

The Exchequer's Guide to Population Ecology and Resource Exploitation in the Agrarian State

Bruce P. Winterhalder, Cedric O. Puleston

University of California, Davis

Abstract

We adopt an imagined exchequer, the functionary responsible in an early polity for securing resources from its agrarian subjects, and we develop a feature-rich demographic and environmental model to explore the population ecology of agricultural production in the context of population growth, Malthusian constraints and economic exploitation. The model system allows us to (i) identify and characterize a peak of surplus production early in population growth, prior to density-dependent constraints and (ii) characterize the taxation potential of a population at its Malthusian equilibrium. For a fixed total level of taxation the exchequer has two options: a small population taxed at a high rate, unstable to small perturbations, or a larger population taxed at a lower rate, which is stable. In a small and growing population it is more effective to tax goods; as the population approaches its density-dependent equilibrium it becomes more effective to tax labor. We likewise show that heavily taxed early agrarian states face an extinction risk dependent on the level of taxation and magnitude of yield variation. Successful agrarian states balanced resource exploitation against dynamic population ecology constraints; we propose that fiscal mismanagement should be among the hypotheses for polity failure.

1. Introduction

The exchequer of our title is a high-level functionary in an early state made up of a ruler, an elite and administrative staff, and a growing population of peasant agriculturists, the subjects of the polity and the producers of its agrarian resources. He, or less likely, she, might be standing atop a peripheral temple at Tikal, Guatemala, looking out over peasant house groups surrounded by fields of corn interspersed with patches of secondary tropical forest (Fedick and Ford 1990). He might also be standing atop a small hill on the north shore of Lake

Corresponding author's e-mail: bwinterhalder@ucdavis.edu

Citation: Winterhalder, Bruce P., and Cedric O. Puleston. 2018. The Exchequer's Guide to Population Ecology and Resource Exploitation in the Agrarian State. *Clodynamics* 9: 40–83.

Titicaca, Peru, viewing thousands of hectares of raised fields interlaced within a reticular network of irrigation canals, the small highly productive plots planted in potato and quinoa (Erickson 1987). Only occasionally have we documentation of an actual figure like the one we envision was common in similar settings in other parts of the prehistoric and ancient historic world:

Aztec provincial *calpixque* reported constantly to the *petlacatl*, or high steward in charge of the *petlacalco* (the main storage facility) of Tenochtitlan. Bernal Díaz del Castillo (1968 [1568]: 168) vividly recalls how the *petlacatl* had a large house full of tribute books and was in charge of all accounting for the entire Aztec tributary system, recording everything in the vast archive of painted assessments and receipts (Gutiérrez 2013: 143).

The exchequer's administrative assignment is to obtain for the state from its territory and its inhabitants the maximum quantity of resources possible, reliably, year after year. On behalf of aspiring elites, exchequers or their equivalents likely attended carefully to the agrarian population's ability to produce resources (Peterson and Drennan 2011: 124), thus securing the goods and labor that underwrite establishment power. The resources garnered will support retinues of craft specialists, the construction of monuments, the holding of ritual displays and other public expressions of centralized authority, the maintenance of powerful military and religious institutions, and the officers who census the polity's subjects and inventory and collect its revenues. The complexity of this task is evident in an ethnohistoric case like that of the Kingdom of Dahomey (Herskovits [1938] 1967: 107–134).

Our exchequer is habitually anxious. He has a trusted bureaucracy but he also faces uncertainties extending well beyond the vagaries of weather and the brittle tolerance by the peasants of their exploitation. He must assess correctly what fiscal policies will be most effective at procuring resources from the landscape before him. The survival of the state rests on his correct evaluation of the mechanisms of population ecology affecting his subjects, their crops and their animals.

Our hypothetical exchequer embodies the decision-making power of the state and its dependence on correctly forecasting demographic and environmental processes, but our results do not depend on the existence of an actual rational, calculating individual to fill that role. Fundamentally, we adopt a modeling approach to assess how the benefits and costs of diverting agricultural production or labor as sources of elite revenue were affected by population ecology and tax policy. Early states that came upon workable or even efficient policies probably did so by trial and error, the emulation of successful neighbors

or, simply, by lucky accident, as well as by calculated insight. Most likely, cultural evolutionary processes subtended whatever design they achieved (Henrich and McElreath 2003; Stoelhorst and Richerson 2013). There must have been many that failed to get it right and struggled or disappeared as a result (Wright 2006); those that succeeded prospered. We confront our imagined exchequer with population ecology questions in order to better understand why some early states thrived and others did not.

Chiefdoms and states are about many things, ranging from personal aggrandizement and social stratification, to public works, to compelling ideology and military power (Turchin and Gavrillets 2009; Earle and Spriggs 2015; Earle 2015). Underlying all of these is the sustained acquisition of economically useful resources (DeMarrais *et al.* 1996; Drennan and Peterson 2011: 65), a key element of what D'Altroy and Earle (1985) term staple finance. Even the expression of power via ideology has steep economic costs, as the beliefs of a large and dispersed population must be inculcated and made palpable through repetitive and impressive material and ritual display. If exploitation is costly to the peasant population, it may require coercive enforcement as well. We begin with the premise that in early chiefdom- or state-level agricultural societies the economic problems facing our exchequer largely are problems of population and agrarian ecology.

Our questions—any of which might have occurred to a prehistoric exchequer—are these: what is the relationship between increasing population density and the potential for surplus income that might be taxed as labor or produce? Among such policy choices as adding territory, increasing yield through technology, adding to the workload or decreasing the consumption of the agricultural producers, which have the greatest potential for enhancing revenue and in what circumstances? What are the trade-offs entailed in securing a steady source of resources from an agrarian peasantry and how stable are the options? Under what circumstances is it better to extract resources in the form of goods or in the form of labor, and which will have the lightest impact on the economic welfare of a potentially restive subject population? What are the consequences for state persistence of fluctuating agricultural yields? More generally, how does the population ecology of producers in the agrarian state determine trade-offs associated with the magnitude and reliability of the resources that might be extracted from them?

However frequently these or similar questions occurred to state-level functionaries in the past, they have seldom occurred in the contemporary writings of anthropologists. While there is a very large literature on the political economy of early states (Trigger 2003; Claessen and van de Velde 2008), to the degree that it considers the material dimension of economics it typically concerns

the collection and *distribution* of resources through institutional networks to ensure continued state hegemony (Smith 2004). Much less has been written about how those resources were generated through the combination of environmental resources and labor. There is good documentation of the crops, minerals or other raw materials involved, sometimes their estimated amounts and values, perhaps the logistical (e.g., transportation) problems of their acquisition, and the manners in which they could be processed or traded to gain value. However, little is known about how they were produced through dynamic mechanisms of interaction among producers, the environment, and the resources being exploited. It is this subject that we take up on behalf of our exchequer and the study of fiscal policy in early agrarian states.

2. Population Ecology as a Model Framework

Our analysis is based on a dynamic, population ecology model (Figure 1) that links together, in order of causal relationships: (1) a unisexual but age-structured population with demographic characteristics representative of a 1:1 sex ratio; from which we obtain (2) an agricultural labor pool with age-specific daily investment in production. That work effort is translated, through interaction with (3) a set of parameters describing the environment available for agriculture and its yield, into (4) agricultural production measured in kilocalories (kcal). The age-structured population also determines (5) the energy consumption requirements of this subsistence population and, thus, (6) the total food required to meet caloric needs at baseline dietary intake. Baseline here is defined as the kcal availability that would be sufficient to sustain fertility and survival rates at their upper bound values, set at $b_0 = 0.0369$ and $d_0 = 0.0192$, respectively (Table 1). The model calculates (7) a *food ratio*, E , or kcals produced divided by those needed to sustain fertility and survival at baseline rates. E then is passed to functions determining (8) age-specific rates of fertility and survival. Finally, these rates then modify (1) the age distribution and population size of the region's inhabitants, initiating a time step and a subsequent iteration of the model.

Parameters of the model are scaled to values realistic for human populations engaged in agricultural production (see Table 1); full details have been published elsewhere (Lee and Tuljapurkar 2008; Puleston and Tuljapurkar 2008; Lee *et al.* 2009). The approach explores underlying features of premodern agrarian populations and is designed to be broadly applicable among such populations, although parameters may differ among cases. This paper builds on the model's dynamic properties, examined in Puleston *et al.* (2014) and Winterhalder *et al.* (2015). The Supplemental Materials contain supporting mathematical details.

Table 1. Model Parameters, Variables, Symbols and Conventions.

Parameter	Interpretation	Value & Unit
Y	Yield/area	21,000 kcal/ha/day
H	Longest age-specific agricultural work day	5 hr/day/ind (or 10 hr/day/worker)
k	Area worked/hr	0.0944 ha-day/hr
ϕ_x	Proportion of hours, H , worked by age x	$1 \geq \phi_x \geq 0$
$\phi_0, \hat{\phi}$	Average age structure weighted hours; effective workers/person given copial or equilibrium structure, respectively	$\phi_0 = 0.738; \hat{\phi} = 0.723$
$m_{x(0)}$	Baseline, age-specific fertility, $E \geq 1$	daughters/woman from age x to $x + 1$
$p_{x(0)}$	Baseline, age-specific survival, $E \geq 1$	probability of survival from age x to $x + 1$
J	Baseline, age-specific kcal requirement for most active age class	2,785 kcal/day
ρ_x	Proportion of consumption, J , by age	$1 \geq \rho_x \geq 0$
$\rho_0, \hat{\rho}$	Average, age structure weighted consumption	$\rho_0 = 0.832; \hat{\rho} = 0.827$
A_m	Arable land available	1,000 ha
F	Fraction of A_m in cultivation	$1 \geq F \geq 0$
a_x	Elasticity at $E = 1$, at age x , for survival rate, ρ_x	$0.00279 \leq a_x \leq 0.156;$ $a_{25} = 0.00464$ at $E = 1$
γ	Elasticity at $E = 1$, at any age for fertility, m_x	$\gamma = 0.135$ at $E = 1$
w	$w = Hk\phi_x$, labor effectiveness, average ha/person; w_0 = copial phase, \hat{w} equilibrium phase	$\hat{w} = 0.3413$ ha/person $w_0 = 0.3483$ ha/person

Table 1 (continued).

