and Russia and three sequential cycles in ancient Rome have been characterized in some detail (Turchin and Nefedov 2009; Baker 2011).

A secular cycle can be considered as a pair of trends, one integrative and one disintegrative, each of which has two phases (Turchin and Nefedov 2009, 19–20). The integrative trend comprises an expansion phase during which most demographic, sociopolitical, and economic indicators are rising and a stagflation phase during which these rising trends gradually diminish. The disintegrative trend begins with a crisis phase, often accompanied by state collapse. Post-crisis recovery is called the depression phase, which lasts until conditions emerge that allow a new secular cycle to begin (Turchin and Nefedov 2009, 19–20). The boundary of a secular cycle typically occurs when the depression phase of one cycle transitions into the expansion phase of the next secular cycle.

To identify a secular cycle, one must quantify oscillations in measureable quantities plausibly related to at least one of four fundamental variables: population, state strength, elite dynamics, and sociopolitical instability (Turchin and Nefedov 2009, 33). Secular cycle boundaries approximately correspond to cycle troughs in these and related variables. In one sense, secular cycles are population cycles (Turchin 2003, 160; Korotayev et al. 2006, 47–50). In another sense, they are cycles of state rise and decline that can be tracked with measures of state strength. Economic inequality exhibits cycles (Turchin and Nefedov 2009, 36, 82) that can serve to characterize elite dynamics. Various measures of sociopolitical instability have been proposed (Sorokin 1937, Turchin 2012) which can be used to identify cycles in these traits. Finally, economic measures such as price level (Fischer 1996) also show secular cycles and may be used as an adjunct to the four fundamental variables.

Recently Peter Turchin, Sergey Nefedov, Andrey Korotayev, and others have developed mathematical models to explain secular cycles. This paper focuses on Turchin’s (2003) modeling work which has made use of Jack Goldstone’s (1991) demographic-structural theory (DST) of periodic state breakdown. DST ties together cycles in population, prices, state strength, and sociopolitical instability in a process in which elite class dynamics plays a central role.

DST describes how rising population relative to available farmland leads to excess workers and falling real wages. Excess agricultural labor implies a shortage of farmland leading to rising rents and land values. Both of these result in rising economic inequality, for which the ratio of per capita GDP to average wage is a good proxy.

Rising land values and falling real wages allow financially-savvy landowners and employers of agricultural labor to earn windfall profits, leading to elite upward mobility and elite proliferation. Heightened social mobility and larger

numbers lead to increased competition and factionalism amongst elites (Goldstone, 1991, 117). Rising rents are not favorable to less financially astute landowners who have leased their land in long-term contracts. They tend to see their real income decline and may be often forced to sell land to maintain their standard of living (Goldstone, 1991, 112). They grow poorer while others grow richer, creating disaffected nobles predisposed to take sides in challenges to state authority.

At some point, population reaches the limits of what the agricultural land can support, causing a declining per capita production. This then leads to intense competition over a no longer growing economic pie. Competition turns to conflict, resulting in state collapse and population decline. The problems caused by excess elites are eventually resolved over a lengthy period, which can contain multiple episodes of temporary state recovery followed by further collapse, until elite conflict is finally brought under control and a new cycle can begin.

This means an important requirement for a successful state is the pacification of elites, which becomes increasingly difficult as elite numbers rise. External wars can perform a pacification role by directing the energies of military elites externally. Wars require money and so a relevant measure of state strength might be government revenue as a percentage of GDP.

Several secular cycle models incorporating aspects of DST have been proposed (Turchin 2003, 2013). The principal objective of this paper is to apply a few of these mathematical models to empirical historical data in order to test the validity of these models. I use England's Plantagenet (1150–1485 CE) and Tudor-Stuart (1485–1730 CE) secular cycles described by Turchin and Nefedov (2009) as the test cases, as these are particularly well-studied and highly documented periods. The models are first order differential equations in time and require initial values, which are the values at the cycle boundaries. To optimize model performance, it is necessary to date the boundaries correctly. Optimally, the periods before and after a proposed boundary should be consistent with the definition of the depression and expansion phases, respectively (see Turchin and Nefedov 2009, 33). For example, the 1485 boundary between the Plantagenet and Tudor-Stuart cycles is very sound, because Turchin and Nefedov have characterized complete secular cycles on both sides of the boundary. Thus, the period before 1485 necessarily fits the description of a depression phase, while the period after fits an expansion phase. To determine sound values for the other boundaries it will be useful to identify secular cycles before and after these two. To this end, an Anglo-Saxon cycle ending around 1070 and a mercantile cycle beginning around 1690 are proposed.

Turchin and Nefedov took a chronological approach, dealing with each English secular cycle in a separate chapter. This approach deemphasizes the definition of

a secular cycle as “historical demographic-social-political oscillations” (Turchin and Nefedov 2009, 5). Such oscillations should be visible in plots of time series data. The empirical data in this paper is presented in terms of long-term plots of time series that span multiple secular cycles so that the oscillations (and the boundaries implied by them) can be seen.

The paper is organized as follows. First, the mathematical models are described. Empirical results follow in four subsections, each of which deals with one of the fundamental variables defining secular cycles defined above. The first empirical section focuses on population and other variables as needed to date the secular cycles. The second looks at the cycle in terms of state strength using revenue/GDP and growth rate in population or state area. Using the results presented so far I make the case for an Anglo-Saxon cycle dated 880–1070. I then evaluate a secular cycle model involving population and state strength for its ability to fit the data for this period.

The third empirical section looks at elite class dynamics. Data on elite numbers are presented and two models for them evaluated. I argue that the Plantagenet cycle begins with the end of the Anglo-Saxon cycle in 1070, not in 1150 as argued by Turchin and Nefedov (2009). The final empirical section deals with sociopolitical instability and a fourth model is tested for its ability to fit the instability data. The paper concludes with a discussion of the model results and comments on what I term the 'mercantile cycle' as a transition between the preindustrial agrarian secular cycle and a possible modern, industrial version of the secular cycle. A brief section summarizing the main arguments of this paper concludes the article.

## Theoretical Review

### Demographic-state Dynamics

The simplest of the mathematical models for secular cycles is the demographic-fiscal (DF) model (Turchin, 2003, 123):

- (1)  $dN/dt = r N (1 - N/N_{MAX})$
- (2)  $N_{MAX} = K + C S / (\kappa + S)$
- (3)  $dS/dt = \gamma (1 - N/ N_{MAX}) N - \beta N$

Here  $N$  is population and  $N_{MAX}$  is the population carrying capacity, or the maximum population that can be supported when all arable land under state sovereignty is under cultivation.  $S$  is state resources,  $\gamma$  is tax rate,  $r$  is the rate constant for population growth in the absence of limiting factors, and  $\beta$  is per capita state expenditure.  $K$  refers to the maximum population that can be

supported in the absence of the state.  $C$  refers to the maximum amount of additional population that can be supported when the state is strong ( $S \gg \kappa$ ). The parameter  $\kappa$  indicates how improvement in  $N_{MAX}$  depends on  $S$ . The value for  $\gamma$  does not affect the operation of the model and so it will be eliminated (set equal to unity). In this case,  $N$ ,  $K$ ,  $C$  and  $\kappa$  have units of persons,  $r$  has units of reciprocal years, and  $\beta$  is dimensionless. State strength ( $S$ ) reflects accumulated state resources in units of labor-power (person-years). Multiplying  $S$  by  $\gamma$  will put  $S$  into money terms.  $N_{MAX}$  is the equilibrium population size, or the maximum population that could be supported when all arable land under state sovereignty is under cultivation. In this paper equation 2 will be simplified:

$$(4) \quad N_{MAX} = K + C * (S > 0)$$

The expression  $(S > 0)$  is a logical function that has the value 1 when true and 0 when false. Thus,  $N_{MAX}$  has two values,  $K + C (= M)$  when  $S$  is positive (during the integrative trend of the secular cycle) or just  $K$  when  $S$  is not positive (during the disintegrative trend). This simplification eliminates an adjustable parameter ( $\kappa$ ) without affecting the utility of the model, as will be seen.

When the state is not functioning ( $S \leq 0$ ) people must live near strongholds for safety and only the land near the stronghold can be farmed. The maximum population that can be supported in this situation is given by  $N_{MAX} = K$ . When a functional state is present ( $S > 0$ ) it establishes order, increasing the size of the territory that can be farmed and increasing  $N_{MAX}$  (Turchin 2003, 120).