Parameter	Interpretation	Value & Unit
j	j = consumption, average weighted per age structure; j_0 = copial, stable age structure; \hat{j} = equilibrium stable age structure	$\hat{j} = 2303$ kcal/day $j_0 = 2318$ kcal/day
E_m, \hat{E}_m	Food ratio for an infinitesimally small population with either stable copial or equilibrium age structure, respectively	$\frac{YHk\phi_0}{J\rho_0} = 3.16, \frac{YHk\hat{\phi}}{J\hat{\rho}} = 3.11$
B	Fraction of A_m the initial population could have cultivated in the absence of density dependence	$\frac{Hk\phi_0 N_0}{A_m} = 0.0070$
r, b, d	Per capita reproductive (r), birth (b) and death (d) rate. Zero subscript refers to baseline rate, observed in copial phase	$r_0 = 0.0176; b_0 = 0.0369;$ $d_0 = 0.0192$
$\hat{r}, \hat{b}, \hat{d}$	Equilibrium growth rate, births per capita and deaths per capita, at \hat{E}	$\hat{r} = 0; \hat{b} = 0.0328; \hat{d} = 0.0328$
S, T	S is copial phase surplus production; T is Malthusian phase taxation	
$\wedge, *, -$	Equilibrium, maximum, & mean value	
\approx	Approximately equal	
	The subscript 0 signifies a baseline value.	

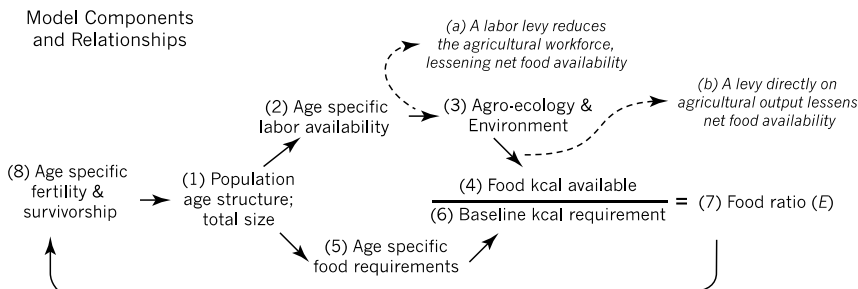


Figure 1. Schematic showing role of taxation in our space-limited population ecology model. Solid arrows indicate the sequence of causal relationships among core components of the model. For instance, the total population is the source of the age-specific labor and thus the agricultural workforce available. That workforce interacting with the environmental parameters determines the food production measured in kilocalories. Exploitation, operationalized as removal by political elites of either productive labor (a) or agricultural product (b), lessens food kilocalories available. A labor levy removes work from the producing population but does not alter consumption; the individuals affected and their dependents must still be fed. Thus, both forms of taxation reduce the food ratio, E , although in a complex pattern due to system dynamics. The food ratio alters vital rates of fertility and mortality, modifying the age-structured population with which the cycle began. An earlier version of this illustration appearing in Puleston *et al.* (2014: Figure 1) depicts in greater detail core features of the population ecology model which serves as the basis for the present analysis of taxation. See text for further explanation.

In a constant environment, model dynamics are determined primarily by the food ratio, E . So long as $E > 1$, food production exceeds the agrarian population's baseline need, fertility is high and mortality is low; neither vital rate changes and the population grows at a constant rate set by $r_0 = b_0 - d_0$ (all terms defined in Table 1). As the environment becomes saturated, land availability is constrained, competitive inefficiencies emerge and E drops below 1. Decreasing per capita food production elevates mortality and depresses fertility. As these terms converge under continued growth, density dependence leads to a stable age structure and an equilibrium at which growth has ceased and a Malthusian equilibrium has been reached. An archival version of R code to reproduce basic features of the model is publicly available at <https://github.com/puleston/spacelim>.

In the present analysis we introduce taxation into the model dynamics by extracting either goods (kcal of agricultural yields) or labor (individual work effort) from the population ecology system (Figure 1). These resources or their products we assume are consumed by elites off-stage, meaning that they disappear from the model accounting. In the case of labor, taxation manifests itself as time diverted from agriculture, for example, into craft production that will be sequestered by elites. The laborer remains a consumer in the agrarian population. Taxation of subsistence production manifests itself as food resource no longer available to meet the consumption needs of the producing population. We do not explicitly model the population dynamics of the elite, assuming their numbers to be small and stable, relative to the producers (Tisdell and Svizzero 2017: 10–12).

For anthropologists accustomed to thinking of chiefdoms and early states as redistributing back to commoners some of the resources they exploit, we in effect are modeling the *net* loss to the productive population due to taxation. We use “tax” in this context to represent the state’s compulsory claim on commoner resources, aware that we are glossing refinements recognized in the use of terms like levy, tallage, tribute, corvée and tithe.

We begin (Section 3) by describing the temporal dynamics of the model, from initiation of growth by a small founding group settling an empty agrarian habitat to its transition to the Malthusian equilibrium regime. We then characterize how the potential for surplus production changes across the early portion of this time series, identifying an ephemeral intermediate optimum of surplus early in the growth trajectory. We show how the timing and magnitude of this optimum is affected by demographic and environmental variables. Section 4 takes up the trade-offs implied in per capita and fixed taxation once the population has reached a density-dependent or Malthusian equilibrium. In Section 5 we examine the relative merits of extracting resources in the form of goods or labor, as a function of population size and density dependence. In Section 6 we analyze the half-life of the agrarian population as a function of stochastic variation in agricultural yields and the degree of their exploitation. Finally, in Section 7 we discuss the model results in light of archaeological and ethnohistoric evidence, to suggest what we can learn about prehistoric economies from complex human-systems models. We conclude with a summary in the form of practical advice to our exchequer in Section 8.

3. Surplus in the Period of Population Growth for which $E \geq 1$

Consider an agricultural region of 1000 ha, newly settled by a group of 20 agriculturalists not subject to taxation. Under our baseline parameters (Table 1),

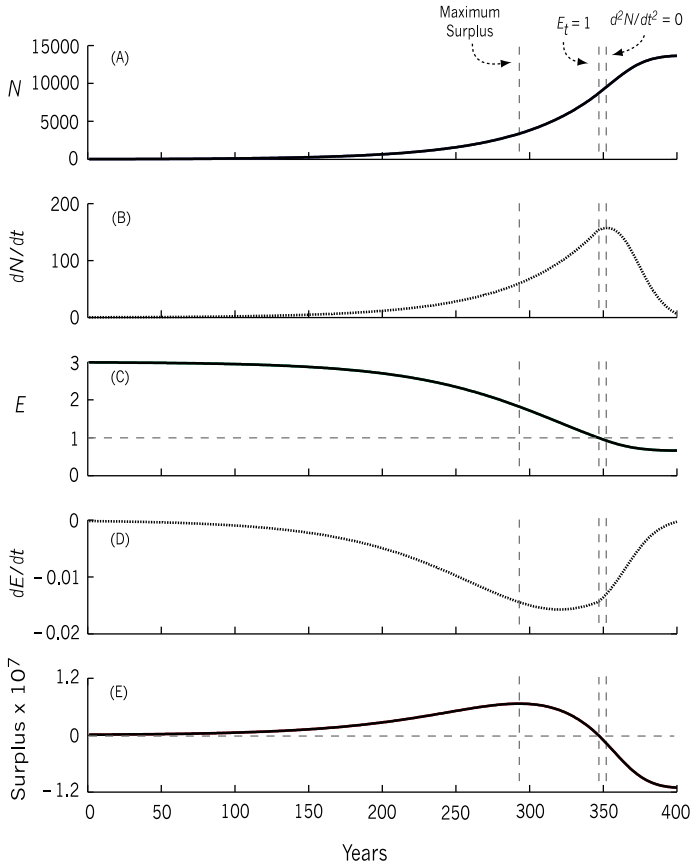


Figure 2. Surplus and related model features as a function of population growth. From top to bottom the panels represent: (A) Population size, N ; (B) Population growth rate, dN/dt ; (C) Food ratio, E ; (D) Rate of change in E as a function of time, dE/dt ; and (E) Surplus, measured as $S_t = N_t(E_t - 1)j_0$. The founding population, N_0 , is 20 individuals. The food ratio E_t has declined to 1 by the year 347, at a population size of 8734. The population growth rate, dN/dt , peaks at 157 individuals per yr shortly after year 352. The equilibrium population size \hat{N} is 13,509. $E_{t=0}$ is 2.99 and at equilibrium, \hat{E} , has fallen to 0.668; E equals 1.82 at the copial maximum surplus, S^* , at year 293. That surplus above population baseline consumption needs equals 6.41×10^6 kcal/day and occurs at population size of $N = 3382$. At equilibrium total production falls 1.03×10^7 kcal/day short of the population food requirement to achieve baseline caloric adequacy.

over a period of about 400 years their population will expand in a shallow sigmoid pattern (Figure 2A). The newly settled inhabitants experience a long period of relative abundance, the *copial phase* (see Puleston *et al.* 2014), in which production of food at the assigned level of moderate work effort exceeds the minimum consumption requirements for baseline vital rates (Figure 2C). However, as the food ratio (E) declines below 1, density-dependent constraints begin (at year 347) to elevate mortality and constrain fertility. This initiates a *transition phase*; growth begins to slow. Fifty years later, per capita mortality effectively balances per capita fertility and the population enters its *Malthusian phase*. It approaches an equilibrium at which $\hat{r} = 0$, the food ratio $\hat{E} = 0.668$ and the subsistence population numbers $\hat{N} = 13,509$.

Three temporal landmarks help to characterize this growth trajectory: (1) surplus food production (definition below) in the copial phase is at its maximum in year 293 (Figure 2E); (2) the food ratio E first drops below 1 in year 347 (Figure 2C); and, (3) the population growth rate (dN/dt) is maximal in year 352 (Figure 2B). Figure 2D shows that the fastest rate of decline in the food ratio (dE/dt) lies between the maximum surplus and $E = 1$. Due to demographic momentum there is a five-year lag between the initial impact of food limitations on vital rates ($E < 1$; yr 347) and a turn-around in population growth rates, $d^2N / dt^2 = 0$ (yr 352; Figure 2B).

A *copial surplus* exists at any point along this growth pathway at which the aggregate amount of food produced is in excess of consumption:

$$S_t = N_t (E_t - 1)j_0 \quad (1)$$

From Equation (1), the copial surplus at time t , (S_t), is determined by the number of individuals (N_t) times the difference between the prevailing food ratio (E_t) and the minimum ratio consistent with baseline vital rates, $E = 1$, times the baseline age-weighted kcal requirement per individual, j_0 . This definition of surplus is rooted in the perspective of the agrarian population, being the difference between what the population requires to avoid hunger and what it has the potential to produce. Although surplus may be exploited through taxes, we do not define S_t in terms of taxation.

Figure 2E reveals an intermediate maximum surplus at year 293, which we designate as S^* , the (*) representing a maximum. So long as the marginal productivity of labor is positive in the founding population (i.e., the food produced by a new individual exceeds the food required to maintain him or her at the $E = 1$ level), there will be an intermediate maximum of surplus. This is a transient maximum because there is nothing in the population growth model to suggest that growth will be arrested at this peak; well fed, the population continues to grow at its maximum rate.

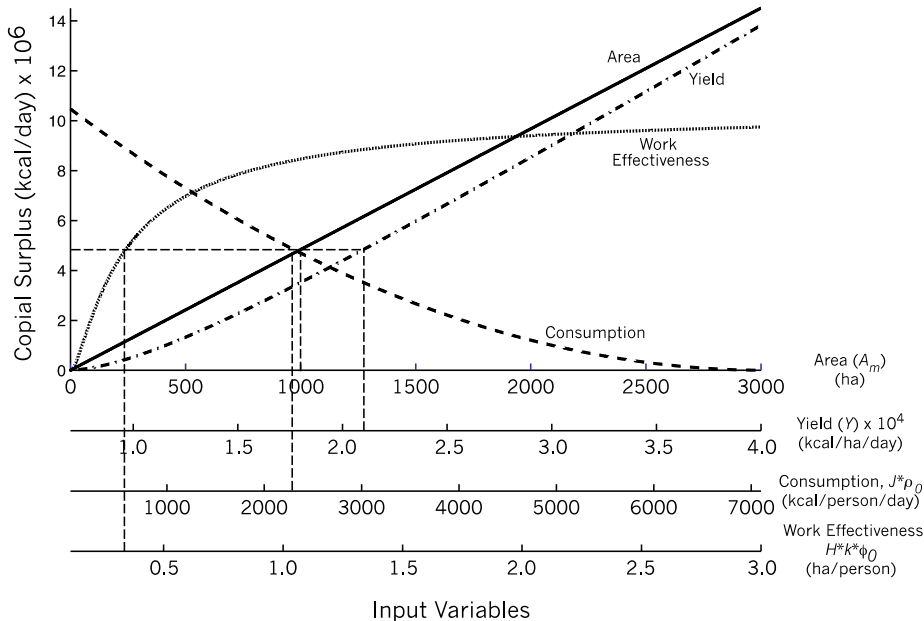


Figure 3. Copial surplus magnitude, as a conditional function of area, yield, work effectiveness, and consumption. This illustration should be read as follows: conditional on the remaining three input parameters or combinations being held at their default values, indicated by the vertical dashed lines, the total surplus available at the copial maximum responds to the fourth parameter as shown by its curve and x-axis values. For example, with consumption, area and yield at their baseline values, increasing work effectiveness in the range of 0.2 to 0.5 ha/person adds rapidly to total tax, but this increase reaches a shoulder and approaches an asymptote above values of 1.0 ha/person. Each input parameter can be read individually in this manner; the illustration cannot be used to represent the joint effects of parameters. Default baseline values are as follows: Area (A_m) = 1000 ha; Yield (Y) = 21,000 kcal/ha/day; Consumption (j_0) = 2318 kcal/person/day; Work Effectiveness (w_0) = 0.3483 ha/person. See Supplemental Materials, Table D1.

S^* occurs 54 years before density dependence begins to affect the producing population, at ($E_t < 1$; year 347); it occurs just over a century before the population approaches equilibrium (Figure 2A). Perhaps more surprising, the maximum potential for copial surplus production occurs when the population is only 25% (3382/13,509) of its Malthusian equilibrium size. In effect, a

population's capacity to generate a surplus that avoids hunger is greatest when it is relatively small; it is not a linear function of population size, as is sometimes assumed. We show in the Supplemental Materials (Sections A and D) that the period of time from initial settlement to this copial maximum is an inverse function of reproductive rates (r), a direct, log-linear function of arable land (A_m) available, and an inverse log-linear function of initial settlement size (N_0).

Figure 3 (also Supplemental Materials, Table D1) illustrates the effects of changing the cultivable area (A_m), agricultural yield (Y), population work efficiency (w) and per capita consumption (j) on the magnitude of the copial surplus maximum. The vertical dashed lines isolate the combination of baseline values represented in the population trajectory of Figure 2, from which we derive a copial surplus maximum of 6.41×10^6 kcal/day. The graphic depicts the form of relationship between each of the four input variables represented on the x -axis, *considered one at a time*, and the copial surplus maximum. Thus, holding the other three input variables constant at their baseline values, increasing area (A_m) results in a linear increase in the maximum copial surplus. Expressed as an elasticity,¹ the effect of yield (Y), all other inputs constant, becomes more linear and approaches 1 ($= w_Y/w_Y$) as yield becomes large relative to consumption (Supplemental Materials, Table D1f). As expected, increasing baseline consumption depresses total tax revenue at the copial maximum.

The response of the copial surplus to changes in work effort w is positive and quite high when population work productivity is low but gains rapidly diminish as w grows in magnitude (Figure 3). With yield per hectare and total area fixed, the effectiveness of increasing the copial surplus maximum by expanding working hours and age classes engaged in agrarian labor quickly begins to saturate as the added production of each new worker approaches zero in the crowded fields. Vital rates do not enter estimates of the copial surplus maximum other than through their effect on the baseline age structure (Supplemental Materials).

Early commentaries show sensitivity to such issues:

The more encouraging view toward trade in the Sanskrit text, the *Sukraniti*, c. A.D. 700, suggests that states should not overburden or over tax agriculture, merchants, or temples; the state must maintain 20 times the cost of a potential crisis in the treasury, but the crisis itself must be paid for by a loan from a merchant consortia or a few large merchants and paid back in a timely manner with interest

¹ Elasticity here represents the instantaneous effect on the dependent variable (copial surplus) of a unit change in the independent variable (w). See Supplemental Materials, Sect. D for discussion.

(Sarkar 1914). The best king enriches by tribute (conquest and control of agriculture), the middle king enriches by trade, and the worst king enriches himself by overtaxing temples (Oka and Kusimba 2008: 359–360).

While a lot is revealed in this text, ostensibly about trade and the possibility of seeking expedient loans from merchants, we focus on the admonitions not to overtax in general and the concluding suggestion that a prudent leader seeks revenue from conquest leading to expansion of agriculture before turning either to trade or to the temples and, presumably, the agrarian population on which the temples may depend.

It might surprise our exchequer that a relatively small, natural-fertility population of agriculturalists—one well below its equilibrium size and, in time, significantly short of the culmination of its growth trajectory—has significant potential for extraction of surplus at a relative high standard of welfare measured in terms of food availability and baseline vital rates. An exchequer intent on hastening access to this transitory surplus would promote a high reproductive rate (r_0). An exchequer intent on increasing its magnitude will find that greater territory and higher yield have a strong, positive and linear or near-linear effect, with no intrinsic constraints. Sharp gains are possible for labor effectiveness (the maximum area a worker can farm) only if it is quite low to begin with; these gains quickly are exhausted. Reducing baseline consumption requirements would also elevate S^* , at least so long as kcal consumption is adequate for a working livelihood, but these requirements are based on physiological responses and therefore assumed to be of limited flexibility.

4. Taxation at the Malthusian Equilibrium

4.1 Per Capita Tax

To this point we have examined the potential for extracting a tax via a transitory surplus of production from a growing population that is well below density-dependent constraints on its size. However, we have no reason to think the population will stabilize around S^* , as conditions remain propitious for growth well after this point (Figure 2). In light of this, we now consider taxation and its effect on producers and system stability for a population that has reached a density-dependent equilibrium.

At the population's Malthusian equilibrium there is an inverse relationship between the level of taxation imposed by our exchequer and the size of the population, \hat{N} . Equilibrium per capita food availability, \hat{E} , determines the point at which births balance deaths; if taxation increases, fewer workers and their

families can be fed on the reduced after-tax kcals available. This effect is shown in Figure 4. The straight lines represent total tax revenues as a product of a per capita tax multiplied by the population size. The curve represents total tax revenues, or total production at that population level less what the population must consume to sustain itself at the equilibrium food ratio, \hat{E} . The line and curve intersect at an equilibrium solution for population size. At zero per capita tax we have the full equilibrium population of 13,509 (see Figure 2). As per capita tax (τ) increases, equilibrium population shrinks. Up to a point ($\hat{N} = 4507$; $\tau = 2120$ kcal/individual/day), increasing the per capita rate increases total revenue to the exchequer. After that point further per capita rate increases result in declining total revenue. A per capita tax rate greater than the initial slope of the total tax revenue curve will reduce the producing population to zero from starvation. In summary, at Malthusian equilibrium, there is an intermediate optimum in total tax revenues as a function of the individual tax rate. This relationship reflects both the competition between the state and its agricultural population for resources and the state's dependence on the productivity of this population. We further characterize these relationships in the Supplemental Materials.

Figure 5 shows how total tax at equilibrium responds to key environmental and production and consumption parameters. All of the relationships are similar in form to those of the copial surplus maximum (Figure 3), so we do not repeat the previous discussion. Note that y -axis values, however, are significantly larger.

4.2 Fixed Tax

A per capita tax imposed on an equilibrium population has only one non-zero solution (Figure 4). Our exchequer might decide instead on a fixed total tax, independent of N . A line representing this form of tax does not pass through the origin except in the trivial case of no tax (see Puleston and Tuljapurkar 2008), creating the possibility of two equilibrium solutions, only one of which will be stable (Figure 6).

Compared to Figure 4, in Figure 6 we rotate tax collection to the x -axis, making total revenue (T) the independent variable. On the y -axis we represent the two linked factors associated with a particular total tax: equilibrium population size (\hat{N}) and the per capita taxation rate (τ) that achieves that equilibrium. This means that the y -axes of Figure 6 are inverse and only the per capita tax is on a linear scale. The concave left curve shows the two equilibrium combinations associated with each level of total revenue. In effect, the same exchequer's assessment can be met from a high per capita tax extracted from a relatively small population or a small tax extracted from a larger population.

The dashed line combinations—high per capita rates applied to low equilibrium population size—are unstable in the face of small random fluctuations of

population; the solid line combinations are stable. For this reason, we almost surely will find our Malthusian phase population somewhere on the solid line.