Growth of  $S$  is described in equation 3 in terms of a fiscal balance. Equation 3 has two terms, which Turchin interprets as state income and expenditure. Both are expressed in units of labor-value per year (persons). The fiscal interpretation of  $S$  implies that it could be measured in terms of money. The early 13<sup>th</sup> century peak in  $S$  given by the model is about 59 million person-years. The wage level in 1209 was about £2 per annum (Clark 2015). If a labor force participation rate of 0.25 and a tax rate of 2% are assumed (Ormrod, ESFD) this works out to about £0.6 million, or about six times the amount of Richard II's ransom payment (Encyclopedia Britannica-A). It is unlikely the early 13<sup>th</sup> century state would have been able to raise sums approaching this level, casting doubt on the fiscal interpretation of  $S$  for the Plantagenet cycle.

The monetary value for the 16<sup>th</sup> century peak in  $S$  was about £1.9 million. Peak state occurred during the 1540's when Henry VIII dissolved the monasteries and sold them to private parties. Over the 1540's the state collected about £3.3 million more than it had in the decade before, presumably because of Henry's action. By the 16<sup>th</sup> century the peak state was now capable of generating surplus funds in excess of the quantity consistent with the size of  $S$  expressed as money.

This suggests a fiscal interpretation of  $S$  is not unwarranted for the Tudor-Stuart cycle.

Another interpretation for  $S$  might be “reserve state legitimacy” obtained by dividing  $S$  by the population, which would have units of years (of continued legitimacy). The value of  $S/N$  at the time of peak  $S$  around 1200 was about 13 years. That is, the legitimacy of the state gave it a minimum of 13 more years of integrity. The interpretation given to  $S$  has no effect on model function because  $S$  operates as a binary variable in the model.

### Elite Dynamics

Although elite dynamics play a major role in DST, the DF model has no explicit role for elites. Demographic structural models in which elites play explicit roles have been proposed (Turchin, 2003, 208–11). These models are more complex. In them, elites perform the same function (extractor of surplus value) as the state in the DF model and show similar cycles, except with a lag. These will not be examined in this paper. Examined instead is a simple economic model (Turchin 2013, 251) that gives total elites ( $E$ ) as a function of relative wage  $w$ .

$$(5) \quad dE/dt = \mu_0 N (w_0/w - 1)$$

Relative wage  $w$  is wage divided by per capita GDP.  $\mu_0$  and  $w_0$  are arbitrary constants that are fit to the data and  $N$  is population in millions. The parameter  $\mu_0$  can be interpreted as a base rate of upward mobility (new elites per million population per year) and  $w_0$  as the relative wage where the economic power of workers and employers is balanced. When unoccupied land is available, the bargaining power of labor should be higher than when land is fully occupied. If this is so, real wages in the early stages of a secular cycle would be maintained at levels close to  $w_0$  and elite population would grow no faster than the population as a whole. As the available land is settled, the labor supply outstrips the amount of land that needs to be worked and worker bargaining power declines. Elites gain the upper hand,  $w$  falls below  $w_0$  and elites proliferate. When population begins to fall as a result of state breakdown and the collapse of  $N_{MAX}$ , worker bargaining power increases,  $w$  rises and elite numbers start to fall because of downward economic mobility.

Another process affecting elite numbers might be the diversion of elites into nonmilitary careers, particularly monasticism, which would serve to directly reduce the secular elite number. Since monks were nominally celibate and their leaders came from the nobility, the monastic project required a continuous influx of elites to maintain it. This influx could be substantial. When St. Bernard joined the Cistercians in 1113, he brought thirty of his friends and relatives with him

(Pirlo 1997). The Servite order was founded in 1233 by seven sons of noble families in Florence (Catholic Encyclopedia). In addition to nobles withdrawn to establish new monasteries, established monasteries required an influx of new members to replace those who had died. For modeling purposes it was assumed that monasteries founded in the last fifty years drew new recruits from secular elites to replace those who had died. Based on an elite lifespan of about 48 years (Cummins 2014, 28) a typical residence time for a monk might be about thirty years, implying a ~3% replacement rate. Assuming an average of ten elites were employed to found and then manage a monastery, the following model was proposed:

$$(6) \quad dE/dt = k_M E - 10 * dM_s/dt - 0.03 * 10 * [M_s(t) - M_s(t-50)]$$

Here  $k_M$  is the natural rate of increase for elites ( $yr^{-1}$ ) and  $M_s(t)$  is monastery number at time  $t$ .

### **Sociopolitical Instability**

State breakdown “occurs when a state crisis leads to widespread overt conflict including a combination of elite revolts, intra-elite struggles, and popular uprisings” (Goldstone 1991, 10). Turchin and Korotayev (2006, 122) have proposed a model for the interaction between internal violence, population, and the state:

$$(7) \quad dN/dt = rN(1 - N/N_{MAX}) - \delta NW$$

$$(8) \quad dW/dt = aN^2 - bW - \alpha S$$

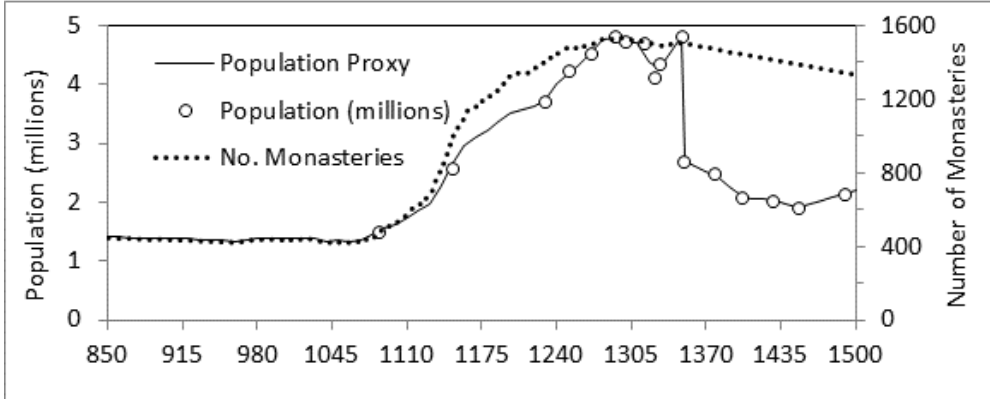
$$(9) \quad dS/dt = [(1 - N/N_{MAX}) - \beta] N$$

$$(10) \quad N_{MAX} = K_{MAX} - cW$$

Here  $N$ ,  $S$ ,  $r$  and  $\beta$  have the same meaning as in the DF model.  $K_{MAX}$  is equivalent to  $N_{MAX}$  in the DF model.  $W$  represents the internal instability associated with state breakdown. The other parameters are arbitrary (adjustable) constants. As in the DF model, population growth is limited by the carrying capacity ( $N_{MAX}$ ) of the state. In this “war model”,  $N_{MAX}$  is an explicit function of  $W$ , but since  $W$  is affected by  $S$ ,  $N_{MAX}$  is still an implicit function of  $S$ . Like the DF model, this model does not include an explicit role for elites. In the next section these models are explored for their ability to explain historical data.

## Empirical Results

### The Secular Cycle in Terms of Population



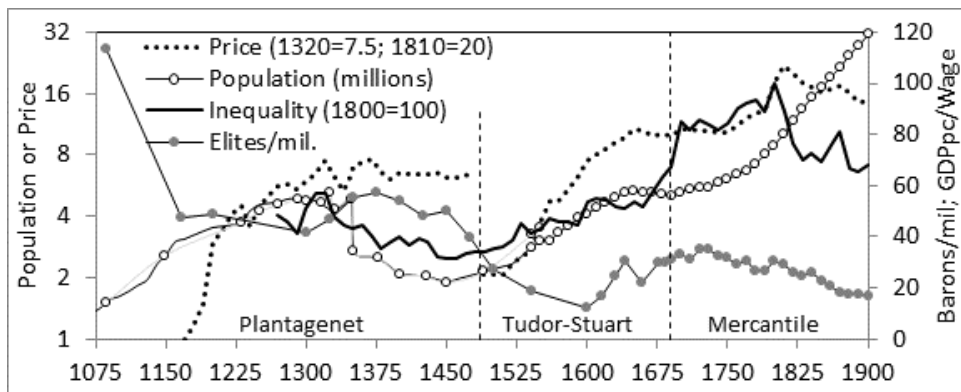
**Figure 1.** Population and monastery growth and the monastery-based population proxy. Monastery trend from a compilation of monastery foundings by author used to interpolate between century-spaced values from Buringh and van Zanden (2006). Population data from Broadberry et. al. (2014); Turchin and Nefedov (2009, 36–7). Population proxy obtained by using the monastery trend to interpolate between population data points.

Since monastery growth required new land to be brought under cultivation and settled by laborers to work the fields, increases in monastery numbers are likely to be associated with increases in population. Monastery numbers and population each increased by the same ratio over the 1086 to 1300 period (Figure 1). Based on this correlation between the number of monasteries and population size, the former was used to interpolate between population data points in Figure 1 to obtain a continuous population trend over the 1086 to 1300 period. The population estimate before 1086 shown in Figure 1 was obtained by assuming population was proportional to monastery number. The monastery proxy suggests that population began to rise shortly after the Norman conquest (Figure 1), implying that the conquest might denote the start of a secular cycle in population.