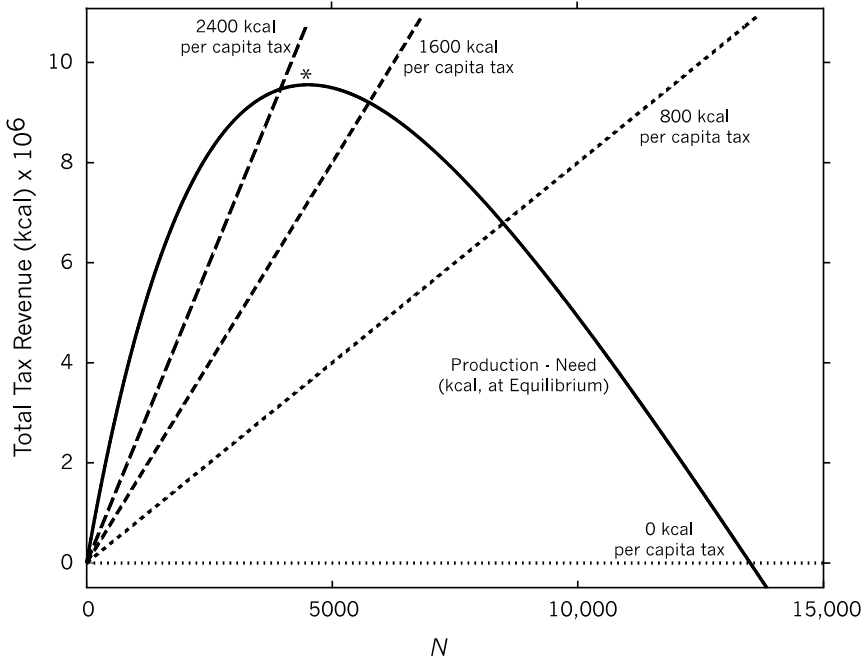


Figure 4. Total tax revenue and population size at equilibrium, as a function of per capita tax rate. The solid, concave downward curve shows all possibilities of total tax revenue as function of population size N . It is determined by total production less equilibrium food requirements ($\hat{E} = 0.668$) at a particular population size. The straight lines show total revenue as a function of per capita tax rate and population. The intersection of these elements establishes the population size and total income at equilibrium associated with a particular rate of taxation. Zero tax produces our default equilibrium density of $\hat{N} = 13,509$ and no net revenue. Increasing tax rates lower equilibrium population size but increase revenue, up to a maximum of T^* . At T^* revenue is 9.55×10^6 kcal/day, set by a per capita tax rate of $\tau = 2120$ kcal/person/day, assessed on a population, \hat{N}_{T^*} , of 4507 individuals.

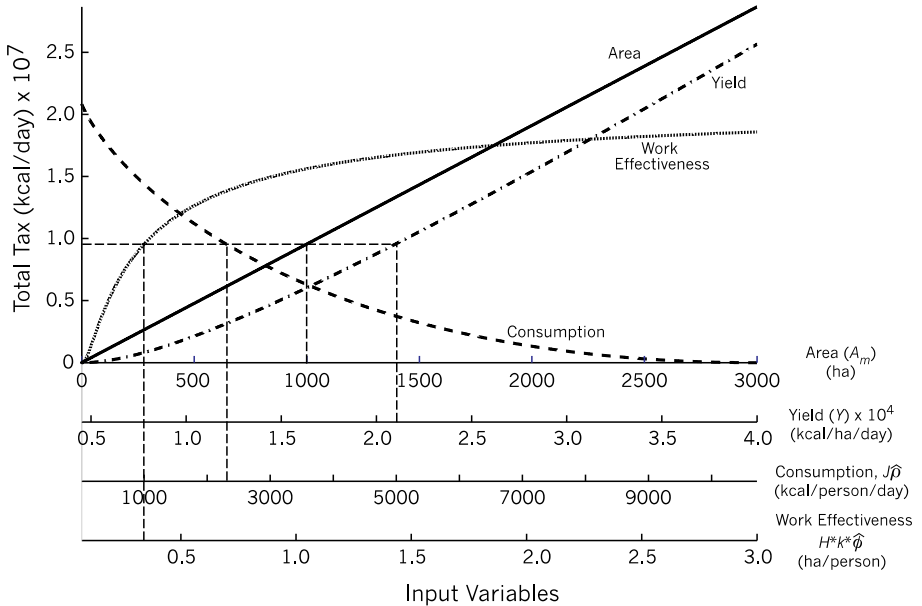


Figure 5. Maximum taxation at Malthusian equilibrium, as a function of area, yield, work effectiveness, and consumption. This illustration should be interpreted by the same conventions as for Figure 3; the baseline values and the form of the conditional responses are the same. Note, however, that the y-axis in this case is an order of magnitude larger (units of 10^7 versus 10^6) than that for Figure 3. See Supplemental Materials, Table D2.

The dynamics work like this: at equilibrium the food ratio \hat{E} has fallen such that the per capita birth rate equals the death rate (and $\hat{r} = 0$); less food implies the population must shrink; more food implies it will grow. Imagine the population rests on the dashed line at point (a). If it is randomly reduced in size from some density-independent cause to point (b), it could form a zero-growth equilibrium only if total tax collections were to fall to the level associated with point (c). Unaware of this dynamic instability, our exchequer continues to insist on collecting the original 7×10^6 kcal/day associated with point (a). He thereby drives E to a level further below replacement and population shrinks again. As this cycle repeats the population collapses, having been drawn to the zero-population boundary at the top of the graph. By contrast, a perturbation above the dashed line implies population capacity to supply more taxes which, being unclaimed by the exchequer, sustain E above replacement, ensuring population growth. This process likewise is reinforcing, and sets the population on a course

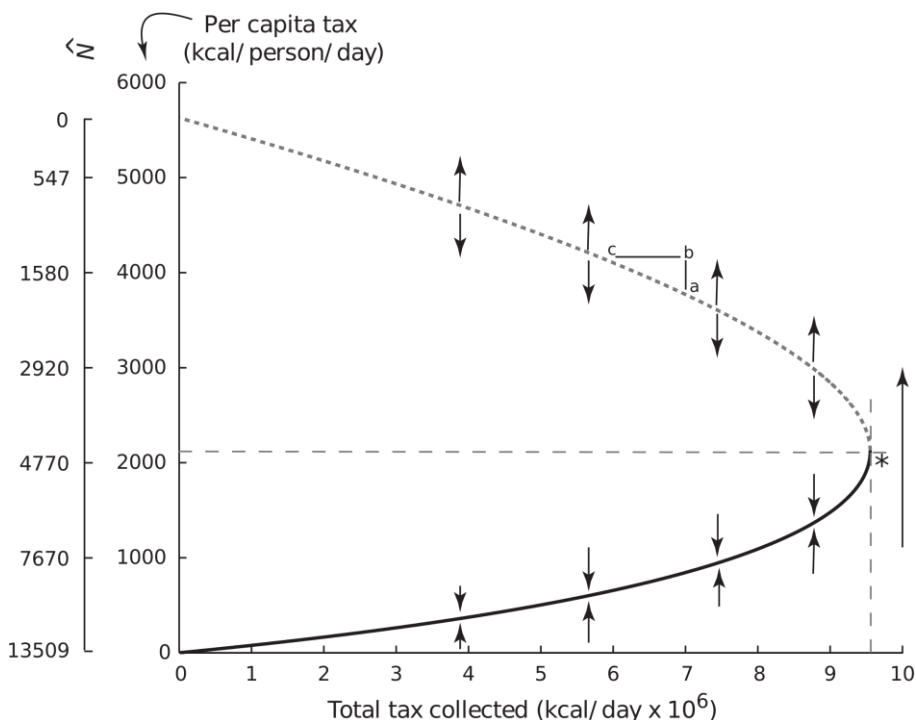


Figure 6. Unstable and stable equilibria for the product (fixed per capita tax rate \times population size) as a function of total tax collected. Note that the y-axis scales are inverse to one another, and that the per capita tax rate scale is linear while the equilibrium population size scale (\hat{N}) is not. Each quantity of fixed-tax collection shown on the x-axis can be generated by a small population assessed at a high rate, or by a larger population assessed at a lower rate. The former (dashed) line is an unstable equilibrium; the latter (solid line) is stable. Unstable equilibria, if perturbed below \hat{N} , decline in size to extinction; if perturbed above \hat{N} , they move to a corresponding stable equilibrium. The arrows show the direction of these tendencies. The maximum sustainable fixed tax is 9.55×10^6 kcal/day, and corresponds to a population of 4507 assessed at a rate of 2120 kcal/person/day. Further details in the text.

toward the solid curve combinations. The same logic applied to the solid curve gives us an equilibrium that is stable to minor population fluctuations.

Even at the stable equilibrium, represented in Figure 6 by the solid line, the exchequer faces a trade-off: greater income entails fewer subjects who are taxed

more heavily. The exchequer can entirely forgo income and have 13,509 subjects or seek the maximum income of 9.55×10^6 kcal/day and have a more modest population of 4507 subjects. A policy choice must balance agrarian revenue against the benefits of a large citizenry. Ironically, an exchequer on behalf of the despot may prefer a relatively high per capita tax and total income in part *because* it reduces the numbers and hence the threat of revolt by the producing subjects. This would be especially tempting if the military forces were part of the elite, maintained by high revenue in sufficient numbers to control discontent because it is expressed by a smaller number of subjects. Unchanged across the full array of stable equilibrium choices is the hunger of the producing population, which is reduced to about two-thirds of its initial diet by the equilibrium food ratio, $\hat{E} = 0.668$ (see Puleston and Tuljapurkar 2008).

The exchequer's margin of error in making these assessments, where error is defined as inducing a trend toward demographic collapse, is reduced as the fixed tax rises. Two kinds of mistakes are possible, even if the environment yields and vital rates are constant: (i) the exchequer may, early in population growth, prematurely impose a fixed tax at such a high level that it rests in the unsustainable zone above the dashed line of Figure 6, setting the population into a spiral to extirpation; or, (ii) the exchequer might impose a tax to the right of *any* stable solution (above 9.55×10^6 kcal/day). In both cases, the margin of error diminishes as the levy increases. It is easy to imagine how the many uncertainties of the exchequer's information—empirical and conceptual—would lead either to significant (and costly) caution or to frequent crises, not to say the anxieties that we mentioned earlier.

4.3 Comparison: Copial Surplus to Malthusian Maximum Tax

In Table 2 we compare the copial surplus at its maximum (S^*) with the largest tax that could be extracted under a Malthusian equilibrium, for our baseline environmental and demographic parameters (Table 1). The per capita tax rates are 1896 and 2120 kcal/person/day, respectively. The maximum transitory surplus available for taxation (S^*) is roughly two-thirds (67.1%) of the maximum levy (T^*) that can be extracted at Malthusian equilibrium. The populations themselves occur in a ratio of 0.75:1 (3382:4507), and both are small compared to an untaxed population that reaches its full equilibrium size of 13,509. However, population welfare is starkly different in the two scenarios: from the copial maximum surplus to the Malthusian maximum tax life expectancy falls from 45 to 30 years, the probability of survival to age five from 77% to 65%, and the food ratio E from a surfeit of 1.82, of which 0.82 could be going to the exchequer, to a hungry 0.668. The population growth rate falls from $r = 0.0176$ to 0.

Table 2. Comparison of Transitional (Copia) Maximum Surplus (S^*) with Equilibrium Maximum Tax (T^*).