Figure 2 shows a plot of population, a price index, the numbers of high-ranking elites (barons) per million total population, and economic inequality (GDP/wage) for the 1075–1900 period. Population shows two cycles, from ca. 1075 to 1485 and 1485–1690/1730, that approximately correspond to the Plantagenet and Tudor-Stuart cycles. The slow population rise following the 1690

trough accelerated after 1730, indicating the start of a new secular cycle, what I term the mercantile cycle.

Turchin and Nefedov (2009, 33) describe the general pattern shown by prices during secular cycles as an initial rise to a variable peak followed by a flat or downward trend. The price index follows this pattern during all three secular cycles shown in Figure 2. Rising prices roughly tracked population growth during the first two cycles and during the first half of the mercantile cycle.



**Figure 2.** Trends in population, price, elite numbers, and inequality 1075–1900. Post-1540 population from Mitchell (1988). Elite numbers pre-1300: Painter (1943), Given-Wilson (1987); 1300–1500: McFarlane (1973); 1500–1640: Mingay (1976); after 1660: Beckett (1986). Price: Clark (2015) and Farmer (1956, 1957, 1969, 1988). Inequality is GDP (Broadberry et al. 2014; Williamson 2015) divided by wage (Clark 2015). Dashed lines indicate cycle boundaries in 1485 and 1690.

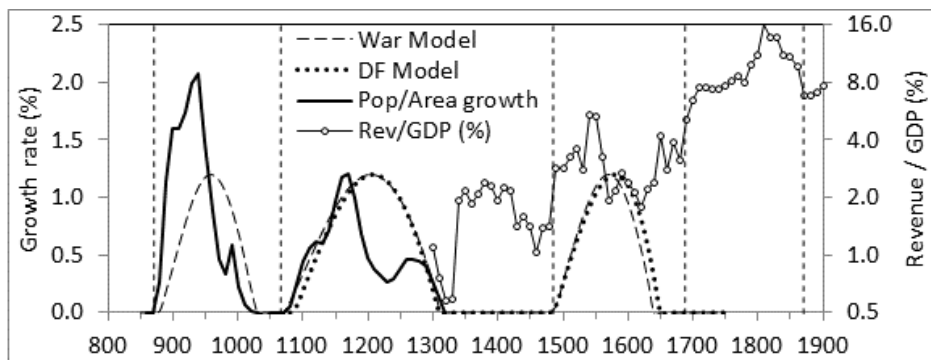
Turchin and Nefedov (2009, 33) describe elite dynamics over a secular cycle as initially declining relative elite numbers followed by a “golden age” of rising numbers, leading to a period of high numbers and rising competition, and ending again with decline. Elite numbers fit this description for both the Plantagenet and Tudor-Stuart cycles: declining in the early stages of the cycle, rising to a peak around the crisis phase and then declining afterward. Elite numbers began to rise after 1660 (Figure 2), which is inconsistent with the 1730 ending date for the Tudor-Stuart cycle proposed by Turchin and Nefedov. Inequality shows a similar pattern, consistent with an earlier boundary between the Tudor-Stuart and mercantile cycles. Population, price, elite numbers, and inequality show respective troughs of 1690/1730, 1690/1740, 1660, and 1660, from which a consensus date of 1690 is proposed.

A 1690 boundary between the Tudor-Stuart and mercantile cycles can be justified by political events as well. The 1642–1651 English Civil War (Wedgewood 1970; Gardiner 2006) is a prime example of state collapse and was one of the base cases around which DST was developed (Goldstone 1991, 63–169). Turchin and Nefedov (2009, 97) identify a crisis phase over 1640–1660 reflecting this fact. The post-crisis depression phase lasts until the elite conflict that led to state collapse is resolved. The restoration of the monarchy in 1660 ended the English revolution crisis by bringing back the Stuart dynasty that had been ousted eleven years earlier. Less than three decades later the Stuart monarch was again rejected in the 1688 Glorious Revolution (Pincus 2009). This time there was no restoration and no further revolution or even a revolutionary situation for more than a century after 1688, implying that the political issues that gave rise to the English Civil War and Glorious Revolution had been settled, making it a state strength/legitimacy (S) indicator for a secular cycle boundary.

### **The Secular Cycle in Terms of State Strength**

Figure 3 shows plots of two measures of state strength (S). One is state revenue as a percentage of GDP, which shows cycles corresponding to the Tudor-Stuart and mercantile cycles. The other is a measure based on the percent growth rate of the population or territorial extent of the kingdom of England. Turchin and Nefedov (2009, 33) characterize state strength as increasing during the expansion phase; high, but declining during the stagflation phase; in collapse during the crisis phase; and intermittent during the depression phase. During the expansion phase, when state strength is increasing, the rate of population growth is also increasing (Turchin and Nefedov 2009, 33). It follows that population growth rate might serve as a proxy for state strength for the period before revenue data are available.

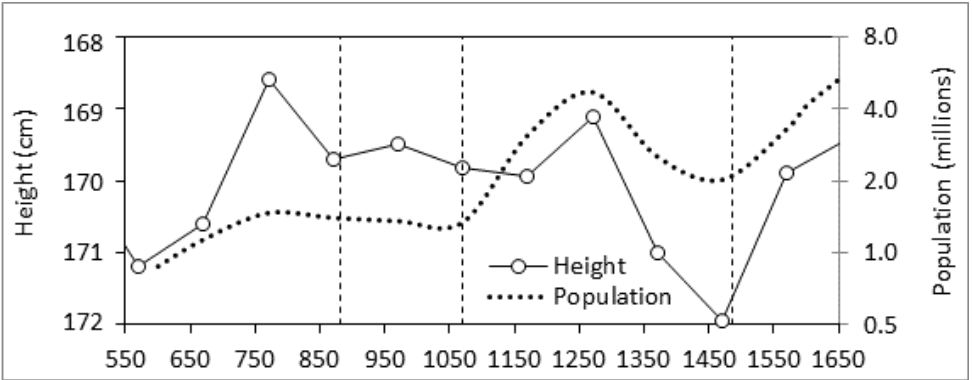
Growth rates were obtained as the slope from a lagging, fifty-year moving regression of the population trend. An analogous argument can be made for growth in territorial extent, since this also increases along with population. During the ninth and tenth centuries when the English were engaged in reconquering England from the Danes, the rate of expansion of the size of the English state was used as a proxy for state strength exactly as was done for population. Figure 3 uses this idea to show two secular cycles in state strength, one for the Plantagenet cycle and one for a possible Anglo-Saxon secular cycle before it.



**Figure 3.** Two measures of the state 800–1900. Area growth is the rate of increase in the fraction of England controlled by Wessex (author’s estimates). Revenue from Ormrod, O’Brien, and Hunt (ESFD) and Mitchell (1988). Population data from Broadberry et. al. (2014); Turchin and Nefedov (2009, 36–7).

*The Anglo-Saxon cycle.* The population during Anglo-Saxon times is unknown. The Malthusian relation between population and living standards that forms the basis for the secular cycle implies that a population’s nutritional status should vary inversely with the population size. Height is an excellent proxy for nutritional status and might be used as a means to estimate population trends. The known population cycles during the high medieval and early modern periods approximately correspond to cycles in height (Figure 4). Another cycle in height and the monastery population proxy can be seen over the late sixth to ninth centuries. The Anglo-Saxon cycle falls into the period between the end of this ostensible secular cycle in the ninth century and the beginning of the high medieval cycle in the late eleventh century.

The cycle is proposed to start when Alfred the Great’s victory at Edington in 878 re-established Anglo-Saxon control over Wessex after the Danish invasion in 865 had overrun East Anglia, Northumbria, and half of Mercia (Keynes and Lapidge 1983: 20, 26). From this base, three generations of Alfred’s family would go on to conquer all of England over the next seventy years (Foot 2011, 19–20, 151; Encyclopedia Britannica-B; Stenton 1988, 324–27). Conquest achieves several of the indicators for an expansion phase: rising population, pacified elites, and rising state strength. The period from 880 to 950 would then correspond to the expansion phase of the Anglo-Saxon cycle.



**Figure 4.** Trends in average height of men and estimated population. Population from monastery proxy before 1300 as described in Figure 1. Height data averages from male skeletons in the UK, Germany, Austria, Benelux, Northern France, and Switzerland from Koepke and Baten (working paper).

The Anglo-Saxon cycle necessarily came to an end with the Norman Conquest. The fact that England had been conquered by Danes half a century earlier suggests a disintegrative trend was already in progress then. That the secession crisis of 957 was resolved without bloodshed (Smith 2012, 95) implies that the country remained in an integrative trend at this time. Sorokin (1937, 596–99) records no internal instability events between 975 and 1015, arguing against the disintegrative trend beginning much earlier than 1013.