Parameter	Transitional (Copia) Maximum	Equilibrium Maximum	Units
Per capita tax rate	1896	2120	kcal/person/day
Total tax revenue	6,413,300	9,553,360	kcal/day
Population (N, \hat{N})	3382	4507	individuals
Life expectancy (e_0, \hat{e}_0)	45	30	years
Probability of survival to age 5	0.773	0.652	
Food ratio (E, \hat{E})	1.820	0.668	
Reproductive rate (r)	0.0176	0	

5. Impact of Taxation on the Subsistence Food Ratio

The state leadership of a growing agrarian population may also want to know whether it is better to extract resources in the form of agricultural output (kcal of food) or labor, and in fixed, proportional or per capita assessments. This is to ask how peasant livelihood and welfare depend on the form and severity of taxation. Put baldly, as might an exchequer, what form of exploitation offers the most abundant state income with the least impact to work effort or size of the peasantry? To answer this question, we examine the consequences of exploitation for the food ratio, E , using the concept of elasticity to capture generally the degree to which a unit change in the tax affects the food ratio, E_t . We track the answer to our question over the full range of population growth, from founding and the copial phase to density-dependent equilibrium.

We consider five forms of taxation shown graphically in Figure 7 (Supplemental Materials, Table B1). The actual levels of taxation depicted in our illustration are arbitrary and altering them will affect the numerical elasticities we report. However we are more interested here in the structural *pattern* of response (see Winterhalder 2002), which is independent of the magnitude of taxation provided that the burden is not so great that the population ceases to exist. The food ratio elasticities are all negative because a unit increase in taxation

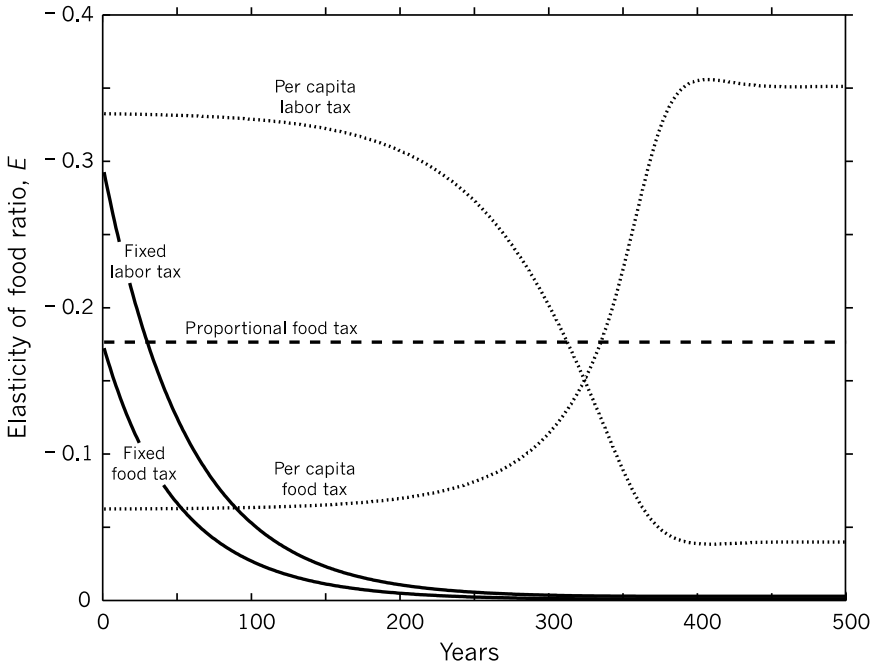


Figure 7. Impact of taxation as a function of population growth. The curves in this illustration represent the instantaneous relative decline in the food ratio, E , due to a unit increase in taxation, technically the elasticity of E , for five forms of tax: a fixed food or labor levy, a proportional food levy, and a per capita food or labor levy. For instance, early in the habitat-limited growth trajectory of the producing population an increase in per capital taxation of labor has a much larger impact on E than per capita taxation of food production; this effect reverses as the population grows toward equilibrium. The elasticities are negative because a unit increase in taxation always causes the food ratio to decline. For the tax rate, and initial year 1 and year 500 elasticities for each form of tax, see Table 3. The per capita labor and per capita food tax curves cross at year 324 for this particular parameter set, 23 years before the first impact of density-dependent constraints on vital rates (Figure 2). See Supplemental Materials, Table B1.

must result in a decrease in the portion of total food production available to the agrarian population.

Fixed taxes in produce which remain in place across the full sweep of population growth always are more burdensome on smaller than on larger populations. In fact, any fixed tax that can be borne by a small population

becomes negligible to a population near its equilibrium size, approaching $T/(YA_m F_t)$, the ratio of the food tax (T) to total production, defined by the yield times the cultivable area modified by F_t , the fraction of that area that may be farmed by the available labor pool.

Calculating the elasticity of a fixed labor tax requires that we convert labor to land worked using the parameter, k , and the number of hours per day each worker provides to the state, Λ . The elasticity then is the maximum amount of land that could have been cultivated with the taxed labor, $k\Lambda$, divided by the amount of land the population can cultivate with the remainder of its labor hours, $A_m F_t^\Lambda$, where F_t^Λ is the fraction of available land the population can cultivate after accounting for a labor tax at time t . Again, a feasible tax when the population is small becomes negligible when it is large, the elasticity falling asymptotically toward the ratio of the adjusted tax to total production, $k\Lambda/A_m F_t^\Lambda$ (Figure 7). At low population size the negative impact of a fixed tax on labor is greater than that on food, a pattern that will repeat for the per capita tax option (discussion below).

Table 3. Elasticities for the five tax scenarios represented in Figure 7.

Tax type	Tax amount	$t = 1$	$t = 500$
Fixed food	20,000 kcal total/day	-0.172	-0.001
Fixed labor	20 hr total/day	-0.292	-0.002
Proportional food	15%	-0.177	-0.177
Per capita food	400 kcal/worker/day	-0.063	-0.351
Per capita labor	2.5 hr/worker/day	-0.333	-0.040

A proportional tax on food production has a constant elasticity. It is equal to negative the ratio of the fraction of production that is taxed to that which is not, or $-f/(1 - f)$. As depicted in Figure 7 for our baseline parameters, a 15% tax on production gives us an elasticity of -0.18 across the full range of population growth. For comparison, the conventional sharecropping payment of 50% of production would entail an elasticity of -1.

Per capita taxes on labor and food have quite dissimilar effects on the food ratio. When the population is small, productive land is plentiful and food is produced in abundance with relative ease. Food availability is limited by labor, not environment. Because labor is so efficient, the marginal cost to the food ratio, E_t , of pulling labor out of production is relatively high. This results in the high negative elasticity associated with the per capita labor tax. By contrast, in the early stages of population growth the impact of an increase in the per capita food tax has a much more modest negative impact on E_t .

As the population grows the relative impact of labor and food taxes inverts. The elasticity of E_t with respect to the per capita food tax (τ) may be thought of as the ratio of the total food tax to total production after taxation. The total tax, τN_t , grows with population more quickly than total production due to the mounting density-dependent effects of competition for increasingly scarce land. As a consequence, as the arable land is filled, the elasticity of a per capita food tax rises dramatically relative to a per capita labor tax. The impact on the food ratio of pulling scarce food from the system is high, but labor is so plentiful that it has become redundant and expendable.

An exchequer, assigned to fine-tune resource taxation policy with the goal of avoiding rebellion while insuring a steady income for the state, would read from these results that it is best to tax a small and growing population on a per capita basis in agricultural produce. When the population begins to saturate the habitat, having put the greater part of its land into cultivation, an effective exchequer will recommend switching to a per capita tax on labor. Across the population growth trajectory, the effect on peasant welfare of food taxes increases whereas that of labor taxes declines. The tipped hourglass form of this relationship counsels the exchequer to switch from one to the other means of exploitation.

So long as the after-tax $E \geq 1$, then whatever the impact of taxes, the population will continue to grow at its maximum rate. The peasant farmers may have to work harder than they would like to meet the demands of the state, but in other respects their vital rates and quality of life measures (e.g., e_0 , life expectancy) are unaffected. Tax payments induce hunger and declines in vital rates to the extent that they augment density-dependent processes that push E_t below 1, perhaps at a smaller population and earlier than would be the case in the absence of taxes. At this point the choice between the forms of taxation become burdensome to producers in the specific sense that it implies fewer of them. As we noted earlier in discussion of Figure 6, whatever the revenue-population trade-off, at Malthusian equilibrium population welfare is poor.

6. Population Survival under a Fixed Tax Regime

We have burdened our exchequer with uncertainty, but so far it is uncertainty only about fixed but difficult to determine demographic and environmental parameter values and their consequences. The reality would certainly be confounded with environmental stochasticity and thus quite a lot more difficult. The effects of random elements of drought, insect infestations and other unpredictable afflictions of agricultural production on state income levels and security would be major sources of anxiety.

To address this, we examine the properties of the fixed total tax options under the assumption of randomly fluctuating agricultural yields. We simulate year-to-

year yields as random draws from a gamma probability density function with a mean of $Y = 21,000$ kcal/ha/day, as in earlier results, and a fixed coefficient of variation (CV) of 0.20. The gamma distribution is symmetrical in this range and not significantly different from a normal distribution; there is no secular trend in the data and yields are not auto-correlated in time. Over a sample representing 600 simulated years the realized mean yield is 20,847 kcal/ha/day and the standard deviation is 4238 kcal/ha/day (CV = 0.203). The lowest yield is 9,253 kcal/ha/day; the highest is 34,283 kcal/ha/day (representative time series dynamics can be seen in Winterhalder *et al.* 2015: Figures 2 and 3).

A fixed tax gains a predictable revenue stream for the exchequer so long as the agrarian population providing it is able to persist. Producer viability can be challenged by internal dynamics that draw the population to zero (Figure 6), acting in concert with yield fluctuations. By repeating simulations at various levels of fixed tax and degrees of environmental variability expressed as yield CV we can examine how these factors interact in affecting population persistence. Because the likelihood of extirpation does not change with the length of time for which a population has survived, we express the frequency of population collapse as a half-life (Figure 8).

As expected, both an increasing tax rate and an increasing yield variance reduce the survival probabilities of the subsistence population. Unexpected, however, is the bent-elbow form of the relationship, which is striking in the sharp opportunities it affords and constraints it places on population viability. For instance, a tax rate of 40% of the maximum deterministic tax offers nearly infinite population survival prospects if the yield CV is below 0.28. It offers almost immediate threat of collapse if the yield CV is above 0.38. If yield CV < 0.20, a 60% tax rate is nearly indefinitely survivable; if yield CV > 0.50 a 20% tax rate is a serious hazard. Assuming that the long term matters, our exchequer has little room for error in crossing over such sharp boundaries of reduced persistence. Environmental variance can sharply constrain the degree of exploitation consistent with survival of the agrarian population and the state it supports.