The political history of England in the 11<sup>th</sup> century is suggestive of heightened intra-elite competition/conflict. The Danish conquest in 1013 was accomplished in less than a year (Anglo Saxon Chronicle). This rapid success suggests that significant political divisions among elites existed at this time. A power struggle between King Edward the Confessor and Earl Godwin almost led to civil war in 1051 (Anglo Saxon Chronicle). Edward had designed his nephew Edward as heir in 1056, but he died in the next year, leaving his son Edmund as the presumed next in line to the throne (Anglo Saxon Chronicle). When Edward the Confessor died in 1066 he was succeeded by Earl Harold Godwinson, who was not of the royal line. His claim was challenged by Harald Hardrada of Norway and William of Normandy, both of whom invaded England (DeVries 1999; Huscroft 2009) which led to the end of the Anglo-Saxon state. These events suggest that 11<sup>th</sup> century Anglo-Saxon England was suffering from a degradation of state legitimacy ( $S=0$ ), which made royal succession difficult and left the country vulnerable to conquest.

The Norman invasion served as a cycle-ending state collapse like the wars of the Roses at the end of the Plantagenet cycle, because it achieved a solution to the central problem of the depression phase: destabilizing elite competition caused by an excessive number of elites. English elites were largely eliminated following the Norman Conquest: almost all of the 1400 men listed as tenants-in-chief in the 1086 Domesday survey came from Normandy (National Archives). In contrast, the Danish conquerors co-opted indigenous elites rather than eliminating them, preventing a resolution of the depression phase. For example, the Anglo-Saxon lords Leofric and Godwin were given earldoms (Lawson 2004, 162, Williams 2004). The reduction of English elites by the Norman invasion resolved the problem of intra-elite conflict and enabled the start of a new secular cycle. The Plantagenet cycle is assumed to have begun with the end of the Anglo-Saxon cycle in 1070 in the next subsection.

*Population and state strength model fit.* The modified DF model is reproduced below:

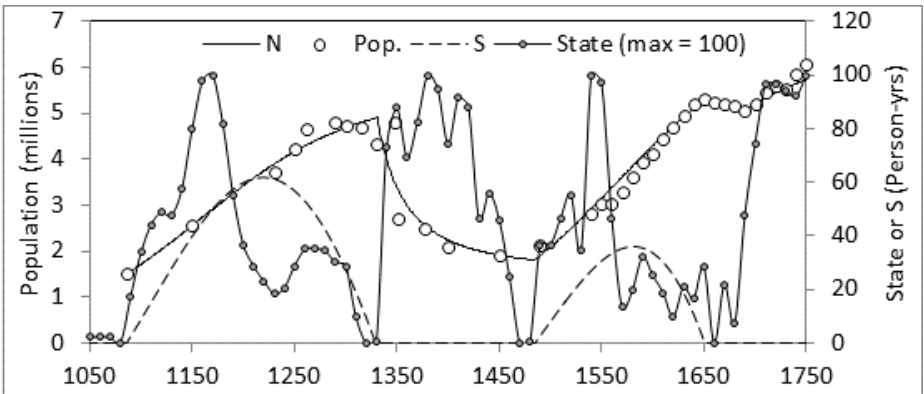
- (11)  $dN/dt = r N (1 - N/N_{MAX})$
- (12)  $dS/dt = (1 - N/N_{MAX}) N - \beta N$
- (13)  $N_{MAX} = K$  when  $S = 0$  and  $M$  when  $S > 0$

Population dynamics depend on  $M$  during the integrative trend and on  $K$  during the disintegrative trend. Values for  $M$  of 5.4 and 7 (million) were needed to fit the population data during the Plantagenet and Tudor-Stuart integrative trends. A  $K$  value of 1.65 was needed to fit the disintegrative trend of the Plantagenet cycle. Because  $K$  refers to the population of an area unaffected by absence of the state, its value will be proportional to agricultural productivity. One factor affecting productivity is the fraction of arable land under cultivation. At the beginning of the second millennium, the common practice was the two-field system in which half of the arable land was left fallow at any given time. The three-field system was gradually introduced over the later Middle Ages (Allen 2001: 44). By the 15<sup>th</sup> century, the fraction of arable land in production had risen to 0.6 (Broadberry et al. 2011: 30). If it is assumed that 50% of arable land was in production during the 11<sup>th</sup> century, the value of  $K$  in 1086 would be five-sixths of 1.65 or 1.38.

Another factor affecting productivity is average yield per acre. Average wheat yields during the 17<sup>th</sup> century were more than twice late medieval values (Broadberry et al. 2011, A30). The fraction of arable land in use increased by nearly a third (Broadberry et al. 2011, 30). These improvements increased  $K$  from 1.65 during the Plantagenet cycle to 4.75 during the Tudor-Stuart cycle. During the 18<sup>th</sup> century, yields rose another 25% and area under cultivation by 7%

(Broadberry and van Leeuwen 2010), which further increased  $K$  to 6.44, which was used for the mercantile cycle.

The  $\beta$  parameter controls the timing of the population peak. Values of 0.31 and 0.45 give population peaks in 1315 and 1650. A single value of 0.013 for  $r$  was found to fit the population trends for the entire 1086–1750 period. In all, six adjustable parameters were determined by curve fitting:  $r$ , two values of  $M$  and  $\beta$ , and one value of  $K$  (the other two were derived from the first and so were not adjustable). The 1070, 1485 and 1690 boundaries provided initial values for what were three sequential runs of the model. Values for these were based on analysis of empirical data and theoretical arguments as described earlier and not by curve fitting.



**Figure 5.** Demographic-fiscal model output for England over 1050–1750. ( $r = 0.013$ ;  $M = 5.4$  or  $7.0$  and  $\beta = 0.31$  or  $0.45$  for before and after 1485, respectively;  $K = 1.65$ ,  $4.75$ , and  $6.44$  for Plantagenet, Tudor-Stuart, and mercantile cycle, respectively). Population from Turchin and Nefedov (2009), Broadberry (2014), and Mitchell (1988).

The model is plotted along with population data in Figure 5. The value for  $K$  exceeded the population for the mercantile cycle, predicting that growth would occur regardless of  $S$ , in which case secular cycles in population stopped happening, as was observed (Figure 2). The model did a good job of representing the population fluctuations. The average relative error was 4.6% for the entire 1086–1750 period, about the same as the fit obtained with a 9<sup>th</sup> degree polynomial (average error 4.8%). The unstructured polynomial model required ten adjustable parameters to do the job of six in the structured DF model. Adding the theoretical structure underlying the DF model gave a much more efficient

representation of historical dynamics, demonstrating real explanatory power for DST.

Figure 5 also shows the model output for  $S$  compared to the empirical measures of the state from Figure 3. The empirical measures were scaled so that the minimum value in the cycle was zero and the maximum value was 100. The model generated one cycle in  $S$  for each of the Plantagenet and Tudor-Stuart secular cycles. In both cases the empirical peaks preceded the model predictions.

According to the DF model, in the absence of population growth state revenue remains below levels needed to pacify elites and the resulting widespread violence suppresses population growth. But revenues did rise in the middle of the 14<sup>th</sup> century and remained high well into the next century (Figure 3). Analysis of instability data shows that the rise in revenues was associated with reduced instability, as expected; the average instability level when revenues were high over 1330–1420 was 1.49, while the value during the rest of the disintegrative trend was 3.12. Population failed to recover after the Plague despite the absence of high levels of sociopolitical instability.

The absence of post-plague population recovery during the period of strong state revenues might be the result of recurrences of plague. Plague recurrences happened about every decade during the second half of the 14<sup>th</sup> century and at increasing intervals afterward (Cohn 2008). Assuming growth resumed after the Plague at the fastest rate seen during the Plantagenet cycle, the observed population decline to the end of the century could be achieved by plague recurrences of 13% mortality spaced a decade apart. This level of mortality is in line with other outbreaks of plague (Gottfried 1983, 131). By the time the frequency and virulence of recurrences diminished, the state had moved into failure mode (Turchin 2009, 72–4) where high sociopolitical instability could restrain growth.

Another possibility involves the interpretation of  $S$  in terms of state legitimacy rather than sound finance. This interpretation holds that in the absence of a legitimate state ( $S=0$ ), small-scale social disorder (e.g. crime) would persist even when the state was fiscally-strong enough to suppress large-scale disorder. Low population numbers meant real wages were high, exerting a negative impact on elites as described by equation 5. Downward elite mobility and the absence of alternate career opportunities could result in elite youth resorting to brigandage with negative consequences for domestic order and population growth.

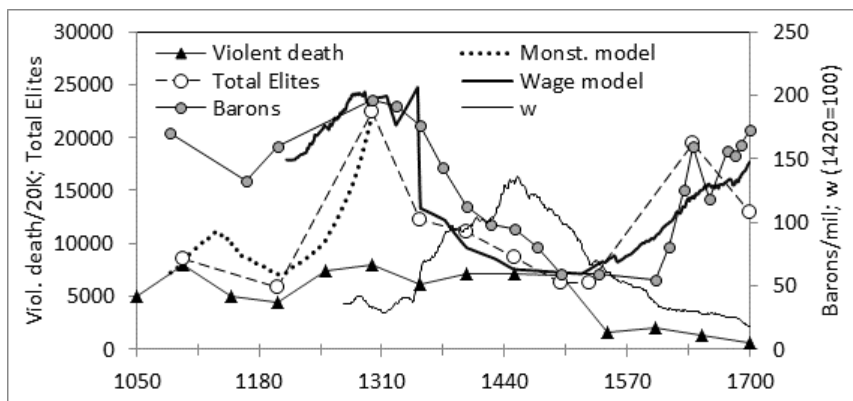
According to this concept, state legitimacy is generated by reduction of elite numbers or influence, not by diverting elite energies away from internal conflict. A fiscally-strong state may temporarily halt elite conflict against the state by buying off or cowing fractious nobles, but it cannot restore law and order (i.e. an integrative trend) until the problem of surplus elites is solved. In this sense, the

fiscal resurgence under Edward III is best viewed as a fathers and sons dynamic (see instability section), not a secular one. As long as there was an effective monarch, there was elite pacification with respect to the state, but not the normative pacification required for a flourishing populace. As soon as the strong monarch was removed, things fell apart. This is shown in Figure 3 by the collapse in state revenues following the death of the capable Henry V in 1422. Sociopolitical instability was still low this time, yet the replacement of a strong ruler with an infant began a process of state decline leading to eventual collapse.

### **The Secular Cycles in Terms of Elite Numbers**

Figure 6 shows plots of total elite numbers and upper-level elites as a fraction of the population over 1086–1700. Both measures of trends in elite numbers show a pattern suggestive of three elite cycles: 1070–1200, 1200–1500, and 1500–1700. Also shown is output from equation 5, referred to as the wage model and equation 6, referred to as the monastery model.

The wage model correctly predicted rising elite numbers to the late 13<sup>th</sup> century, high levels until the mid-14<sup>th</sup> century, and falling elite numbers during the century after. The model also correctly predicted rising elite numbers over ca. 1530–1640. The wage model failed to predict the decline in elite numbers associated with the English Revolution, because relative wages did not rise at this time. Relative wages did not rise during the disintegrative phase of the Tudor-Stuart cycle because population did not decline to the degree it had in the Plantagenet cycle. An important component of declining elite numbers after 1350 was the downward economic mobility of elites arising from increased labor bargaining power caused by the contraction of the labor force. This was not the case for the Tudor-Stuart cycle. The decline in elite numbers likely reflects factors not captured by wage trends. This is probably also the case for the 12<sup>th</sup> century decline.



**Figure 6.** Trends in elite number and model output 1086–1700. (Wage model:  $\mu_0 = 20$ ;  $w_0 = 0.39$ . Monastery model:  $K_M = 1.9\%$ ). Violent deaths is an estimated frequency of European elite male deaths from violence (Cummins 2014, 12). Number of high-level elites (Barons) for pre-1300: Painter (1943), Given-Wilson (1987); 1300–1500: McFarlane (1973); 1500–1640: Mingay (1976); after 1660: Beckett (1986). Lower-level elite numbers from Turchin and Nefedov (2009, 56, 84).  $w$  is wage (Clark 2015) divided by GDP per capita (Broadberry et al. 2014; Williamson 2015). For 1209–1270, real GDP per capita assumed to be a constant equal to its value in 1270.

The projection given by the monastery model is interesting. This model predicts a rise in elite numbers from 1086 to about 1130 and a decline to the end of the century with a strong rise over the next century. According to the monastery model, the cause of rising elite numbers is natural increase. The 1.9% value for elite growth rate ( $K_M$ ) is in line with growth rates seen for pre-industrial agrarian populations when they are adequately provided with resources (Korotayev *et al.* 2006, 49) as would be expected for elites. Elite decline reflects large numbers of elites drawn into religious life, particularly the popular Cistercian movement, which had spread to England in 1128 (Catholic Encyclopedia). As the force of this movement dissipated, natural increase overtook monastic divergence and elite numbers grew. This model provides a mechanism through which elite proliferation was suspended for nearly a century, allowing the expansion phase of the Plantagenet cycle to be lengthened from 110 to 190 years. Once monastic elite reduction had run its course, the secular cycle continued as described by the wage model.

*Justification for dating the Plantagenet cycle as 1070–1485 instead of 1150–1485.* Plantagenet cycle dates of 1070–1485 have been assumed in the previous

sections. Turchin and Nefedov date the beginning of this secular cycle to 1150. Population trends argue against a secular cycle beginning in 1150, however. As implied by equations 2 and 4, population at the end of a secular cycle should return to about the level at which it started. This has been illustrated, for example, with Chinese population dynamics (Korotayev et al. 2006, 47–63). China displayed three secular cycles from 210 BCE to 760 CE in which population rose to a peak and then returned to about the level at which it had started. The population cycle peaks averaged 59 million while the average value at cycle troughs (boundaries) was 14.5 million. The standard deviation of the boundary populations for these cycles was 2.2 million, or about 5% of the 45 million cycle amplitude. During the Song cycle (950–1125), population rose to a peak of 54 million, close to the prior peaks, at which point China began to show “evidence for all the symptoms of sociodemographic crisis preceding demographic collapse: under-nutrition, rising rebellions etc.” (Korotayev et al. 2006, 55). Demographic collapse was prevented by technological innovations which increased the carrying capacity of land. In this way, the Song cycle was like the Tudor-Stuart cycle in that rising agricultural productivity prevented demographic collapse. Unlike in England, rising productivity did not continue and all that was achieved was a new secular cycle built on a higher base population ( $K$ ) of 57 million on average with a higher average peak population of 113 million. The standard deviation of these higher base populations was about 12% of the cycle amplitude.

The same analysis was done for the English Plantagenet cycle. The standard deviation of boundary populations in 1150 and 1485 was 22% of the cycle amplitude, whereas the corresponding figure for boundaries in 1070 and 1485 was 9%. The later value is more consistent with the 5–12% range seen with the Chinese secular cycles. Another way to show this is to scale the data by  $K$ . According to equation 13, the population minimum near a secular cycle boundary should be close to  $K$ , which is the case for the population minimum in 1086, 1450, and 1690, which averaged  $1.10 \pm 0.05$  times  $K$ . In contrast, the mid-12th century population was 1.57 times  $K$ —too high to be a secular cycle boundary according to the DF model.

Turchin and Nefedov (2009, 35) base the start of the Plantagenet cycle in 1150 explicitly on the 1138–1154 civil war known as the Anarchy (Bradbury 2009; Barlow 1999; Davis 1977), whose turbulence is attested by the large number of coin hoards found from this period (Thompson 1956). The Anarchy resembles an episode of state collapse and reformation like the Wars of the Roses that ended the Plantagenet cycle. Turchin and Nefedov (2009, 37–38) note that tax receipts collected in the early years of Henry II’s reign were 25% below 1130 levels and that it was only at the end of his reign that Henry II’s revenues matched those of Henry I, suggesting a decline in state strength occurred. The trend in

elites shown in Figure 6 displays the pattern expected for a cycle boundary in the mid-12<sup>th</sup> century.

According to the monastery model, elite decline around the time of the Anarchy was caused by rapid monastery formation, which might be thought of as an elite response to the instability of the times. According to Thomas (2008), founding a monastery, like hoarding coins, is a logical response to unsettled times because it secured one's assets against the threat of seizure. Regardless of the motives, rapid monastery formation would exclude some of England's wealth from taxation, resulting in a reduced ability of the state to obtain revenues. In this case, both the elite and state revenue trends that supported a secular cycle boundary in 1150 can be explained by the monastery model without large-scale elite mortality. That the royal line continued through Henry I's designated heir shows continuity with the pre-existing state. Because the elite proliferation problem could be handled by the monastery mechanism, the Anarchy does not need to function as a cycle-ending conflict to explain the data.

Figure 6 shows an index of the frequency of violent death for European elite men (Cummins 2014, 12). Boulton (1995, 45) writes: "by around 1100 the professional knightage had evolved to the point where it constituted an international corps, though not a true corporation". Medieval military elites as an international class suggests that the European trends should be representative for England. The Cummins index shows relatively high mortality over 1050–1100 and 1400–1500, which is indicative of elite-reducing warfare consistent with secular cycle boundary conflicts at these times. The lower value of this index during the 12<sup>th</sup> century suggests that excess elites were not culled by the Anarchy to the degree they were by the Norman Conquest and Wars of the Roses. Rather, they diminished because of the appeal of an alternate career.