7. Discussion: Financing and Sustaining the Peasant State

7.1 Surplus

Surplus has a convoluted and fraught intellectual history in anthropology (e.g., Pearson 1957; Harris 1959). It nonetheless remains an important concept in explanations of origins of centralized political governance (Tisdell and Svizzero 2017: 10–12). We have defined surplus as the difference between copial phase production possibilities and the amount of food required to maintain the agricultural population's baseline vital rates. That is, surplus is any production in

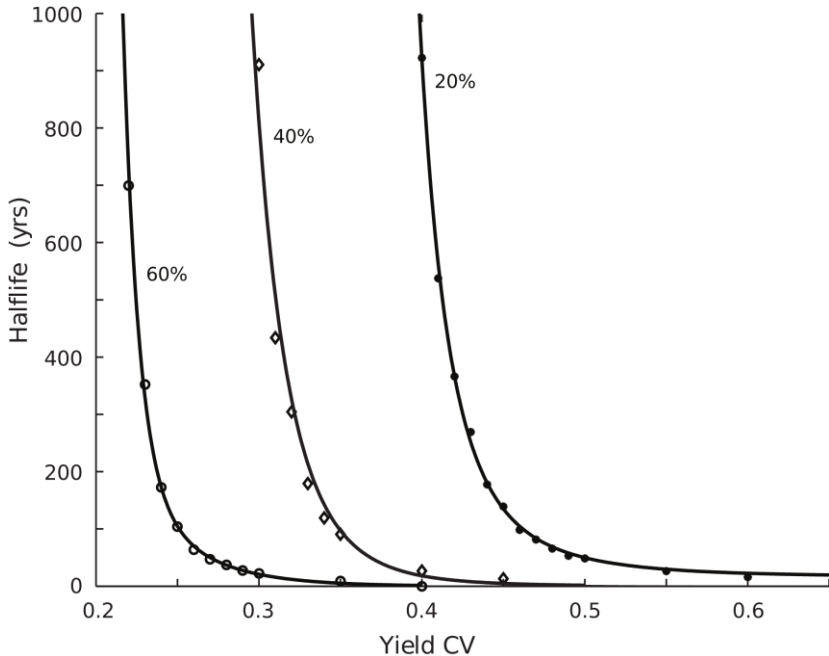


Figure 8. The half-life of an agrarian population as a function of yield variation and rate of taxation. The filled circles (20%), open diamonds (40%) and open circles (60%) represent rates of fixed taxation as a fraction of the maximum deterministic fixed rate (see Figure 6), determined by repeated simulation; the solid lines are best fit polynomials. Yields are represented by random draws from a symmetrical gamma distribution with a mean of $Y = 21,000$ and a coefficient of variation (CV) as shown. Increasing the yield CV for a particular tax rate shortens the expected persistence of a population; raising the tax rate for a particular coefficient of variation has the same effect. The angular or “elbow” shape of the curves implies that small changes in yield variation or taxation can dramatically shorten or lengthen population half-lives, e.g., elevate or decrease year-to-year odds of extirpation.

excess of that needed to avoid hunger. We obtain a copial surplus by reasonable parameter assignments determining which age groups work, for how long, and with what effectiveness in producing food calories from the environment. We have assumed these to be constant, however much food is produced. A time-minimizing (Smith 1987) or Chayanovian (1977) approach would have our agrarian producers continuously adjusting their work effort so as to just maintain

$E = 1$ for as long as possible and there would be no copial surplus. Within the context of our model, identifying production above $E = 1$ as surplus is a conceptual matter and not an endorsement of a particular political economy.

7.2 Copial Surplus, Malthusian Tax

A population might be positioned in the early phase of copial growth for several reasons: because it is recently settled in a new habitat, because it is long established there but recovering from an episode of depopulation, or because it is experiencing a density-dependent release due to a technological innovation or favorable environmental change. If the population is small with respect to its equilibrium size it will experience a prolonged peak of potential surplus production relatively early, well before density-dependent limitations come into play. This maximum (S^*) is transient, but elevated surplus production can be lasting (e.g., 100+ years for our default scenario; Figure 2E). During this phase, taxation can be extracted without having an impact on the agrarian population's vital rates or demographic quality of life variables. The food producers enjoy abundance and well-being, although their work effort is above the level that would sustain these amenities in the absence of taxation.

The effect of taxation on an equilibrium population is of a different nature. Resources are obtained from a population suffering the Malthusian duress of suboptimal food intake, low fertility, high mortality and a shortened life expectancy (Table 2). Under such circumstances there is no surplus as we have defined it in the copial phase. Taxation under the conditions of Malthusian equilibrium entails a direct trade-off: state revenue in goods or labor versus subject population size. Under our default conditions and focusing only on the stable outcomes (Figure 6), each per capita tax rate from 2120 kcal/individual/day down to zero determines an equilibrium population density ranging from 3875 up to 13,509 individuals and a total tax collection ranging from 9.55×10^6 kcal/day down to zero. Perhaps counter-intuitively, although individual tax rates and population size vary across this set of revenue expectations, hunger and vital rates (fertility, mortality and life expectancy) do not. Life under Malthusian equilibrium conditions is equally punishing, whatever portion of production is redirected from feeding the agrarian population to elite income.

Anthropologists debate the causal role of population processes in the origins of the state, but they typically agree in conceptualizing that causation in terms of "population pressure." For instance, in Carneiro's (1970) well known circumscription model, population growth in an over-full landscape provokes conflict and subjugation of weaker by stronger political units, leading to wider political integration. But population *pressure* is not the only circumstance in

which population might be important. Surplus co-opted by authorities can be used to build powerful state-level institutions, and it can be produced in abundance by a population that is well below the size that invokes ideas of population pressure and density-dependent constraints on labor and vital rates.

Upper Egypt, beginning around 3000 BC, gives us a case example. In a comprehensive review, Allen (1997) evaluates five explanations advanced for state origins in light of the shift in Egypt to agricultural food production and state consolidation. Specific to our purposes, he notes that theories based on population pressure fail in Egypt because “the Nile Valley was underpopulated—not overpopulated—at the time the Egyptian state was created. The Pharaoh’s rule was not the result of diminishing returns to labor in Egypt” (Allen 1997: 136). Allen instead builds on Carneiro’s circumscription model, amended by the observation that full occupation of the productive landscape was not essential to elite imposition of control. In Allen’s view, agricultural production entailed the essential preconditions of state formation: amenability to storage, high per hectare yields, high returns to labor and seasonality of labor requirements. However, the “salient fact of Egypt is that state formation occurred when the population density was low” (138), and subsistence farmers could escape taxation simply by moving downriver. Here geography came to the aid of elites. The very narrow corridor of fertility along the Nile, bounded to the east and west by inhospitable desert, provided a tight geographic constraint, allowing elites to design tax administration that immobilized producers and captured their surplus resources. Unoccupied land was available but the frontier beyond which it lay was easy to close. Allen’s amendment of Carneiro thus focuses on the conditions that allowed surplus to be captured despite low density; our model provides the complementary insight that considerable surplus likely was available and perhaps easier to capture *because of low density*.

The Egyptian Kingdom is notable for persisting nearly 3000 years, through remarkable population growth. Estimated to be 350,000 around 4000 BC, the Nile Valley population had risen to 5 million by 150 BC (Allen 1997: 145). Our modeling suggests that it would have been a significant challenge to negotiate a shift from a tax regime of copial surplus extraction (Figure 3) to exploitation under more Malthusian conditions (Figures 5 through 8) as the Nile Valley population grew. This case and likely others offer many opportunities to test the mechanisms we propose to be important to state development and likelihood of persistence.

7.3 Taxing Goods, Taxing Labor

We have shown that the impact on E of taxation in goods (kcal) is low relative to a tax on labor in the early, copial stages of population growth, the relative impact

of exploiting goods and labor switching as the population approaches equilibrium (Figure 7). Early in a growth trajectory returns to labor are high in the form of per capita productivity. Food is abundant but labor is in short supply. Late in the growth trajectory food is more dear. Under Malthusian conditions the marginal cost of taking kcals is relatively high but the marginal cost of removing labor from a production system in which it has little more to contribute is low.

It is important here to consider what these impacts do and do not imply. In the copial phase, a reduction in the food ratio, E , for instance due to incremental population growth, means greater work effort to meet tax and baseline consumption needs. But, so long as $E \geq 1$ it does not matter to vital rates and commoner well-being. At Malthusian equilibrium, differential impacts are expressed through their effects on population size, not life expectancy and vital rates. Taking taxes in kcals suppresses the population size to a greater degree than a labor tax, but it will not further diminish the food ratio, which is determined solely by Malthusian constraints.

Correctly assessing the population-dependent impacts of different forms of elite taxation matters to archaeological problems. For instance, Drennan and Peterson (2011: 75) state that the magnitude of a population's investment in public works "can be thought of in terms of labor, irrespective of whether the population contributes labor or goods..." While it is true in the abstract that goods and labor can be converted one into the other using kcal equivalents of consumption, work effort and output, as we have done in various isolated steps of Figure 1, the population ecology dynamic that results when these steps are taken together shows that goods and labor are not interchangeable. In a system context, the relative scarcity and abundance of production factors interact with vital rates to differentiate their impacts. Revenue sources, whether tapped from external wealth or internal taxation, have become a marker in the attempts to distinguish and characterize archaic states organized according to autocratic or collectivist principles. Early states tending toward more republican forms of government typically rely to a greater degree on taxation for the provision of public goods (Wade 2017; also Feinman and Nicholas 2016: 283–287).

Or, to take another example, Culbert (1988: 99) has proposed that the collapse of Classic Period Maya polities was due to an agricultural labor shortage, as agrarian workers were unwisely diverted to build monuments or serve the state in other capacities. Our results cast doubt on this hypothesis. The Terminal Classic collapse capped a long period of population growth to high densities, precisely the circumstance in which the marginal agricultural return to labor is minimal. With a significant part of the population redundant so far as its capacity to produce yields from agrarian work, it safely could be diverted to state activities with little or no effect on total production.

Issac (2013: 442; D'Altroy 2015) summarizes a well-documented instance in which states diverge on this question: "The Aztecs taxed mostly in material items ... The Incas, in contrast, taxed almost entirely in labor," with the Inca state itself supplying the materials on which the laborers worked. Isaac suggests broad implications followed from this divergence in elite resource acquisition policy. The Aztec requirement that resources be provided in goods promoted commodification and the development of markets (see also Gutiérrez 2013) and a more urbanized settlement pattern, whereas the Inca policy entailed more coercive intervention in the day-to-day lives of state citizens. The Inca instituted their labor tax in the form of *mit'a*, an obligation by turns similar to the feudal corvée (Stanish and Coben 2013). More generally "the control of labor was the fundamental basis of economic power in the pre-Hispanic Andes" (Topic 2013: 343). We know much less about the means by which Maya elites, for instance, extracted resources from the agrarian producers they taxed (McAnany 2013).