Turchin and Nefedov (2009, 5-6) note that the manner in which elites treated vanquished elites differed during disintegrative versus integrative trends of the Plantagenet cycle. Four monarchs were deposed and killed over 1315–1485, whereas before the early 14<sup>th</sup> century defeated opponents were scarcely ever killed (Bartlett 1980, 60). The harsh treatment of vanquished rivals during the disintegrative trend is consistent with an elite reduction function. The Anarchy did not show this harsh treatment. Early in the conflict, Stephen captured Matilda and then released her (Bradbury 2009, 8). When a teenaged Henry Plantagenet invaded England without sufficient resources, his troops subsequently abandoned him, leaving him at the mercy of his enemy. Stephen magnanimously paid the cost for him to return to Normandy (Barlow 1999, 80). These observations are consistent with the idea that the Anarchy was an integrative trend conflict.

In summary, the Norman conquest is to be preferred to the Anarchy as the beginning of the Plantagenet cycle because: (1) the population at the time of the Anarchy was inconsistent with a secular cycle boundary; (2) the Norman Conquest solved the problem of excess elites, which is characteristic of a boundary conflict; (3) the monastic mechanism can explain elite and state revenue dynamics after the Anarchy without it being a boundary conflict; (4) twelfth century trends in elite mortality through violence and the behavior of top actors during the Anarchy was not characteristic of a disintegrative trend conflict.

### **The Secular Cycles in Terms of Sociopolitical Instability**

Sociopolitical instability tends to be higher during the disintegrative secular trend (Turchin and Nefedov 2009, 3). This idea was explored using the methods of Sorokin (1937, 96). Figure 3 shows the proxy for state strength reaching zero in 1010 and 1320, indicating that the shift from the integrative to disintegrative secular trend for the Anglo-Saxon and Plantagenet cycles, respectively, occurred around these dates. The 1010 date for the Anglo-Saxon cycle is consistent with the conclusion above that the shift did not occur much before 1013. The 1320 date obtained from Figure 3 is likewise consistent with the 1315 date given by Turchin and Nefedov for the Plantagenet cycle. For the Plantagenet and Tudor-Stuart cycles, both Turchin and Nefedov's boundary dates and the alternates proposed in this work were used to designate the secular trends.

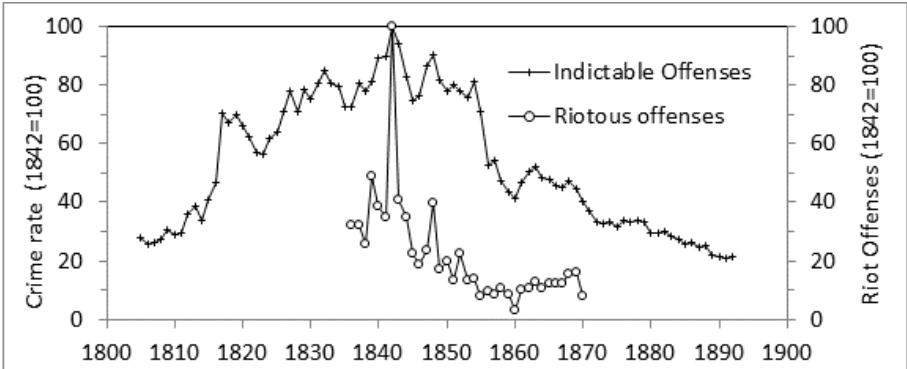
The trends in price, inequality, elite numbers, and state strength (Figures 2 and 3) suggest that the mercantile cycle shifted from an integrative to disintegrative secular trend around 1800. The period around 1800 has the earmarks of a crisis phase, although there was no state collapse. British radical sentiment burgeoned following the French Revolution. Particularly alarming to the government was "the connection between the plotting and arming of a small minority, inspired and possibly aided by the French, and openly seditious pro-French talk in the debating societies" (Royle 2000, 7). The landing of a French force in Wales in February 1797 precipitated a financial panic and a sense of crisis, which deepened with the April naval mutiny. At the same time the United Irishmen were threatening subversion, while Wolfe Tone was in Paris trying to arrange a French invasion in support of an Irish revolt (Royle 2000, 4). The revolt broke out the following year without French support and was suppressed in a matter of months. It was followed by a coup attempt in 1802 and another rebellion in 1803.

The mutinies, invasions, and armed uprisings over 1797–1803 suggest that revolution was possible, even likely, at least in the eyes of the authorities (Royle 2000: 5). Tilly lists this episode as a revolutionary situation (Tilly 1995, Table 4.2). The realization that measures were needed to control Ireland and to reduce

tensions led to the one major constitutional reform of the period, the 1800 Act of Union with Ireland (Royle 2000, 5). The beginning of this crisis in 1797 was used to date the shift from integrative to disintegrative trend for the mercantile cycle.

Crime rates soared during the second decade of the 19<sup>th</sup> century to an initial peak in 1817–19 (Figure 7). At about the same time (1816–20), there was a spate of sociopolitical instability twice as severe (by the Sorokin measure) as one of the Jacobite revolts of the 18<sup>th</sup> century (an integrative trend conflict). After a brief decline, crime rates rose to a second peak, which occurred at about the same time as another outburst of sociopolitical instability over 1829–32 that was even more severe than the post-war violence. Over this same period, two more constitutional reforms were adopted: Catholic Emancipation in 1829 and the Reform Act of 1832. After a decline to 1836, crime rates rose to a 19<sup>th</sup> century peak in 1842. A sharp rise in riotous offenses were also seen in that year, presumably associated with the Chartist-inspired 1842 general strike (Figure 7). Chartism was a working class movement to gain political rights and influence for the working classes. The disturbances of 1842 were the most intense of any that occurred in Britain from the time of the French Revolution to the Chartist détente of 1848 (Royle 2000, 13).

The 1848 revolution on the continent spurred the last major expression of Chartist protest. Economic downturn had spawned riots in London, Glasgow, Manchester, and other cities. The Chartists had decided to convene a mass demonstration on Kennington Common to submit to Parliament their petition for political rights. About 150,000 gathered on April 10 for the demonstration and the leaders submitted the petition, which was ignored (Royle 2000, 26). Popular anger did not dissipate. There were reports of working men forging pikes, conducting military drills, and dangerously inflammatory rhetoric at political meetings (Royle 2000, 29). Secretary to the Privy Council Charles Grenville recorded in his diary on June 3 that “it was quite new to hear any Englishman coolly recommend assassination” and that “it was impossible not to feel alarm when we consider the vast amount of the population as compared to any repressive power that we possess” (Royle 2000, 130). The Chartist movement was infiltrated with spies and the government was able to thwart the “1848 conspiracy” with mass arrests, ending the last revolutionary attempt of the era (Royle 2000, 34).



**Figure 7.** Trends in crime and disorder rates in 19<sup>th</sup> century Britain. Crime rate from Gatrell and Hadden (1972); Riotous offenses from Stevenson (1979, 95).

Sociopolitical instability diminished after this; the average level during the second half of the 19<sup>th</sup> century was less than half that in the first half. Rates of crime and riotous offenses declined as well (Figure 7). Based on this series of events, the end of the mercantile cycle from an instability perspective was set at 1850.

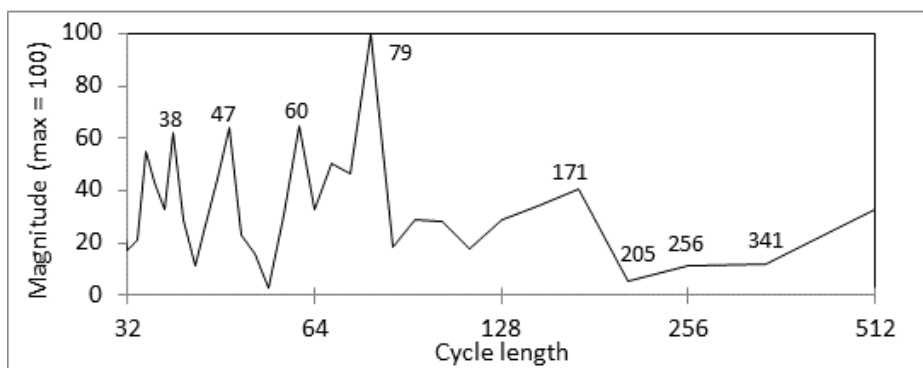
With these dates, average instability levels were assessed for the integrative and disintegrative trends of all four secular cycles (Table 1). Instability was higher during the disintegrative phase for all four cycles. This finding was not statistically significant for the individual secular cycles in isolation. When the Plantagenet and Tudor-Stuart cycles were analyzed together the result was just barely significant ( $p = 0.05$ ) for the modified cycle dates, but not for Turchin and Nefedov’s dates ( $p < 0.07$ ). When all four cycles were analyzed together the result was highly significant ( $p < 0.013$ ).

**Table 1.** Analysis of instability patterns for three English secular cycles

Cycle	Turchin & Nefedov 2009		This work	
	Dates	Value	Dates	Value
Integrative (Anglo-Saxon)	--	--	880-1009	1.17
Disintegrative (Anglo-Saxon)	--	--	1010-1070	2.74
Integrative (Plantagenet)	1150-1314	1.34	1071-1314	1.31
Disintegrative (Plantagenet)	1315-1485	2.28	1315-1485	2.28
Integrative (Tudor-Stuart)	1486-1639	1.49	1486-1639	1.49
Disintegrative (Tudor-Stuart)	1640-1730	2.78	1640-1690	3.75
Integrative (mercantile)	--	--	1691-1796	1.57

Disintegrative (mercantile)	--	--	1797-1850	3.02
Overall Integrative $\pm$ std. error	1.41 $\pm$ 0.27		1.37 $\pm$ 0.18	
Overall Disintegrative $\pm$ std. error	2.46 $\pm$ 0.50		2.70 $\pm$ 0.45	

The need to combine cycles to find a significant effect of the secular cycle on instability shows that the effect was weak. Figure 8 presents a plot of a Fourier transform of the instability data over the 850-1873 period. The dominant wavelength was around 79 years. Smoothing the data with an exponential average ( $\alpha=0.04$ ) and plotting it at decade intervals yielded peaks at 880, 960, 1020, 1070, 1150, 1230, 1325, 1410, 1485, 1550, 1650, 1690, and 1820, which are spaced an average of 78 years apart. These cycles probably correspond to “fathers and sons” cycles, “a general dynamical pattern of alternation between very turbulent and relatively peaceful spells” (Turchin and Nefedov 2009, 27). Frequencies corresponding to oscillations of multi-century length corresponding to a secular cycle were not evident in Figure 8.



**Figure 8.** Frequency analysis for sociopolitical instability in England 850-1873. Instability level from Sorokin (1937) supplemented by events from Stevenson (1979), Tilly (1979), Royle (2000), and the University of Oregon Mapping History project analyzed with Sorokin’s method. Resulting data was analyzed by Fourier transform to yield the above spectrum.

Fathers and sons cycle outbursts of elevated sociopolitical instability occurred during all parts of the English secular cycle. There was a difference in political behavior during these outbursts depending on whether they occurred during an integrative or disintegrative trend. The monarch retained his throne in all four outbursts during an integrative trend. In contrast, the typical outcome of a disintegrative-trend outburst can be described as state collapse. During the

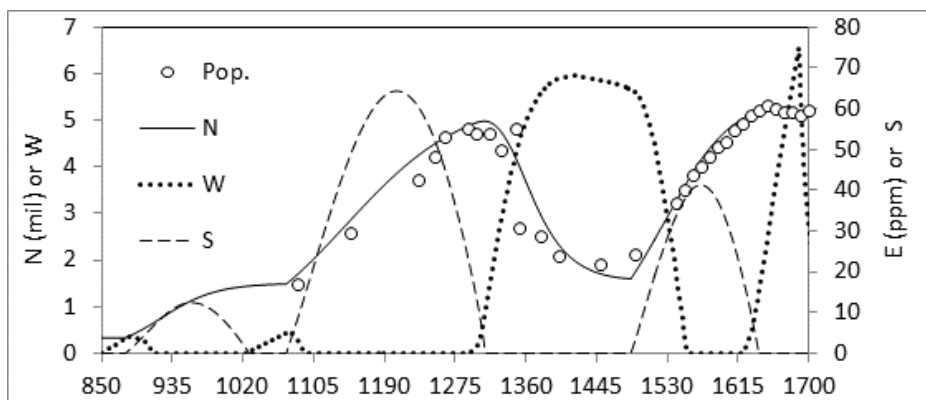
Anglo-Saxon disintegrative trend, there were two outbursts, both of which involved conquest of the country. The Plantagenet disintegrative trend saw three outbursts, all of which saw the monarch deposed and killed. Finally, the Tudor-Stuart disintegrative trend saw two outbursts of political instability that have been described as revolutions.

According to the DF model, the defining attribute of an integrative trend is that  $S$  is greater than zero. Reserve state legitimacy ( $S/N$ ) appears to protect the monarch and state from extreme outcomes during the integrative trend. Presumably, internal war would have negative effects on state legitimacy. This was explored by setting the first term of equation 12 equal to zero at the beginning of an integrative trend conflict. Using this approach,  $S/N$  fell from 13 to 7 years during the Anarchy, and from 10 to 5 years over the 1549–57 period of elevated instability during the “mid-Tudor crisis” (Jones 1973). In both cases reserve state integrity remained positive at the end of the conflict, preventing state collapse.

Fathers and sons episodes of instability reflect the periodic appearance of weak monarchs (typically those who came to the throne as children). When this happens during an integrative trend, the positive value of  $S$  prevents the increased instability from inducing state collapse and the weak monarch retains his throne. During a disintegrative trend, there is no built-in legitimacy for the monarch or state. Strong kings can achieve a resurgence in state power as measured by strong revenues and an absence of instability, but the instability that comes with a weak monarch typically results in state collapse, with unpleasant consequences for the monarch.

*War model implementation.* Figure 9 presents the war model output for a set of parameters adjusted to fit the population data over 1086–1700. Equation 7 contains two separate feedback effects of  $W$  on population growth.  $W$  exerts a direct negative effect on  $dN/dt$  through the parameter  $\sigma$ .  $W$  also exerts an indirect effect through its effect on  $N_{MAX}$  in equation 10. Having two parameters seemed superfluous, so  $\sigma$  was eliminated by setting it equal to zero. Doing this made the model more tractable and a reasonable fit to the population data and (qualitatively) to the observed trends in state and instability was obtained by fitting separate values of  $K_{MAX}$ ,  $c$ ,  $r$ , and  $\beta$  for the Anglo-Saxon, Plantagenet, and Tudor-Stuart cycles. A single set of values for  $a$ ,  $b$ , and  $\alpha$  in equation 8 was used for all three cycles except for the 850–950 period when  $a$  and  $\alpha$  had different values and the post-1690 period when  $b$  was substantially increased in order to prevent a runaway rise in  $W$ . Thus, the implementation shown in Figure 9 employed 18 adjustable parameters.

The population fit by the war model was unimpressive, since an equally good fit was obtained with the DF model which used a third of the adjustable parameters. The predictions for  $S$  from the war model and the DF model were plotted in Figure 3. The two models gave near-identical results for the Plantagenet and Tudor-Stuart cycles. Therefore, the DF model gave a far more efficient account of population and state trends as the war model. Unlike the DF model, however, the war model has the potential to model sociopolitical instability through the value of  $W$  and it does indeed show some capability in this area. The average value of instability when  $W$  is zero was  $2.24 \pm 0.35$  compared to  $1.33 \pm 0.26$  when  $W$  is greater than zero. This difference is statistically significant ( $p < 0.04$ ). Thus, the war model is capable of representing the broad trends in population, the state, and instability called for by DST (see Turchin and Nefedov 2009, 3). The large number of adjustable parameters that must be fit to the data detracts from its utility, however.



**Figure 9.** War model for 850–1700. Model parameters: 1<sup>st</sup> cycle,  $K_{MAX} = 1.6$ ,  $c = 0$ ,  $r = 3\%$ ,  $\beta = 0.25$ ; 2<sup>nd</sup> cycle,  $K_{MAX} = 5.5$ ,  $c = 0.7$ ,  $r = 1.4\%$ ,  $\beta = 0.3$ , 3<sup>rd</sup> cycle,  $K_{MAX} = 5.5$ ,  $c = 0.1$ ,  $r = 2.5\%$ ,  $\beta = 0.25$ . All 3 cycles:  $\delta = 0$ ,  $a = 0.0045$ ,  $b = 0.003$ ,  $\alpha = 0.005$ , except for 850–950 when  $a = 0.1$  and  $\alpha = 0.01$ . After 1690 all parameters same as 3<sup>rd</sup> cycle, except  $b = 0.1$ .

## Discussion

### Summary of Model Evaluations

Four models were used to fit historical data defining two secular cycles. Since the cycle boundaries serve as parameters in the DF model, it is necessary that the proper boundaries were used. A variety of measures of variables that show

secular cycles were plotted over seven or more centuries so that the oscillations that define secular cycles could be visualized. The best fit of the data visually were cycles over 1070–1485 and 1485–1690. Turchin and Nefedov (2009) date these cycles as 1150–1485 and 1485–1730. Consideration of theory, empirical data, and model requirements support the first set of dates.