Our results add an implication to those noted by Isaac: the economic effectiveness of taxing agricultural goods or labor changes as a function of population size relative to limits on agrarian space. Evidence gathered by economic historians shows that the form and impacts of resource extraction may shape societal evolution for a considerable time after a particular tax and perhaps even the polity imposing it have ceased to exist. We cite two empirical cases. From 1573 to 1812 the Spanish colonial government of Peru and Bolivia instituted a rotating *mit'a* that levied a district-by-district compulsory labor contribution. It required that one in seven adult males work in the mines of Potosí and Huancavelica. An econometric and historical analysis by Dell (2010), comparing districts subject to this tax with those geographically matched but not affected, demonstrates that it has measurable *contemporary* consequences, among them a 25% reduction in household consumption and 6% increase in childhood stunting in the affected districts. Dell isolates land tenure and the underdevelopment of public goods as the structural factors that give the *mit'a* continuing impact two hundred years after the silver mines were depleted and the labor levy abandoned.

In a similar example, land taxation and collection regimes instituted during the 18th and 19th centuries by administrators of British colonial India varied regionally in a pattern that today, long after independence, accounts for differences of up to 25% in wheat yields. In their explanation, Banerjee and Iyer (2005) cite enduring political factors that shaped the potential of local populations for collective action and public investment. Extractive policies like those we model may have institutional effects long after the policies themselves and their state sponsors have disappeared, imposing changes on subsequent institutional evolution in the aftermath of state collapse. Dynamic models such as

developed here may help us to understand why collapse is followed by a resurgence of political consolidation in some but not in other cases.

7.4 Options for Affecting the Magnitude of State Revenue

The amount of surplus an exchequer might expect is a function of consumption, labor productivity, agrarian habitat area, and yield. We review the exchequer's options, working from the least to potentially the most effective (Figures 3 and 5).

Consumption ($j_0 = J\rho_0$). A unit increase in per capita baseline agriculturalist consumption (the daily caloric requirements of the neediest age class, J , scaled to reflect the population's age structure via ρ_0) causes a decline in maximum copial surplus or equilibrium tax. The elasticity describes a near-linear relationship, which may be approximated $-2j_0/w_0Y$, or equivalently, $-2/E_m$, so long as density-independent output (w_0Y) is significantly greater than consumption (j_0) (Supplemental Materials Table D1h). E_m is the theoretical maximum of the food ratio in a given context, achieved when the population is infinitesimally small and thus competition is minimized. Our parameterization of caloric requirements assumes an active agrarian lifestyle, but we do not vary consumption by work effort, nor do we track the effects on effort of suppressing consumption. Overall, the physiological range over which consumption can vary is quite constrained, making it a poor candidate for extracting enhanced surplus.

Labor productivity ($w_0 = Hk\phi_0$). The elasticity of revenue to labor effort is the same as for consumption but is positive instead of negative (Supplemental Materials, Table D1g). Elevating labor effectiveness—achieved through raising the length of the workday for the age-class working the most (H), the area worked per unit time (k), or the commitment of a greater range of age classes to longer work-days ($H\phi_0$)—increases surplus. But this variable can have a dramatic impact on surplus only if the initial investment of effort is quite small (Figures 3 and 5). If the exchequer is dealing with a population that works short hours, cultivates a small area, and employs a small percentage of its age distribution in agricultural work, there are significant gains to pressing for greater effort from a wider array of individuals in the population. The gains are, however, asymptotic and fairly quickly exhausted. Even labor-saving innovations as seemingly effective as the introduction of the ox-drawn plow—which in our model would operate through k —have to be appraised in this context of rapidly diminishing returns. That is, it would take many fewer workers to farm the available land, but total output would be capped by the extent of the population's holdings.

Habitable agrarian landscape (A_m). Increasing the area in agrarian production is a sure means of increasing surplus. The elasticity, for instance, of maximum copial surplus production S^* to A_m is 1, meaning a 10% increase in the agrarian

territory yields a 10% increase in surplus, irrespective of scale (Supplemental Materials, Table D1e). The positive elasticity of return on annexing habitat, its constancy, and perhaps its simple observability may be one reason that archaic states regularly were expansionist in character (Spencer 2010).

Yield (Y). Yield, independent of labor, likewise has a positive and near linear effect for values of production (w_0Y) that are significantly greater than consumption (j_0) (Supplementary Materials Table D1f). Yield-increasing technologies like irrigation, terracing, raised fields, manuring or improved cultigen productivity have high potential to increase surplus. Limits to these gains would be technology-specific; irrigation of drylands might double yield, but twice as much more irrigation is unlikely to double it again.

Although the numerical results for our baseline dataset are different (compare Supplemental Materials, Table D1 and Table D2), the elasticity graphs for the Malthusian equilibrium tax have the same general form as those for the copial surplus (compare Figure 3 with Figure 5). The generalizations we have just cited for the copial surplus will also apply to the Malthusian equilibrium tax.

Our exchequer might through experience or analysis come to rank options as we have done. But, our model also points in the direction of a much less conscious or agent-oriented approach. All else equal, states with agrarian producers that consume relatively little and work hard will have access to greater copial surpluses and Malthusian tax than their counterparts without these advantages. However, the gains available through reduced consumption and enhanced labor productivity are modest compared to the reliability and expandability of adding arable land or increasing its yield through technology.

7.5 The Perils of Being and Serving an Agrarian State

An exchequer intent on a fixed resource stream in the service of the state will probably be unaware that this option entails two equilibrium points for each feasible level of state income (Figure 6). A high tax rate applied to a small population is an unstable option, which will either collapse to extinction through failure of the agrarian population to replace itself or move to the second and stable equilibrium combining a large population with a lower effective tax rate. There also is a tax limit above which the agrarian population simply cannot persist. The exchequer who seeks a fixed (as opposed to per capita) but increasing revenue stream courts an ever greater risk of misjudging and crossing the boundary from persistent tax and population combinations to those that are doomed to collapse from endogenous dynamics.

Our equilibrium results provide a dynamic, population ecology perspective on estimating a feasible degree of exploitation, or what the development economist Milanovic and colleagues (Milanovic *et al.* 2011; Milanovic 2013) term the

inequality possibility frontier (IPF). The IPF establishes the largest possible Gini index for a particular society by determining its maximum exploitable surplus, defined as total societal income less what is required for physiological subsistence of the population. For a population of a fixed size, the IPF/Gini increases to an asymptote of 1 as a function of general societal affluence. The IPF makes it possible to compare the observed level of income inequality to that theoretically possible, a relationship termed the *inequality extraction ratio* (IER). The IER normalizes degree of exploitation for comparative purposes, increasing its relevance to a wide variety of issues of political economy. For instance, historical data on class and occupational income ("social tables") establishes that Moghul India in 1750, Nueva España in 1790, Kenya in 1927 and the Maghreb in 1880 all faced IERs of 100% or slightly greater. Our model shows similar limits to the surplus that might be extracted by elites, but in a context that engages the issue at the level of time-series representing demographic and agro-ecological processes.

Adding yield variation exacerbates the possibility of collapse to such a degree that we can characterize early states dependent on a peasant agrarian population for resources as having a half-life. The form of this relationship (Figure 8) implies that even small changes in yield variability or resource extraction rates can potentially entail dramatic changes in half-life and thus the odds of a polity persisting. Astute administrators of agrarian populations living in zones with high yield variability such as the Andes (Goland 1993) quickly learned this. The *visitas* (inspection tours) initiated by Viceroy Toledo shortly after the conquest of the central Andes illustrate awareness of this problem on the part of high-level state representatives:

Spanish officials blamed demographic collapse on excessive, unregulated rates of tribute extraction by local Hispanic elites (*encomenderos*), who received the right to collect tribute from the indigenous population in return for their role in Peru's military conquests. Thus Viceroy Francisco Toledo coordinated an in-depth inspection of Peru, Bolivia, and Ecuador in the early 1570s to evaluate the maximum tribute that could be demanded from local groups without threatening subsistence (Dell 2010: 1874).

In an earlier application of this model (Puleston *et al.* 2014), we noted that the transition from a copial growth phase to Malthusian density dependence can be abrupt. It is typically more rapid and severe if the circumstances preceding it were benign. We suggested in that paper that the abruptness of Malthusian constraints should be considered among the causes of state collapse (Drennan and Peterson 2011; Turchin and Gavrillets 2009; Trigger 2003). Analyses

presented in the current paper add fiscal mismanagement to the list of potential causes of state collapse (Middleton 2012). We began by assigning our fictional exchequer an anxious mien; our reasons for doing so should now be apparent. Selecting the form of revenue to extract from an agrarian population, setting the level of that taxation, and then adjusting it to balance demands for state income with the environmental and demographic exigencies that can threaten the producing populations are complex problems fraught with high levels of uncertainty, risk and the potential for surprise.

Of course, elites, whom we are representing through the decisions and thus agency of the exchequer, may have concerned themselves more with immediate gains than with long-term sustainability (D'Altroy 2015: 62–63). Drawing on the *Información de 1544*, Gutiérrez (2013: 158–163) calculates that the tribute obligations of the Aztec Province of Tlapa increased over the period just prior to the conquest (1487–1521) by 947%. The greater part of the leap in exploitation occurred rapidly after 1510. Gutiérrez (2013: 161–162) attributes the accelerating demands to centralized exploitation, suggesting that:

the Aztecs were in need of more resources, perhaps due to the formation of a larger imperial bureaucracy, more ambitious construction programs in Tenochtitlan, or the never-ending elaboration of ritual life in the imperial capital ... High sumptuary costs at the courts of the Triple Alliance during the rule of Moctezuma II may explain the external push for large tributary increases after 1517.

We do not know in this case if the increases were sustainable because the conquest terminated the records and disrupted the demands, at the least from the Aztec overlords. We are unaware of other empirical work by which we might assess the fiscal stress hypothesis, but the ability of archaeologists to indirectly estimate taxation (below) suggests that it is feasible. The resurgence in archaeology of more formal, quantitative and market-oriented study of early state economies is likewise promising (Blanton 2013; Feinman and Garraty 2010; Garraty and Stark 2010; Hirth and Pillsbury 2013; Mayer 2013).

7.6 The Archaeology of State Revenue

Effective prehistoric tax rates can be calculated from data on the labor and materials requirements of public works and the size and consumption of non-producing political elites (Drennan and Peterson 2011: 75–76). Peterson and Drennan (2011) estimate population size and labor investments required for the construction of ancient monuments. Their analysis covers the trajectories of eleven cases that begin with low-density Neolithic communities assessed as too

small to place “any pressure on subsistence resources” (91) and then develop toward large-scale (10,000 or more people) social formations. They observe that population growth and social reorganization appear to be punctuated or episodic. While conflict is common across cases, it appears to be at population densities too low to be provoked by resource shortages (122). Because monumental construction of some form—elite residences, burials with high social salience, defensive constructions—occurs in all cases, Peterson and Drennan are able to estimate (122–127) labor tax rates required for these public works projects of less than a day to a week per capita annually, a burden significantly lower than commonly perceived (see also Webster 1985; Hunt and Lipo 2011). Only one of the cases (Hohokam) shows evidence of significant investment in agricultural infrastructure.