With the modified cycle boundaries, a simplified version of the DF model did a good job of representing the population trend over 1086–1700, performing as well as an unstructured model with two-thirds more adjustable parameters. The DF model also gave a partial account of trends in state strength with no additional parameters. There was, however, a discrepancy; an apparent resurgence of the medieval state (as measured by increased revenue and decreased sociopolitical instability) occurred in the midst of a disintegrative trend, when the DF and war models say this is not supposed to happen. This resurgence reflected the fathers and sons dynamic, a phenomenon beyond the scope of existing secular cycle models.

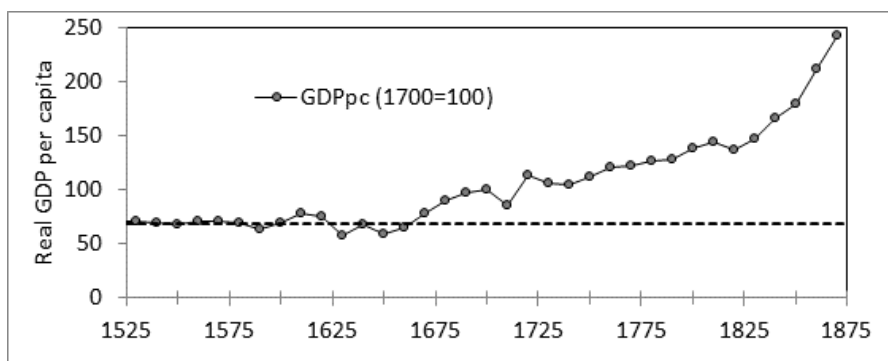
The wage and monastery models for elite numbers together provided a good description of trends in elite class dynamics from the 11<sup>th</sup> through 17<sup>th</sup> centuries. The two approaches are complimentary. The basic DST mechanism represented by the wage model remained valid throughout. Monasteries merely provided a temporary respite to normal DST dynamics by absorbing excess elites, permitting an unusually long expansion phase. An analogous thing happened during the long expansion phase of the Roman Republican cycle (350–180 BCE). The Second Punic War (218–201 BCE) resulted in a severe decline in elite numbers. After growing at a 1.3% rate over 330–280 BCE and holding roughly steady to 265 BCE, elite numbers fell by more than a third to 200 BCE (Turchin and Nefedov 2009, 86). In both cases, a non-economic mechanism reduced elite numbers, extending the length of the integrative trend.

The war model shows that a different model structure can replicate the results of the DF model with respect to population and state strength using instability as the driver for the cycle. This model incorporates an explicit instability-suppression mechanism by the state to model the positive effect of S on population. The model calls for high average instability at certain times and low levels at others. The empirical data correlated with the predicted periods in a statistically-significant fashion. This provides some support for the idea that state-induced cyclical variations in instability can approximately account for secular cycles in instability. But these cycles were weak; the dominant oscillation was the fathers and sons cycle.

As previously discussed, instability was relatively low for 70 years after 1350, yet there was no post-plague population recovery. This observation argues against the inverse relation between instability and population growth given by

equations 10 and 7 in favor of the direct positive effect of  $S$  on growth posited by the DF model. In the DF model, the resurgence of  $S$  is governed by the boundary dates, which in turn are based on the resolution of the excess elite problem, which restores state legitimacy.

Solutions to excess elites seem to fall into two categories, reduction and dilution. Reduction was achieved through attrition in war. Examples include the Norman Conquest and Wars of the Roses. Dilution was achieved by diversion of elite energies into non-military/political career paths. Examples include the Anarchy, when elites were diverted into religious careers, and the Glorious Revolution, when rising real per capita GDP provided a way to extract additional wealth to support more elites, eliminating the need for a fratricidal conflict (Figure 10).



**Figure 10.** Real GDP per capita 1525–1870. Data from Broadberry et al. (2014).

*Comments on the mercantile cycle and the end of agrarian cycles.* Turchin and Nefedov describe French Capetian and Valois secular cycles that are parallel with the English Plantagenet and Tudor-Stuart cycles. They also propose a Bourbon cycle that follows the Valois (Turchin and Nefedov 2009, 43). The mercantile cycle can be thought of as the English parallel to this cycle. The Valois cycle saw two waves of political instability during its disintegrative trend. The first was associated with the Wars of Religion and the second culminated in the Fronde of 1648–1653 (Turchin and Nefedov 2009, 56). The Tudor-Stuart cycle also saw two waves: the English civil war and Glorious Revolution (1688). The Bourbon cycle began in 1660, just after the final Valois wave of instability. Similarly, the mercantile cycle began in 1690, just after the final Tudor-Stuart wave of instability.

As described earlier, trends in price, inequality, relative levels of top elites, and state strength are consistent with an integrative trend running from 1690 to

about 1800, while sociopolitical instability trends suggest a disintegrative trend running from 1797 to 1850. There was no state collapse, although several constitutional changes of the sort that often require revolution had taken place by 1850. If the sort of change that once required internal war and state collapse happens without state collapse, might that be classified as a boundary-defining political event? On the other hand, the trends in price, economic inequality, and state strength reached troughs in the 1870–1900 period, suggesting 1850 is too early.

More importantly, the mercantile cycle does not have an associated population cycle, which is the defining characteristic of secular cycles. The uncoupling of population from all the other cycle measures shows that demographics were not the driver of the mercantile cycle. In an agrarian economy, GDP and demand for labor are fixed by the amount of cultivated land, while supply of labor and economic demand are proportional to population. Growing labor supply relative to fixed demand leads to lower real wages, while growing economic demand relative to fixed supply gives rising prices. Ultimately, price, economic inequality, elite numbers, and state strength are all consequences of population dynamics. The end of population cycles means some new mechanism was now responsible for cycle dynamics.

A likely reason for the uncoupling of price from population was the establishment of a central bank at the beginning of the mercantile cycle. Central banking allows the state to manage the money creation that leads to price inflation, instead of accepting the expansion dictated by the market. Price inflation occurred when the state had need for large expenditures, typically during wartime, and so it can be thought of as a manifestation of state strength.

Central banking does not change the agrarian labor market relation. Rising population should still shift bargaining power towards employers regardless of price trends. In practice, however, it is much easier to resist calls for nominal wage increases during inflationary periods than it is to decrease nominal wages during deflationary times. Therefore, the tendency is for inequality to track price rather than population when price deviates from the population trend. Elite numbers trends will reflect inequality trends according to the wage model. Thus, the driver for the mercantile cycle was state finance, which gives rise to a price trend, from which the other trends are derived.

The relation described above will be true as long as the only change is central banking and the economy is otherwise still largely agrarian. Industrialization greatly complicates the system dynamics. In an industrial economy, GDP and demand for labor are not fixed by land, but rather by economic demand (correlated with population). Wages are set by the interplay between supply and demand for labor, both of which are correlated with population making wage

trends no longer predictably related to population, price, or anything else. Inequality measured by GDPpc/wage is then also unpredictable, as are elite numbers. That is, in an industrial economy the secular economic relations characteristic of agrarian societies are no longer valid.

It is probably safe to assume that industrialization did not significantly affect secular cycle dynamics before 1800 and so the designation of the 1690–1797 period as an integrative trend is valid. It is less likely that this assumption would still hold in 1850. For this reason, what constitutes a secular cycle may have changed as the 19<sup>th</sup> century progressed. In the absence of a suitable theory of post-agrarian secular cycles, it may not be possible to date the latter boundary of the mercantile cycle with confidence. Turchin and Nefedov (2009: 43) reached a similar conclusion for the Bourbon cycle in France.

## **Conclusions**

The DF and wage models show promising approaches to characterizing secular cycle dynamics. Several other conclusions can be drawn: (1) there is an Anglo-Saxon cycle (880–1070) before the Plantagenet cycle and a mercantile cycle beginning in 1690 after the Tudor-Stuart cycle. (2) Monastic growth was of sufficient magnitude to produce the decline in secular elite number during the 11<sup>th</sup> century and provides an alternate elite-reducing mechanism. (3) The Anarchy appears to be an integrative trend conflict like Republican Rome's Second Punic War. (4) The dominant cycle in sociopolitical instability (as measured by Sorokin's method) is the fathers and sons cycle, not the secular cycle. (5) The mechanism by which elite proliferation suppresses population growth does not seem to be increased sociopolitical instability of the type measured by Sorokin.

Future study of post-industrial secular cycles, if they exist, is complicated by breakdown in the old relations between population and economics. Some of the old indicators such as economic inequality still exist and might be useful as a starting point for future research. Future work might involve modeling of the fathers and sons dynamic, which appears to be the most important cycle in sociopolitical instability, at least in England.

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