The Peterson and Drennan analysis reinforces the view that monumental constructions need not impose heavy burdens on the producing population. Nonetheless, given other necessary or potential demands of elites, and the possibility that construction itself was episodic and intensified, we should view these estimates as minimums, a point that they recognize: “estimates of the labor required for large-scale construction may not represent the entire burden” (124). Of equal interest, it appears that these rates were imposed on populations that continued to grow throughout the periods analyzed. This suggests in our population ecology terms that the trajectories being observed are occurring during copial periods of population growth, during which surplus extraction has a limited impact on population welfare, and does not incur trade-offs with population size.

The case with the highest tax rate, that of the Black Warrior Valley in the American Southeast, suffered something like an elite collapse after two centuries. Peterson and Drennan (2011: 126) suggest in this instance: “...that the tax rate reached levels not seen in any other sequence makes it possible to suggest that elites placed heavier burdens on their populations than they could continue to enforce over the long term.” In our terms, this may have been a case (a) in which growth had reached the end of an easily mobilized copial surplus, faltering as it approached or reached a transition to more Malthusian conditions (Figure 2E), perhaps (b) augmented by mistaken assessments of the best means of elevating income (Figures 3 and 5).

Political centralization and the relative size of the non-producing population also may be archaeologically visible by assessing the extent of public state-level architecture relative to rural settlement and agrarian habitat. For instance, Steponaitis (1981) has shown for the Middle to Terminal Formative Period in the Valley of Mexico that data representing settlement size (a proxy for population) and the catchment area of the surrounding agricultural habitat (representing

potential food production) can be used to calculate estimates of political centralization and the relative amount of food which state functionaries and elites at higher level sites must have mobilized from the productive population at lower levels in the settlement hierarchy. Transfers of tribute from the egalitarian nucleated-village level to local centers, and from local to regional centers, both suggest that about 16% of production was directed to maintenance of the political establishment. For comparison, our 60% tax rate (assessed as a percentage of the maximum sustainable fixed tax) converts to a 28.5% rate if applied to total production, the metric used by Steponaitis.

7.7 Topics and Issues Neglected

We well are aware of the caveats associated with the use of simplifying models in historical study (Winterhalder 2002; Kohler and van der Leeuw 2007); we also contend that models are essential aids in understanding the population ecology dynamics that may be consequential but difficult to discern within archaeological and even historical records of socioeconomic evolution. In this paper we have kept to a tight focus on demography, population, environment and economy, the latter to include the extraction of taxes by elites that otherwise are kept off-stage. That focus has prevented us from acknowledging and commenting on a variety of prominent issues that engage archaeologists and other evolutionary anthropology theorists. This neglect is expedient to our narrow purpose, but it is not meant to diminish the relevance or importance of those issues in analyses of prehistoric political economy. We comment briefly on several of them.

Boserup. Our space-limited modeling privileges the Malthusian side of demography, but Boserup (1965) has a like claim on anthropological attention (Bayliss-Smith 1974; Wood 1998) for her emphasis on the ways in which population growth might spur behavioral changes and technological innovations that lead to agro-ecological intensification and improved yields per unit area of land. Intensification as envisioned by Boserup and related scholarship has a prominent role in studies of socioeconomic development (review in Morgan 2014). In the present study intensification is represented only to the degree that we analyze how increasing yields would affect the potential for surplus and exploitative taxation (Figures 3 and 5). Puleston and Winterhalder (n.d.) explore in greater depth how the space-limited model could be modified to accommodate a Boserupian perspective.

Benign despotism. As we noted earlier, the taxation of our model measures net loss of kcal or labor to the producing population. We thus allow for redistribution in the abstract, but there are no mechanisms in our present model that explicitly represent the effects of such redistribution on model dynamics. If redistribution simply is a return in kind, the effect is neutral, allowing for delay and losses from

whatever inefficiencies such a scheme introduces. But, if benign despots or elites redistribute by developing social capital among the producers or by underwriting infrastructure projects such as irrigation or drainage that enhance productive capacity, then the population ecology impact may be significant. Sidky (1996) gives an excellent ethnohistorical example.

Surplus. The concept of surplus is receiving renewed attention in archaeological studies of the development of complex social formations (Morehart and de Lucia 2015). Our present contribution to this literature rests in three features of our analysis: (i) we show how potential magnitudes of surplus can be estimated using realistic parameters for human demography, work, consumption and premodern agro-ecological conditions; (ii) we demonstrate that this estimate is a dynamic property of a population's growth trajectory with some counterintuitive features; and (iii) we make an important distinction between a copial phase surplus requiring extra labor investments but without impact on positive demographic measures of agrarian welfare, and a Malthusian phase tax which does not lessen the dismal welfare conditions of the population but does reduce its equilibrium size and may threaten its persistence.

Agency. We have framed our study such that a hypothetical exchequer represents the agency of elites making decisions about the production of state revenue. Our elites are a stable resource sink, and their interests and actions are quite simple. This assumes that elites and state bureaucrats like our exchequer are of the same mind, although history is replete with examples of governors, viziers and deputies who pursued their own interests in conflict with those of their masters. And, while we consider the implications of such decisions for the demographic welfare of the producing population, our recognition of their agency is indirect at best. Blanton and Fargher (2008) make the case with historical data that it is unreasonable to regard commoners as an inert economic unit, and they oppose the idea of a "a dual structure consisting of an aristocratic governing class and a subjugated class that loyally accepted aristocratic overlordship" (9). Our population ecology approach is useful in that it can highlight the implications of elite decisions for commoner well-being. However, in its present formulation it does not include specific mechanisms by which the commoners may through collective action put pressure on the exchequer and ruling class.

Model limitations. In its present form, our modeling effort neglects several kinds of structure that may affect the potential for agrarian resource exploitation by elites: environmental heterogeneity in production options and yields, and social population structure such as corporate kinship, both of which are being explored by Kohler and colleagues (e.g., 2012). Agent-based models like those used by Kohler offer insights complementary to the more analytical, simulation focus of the efforts presented here (Lake 2014). Our focus on staple finance also

leads us to ignore elite finance (D'Altroy and Earle 1985) along with the potential for elite capture of resources via the imposition of taxes on trade, a source of revenue prominent, for instance, in Herskovits' ([1938] 1967) description of tax policy in Dahomey. The "structural-demographic" model of state instability (Turchin *et al.* 2017; Goldstone 1991) addresses some of these concerns by expanding the scope to include three interdependent actors: the general population, elites and the state. These groups have their own, often conflicting, interests and their actions affect state stability through unrest over real wages, elite resource acquisition, trust in institutions and the balance of production and state debt. The approach differs from food-limited demography in that its aim is to forecast revolutions and collapse, rather than understand the evolution of human behavior.

Assessment. Finally, it is worth noting that while we offer occasional points of evidence in support of our model formulation, we do not claim that they constitute a comprehensive empirical defense of its utility. That will require more systematic and detailed case study by or in collaboration with prehistorians. We do claim, however, that population ecology modeling will make important contributions to the revival of formalist economic methods in the study of premodern societies (Feinman and Garraty 2010).

8. Conclusion

Research on the political economy of chiefdoms and the early state has focused almost entirely on resource distribution and the means by which state power is consolidated and exercised through elite manipulation of what has been extracted from the producing population. The production of those resources is seldom examined and, in this respect, the extant literature is seriously incomplete. Even a basic examination such as we have given of the population ecology of agrarian producers subject to taxation reveals mechanisms and dynamics essential to analyzing the origins, persistence and eventual decay or collapse of centralized political economies. It also may be essential to an understanding of their lingering consequences for subsequent political economic developments in the regions they previously controlled.

Although there is much to explore in them, we have put the mathematical details off-stage, in the Supplemental Materials. Consistent with our title, we summarize our text in the form of a general guide, documenting "best practices" as they might be addressed to an exchequer of the agrarian state. Our summary points might as easily be formulated as hypotheses about the effectiveness of institutional fiscal practices in early agrarian states. In conditions like those we model:

(i) Your best opportunity to exploit surplus production from a hard-working peasantry without causing them to suffer hunger or cease expanding occurs when that population of producers is relatively small in numbers, only a fraction ($\sim 25\%$) of its maximum potential, perhaps long after it has been established and nearly a century before it reaches carrying capacity.

(ii) Exploiting a peasantry suffering the hunger of a Malthusian equilibrium entails a stark political trade-off: you cannot have both a large citizenry and high revenue, contrary to the common supposition that you can generate income in direct proportion to the size of your citizenry. Furthermore, pushing up resource exploitation invites demographic misjudgment by increasing the odds that the peasant population on which the state depends will crash.

(iii) Your most effective options for increasing returns—either from an intermediate optimum of surplus production, or from equilibrium extraction of resources generated by a population at Malthusian equilibrium—are to expand the area of agricultural production and/or to implement high-yield technologies. Although perhaps tempting, there are very likely only limited gains to be achieved by making the peasants eat less or work longer hours.

(iv) To minimize impacts on the peasantry while optimizing state income, you are best advised to tax a small growing population on a per capita basis in produce and then switch to a per capita tax in labor as the arable habitat is filled. Your agrarian commoners will have to work less hard when their numbers are growing, or will suffer a smaller diminution of numbers when at equilibrium.

(v) Do not be overconfident about your job security, or the persistence of the institutions you support; hard-to-discern population and ecological dynamics are not your only worry. The probability of your society surviving a fixed regime of exploitation is highly sensitive to small changes in the tax rate or in environmental stochasticity affecting agricultural production. You will be lucky to avoid the fiscal mistakes and fates of those who have come before you and will follow you.

More generally, advancing our understanding of the complex evolutionary dynamics linking natural and human systems in the pursuit of issues like the origins and persistence of stratified political institutions requires a variety of

Winterhalder and Puleston: The Exchequer's Guide. Cliodynamics 9:2 (2018)

modeling and empirical approaches (Kohler and van der Leeuw 2007; Turchin 2008; McConnell *et al.* 2011; Kirch *et al.* 2012; Lake 2014). In this and in earlier work (Puleston *et al.* 2014; Winterhalder *et al.* 2015; Demps and Winterhalder 2018) we seek to advance the general objectives of this literature from a perspective based in human behavioral and population ecology models.

Compliance with Ethical Standards and Conflict of Interest

The authors declare that they have no conflict of interest; the research described in this manuscript did not involve human or animal subjects and was therefore not subject to IRB approval.

References

- Allen, R. C. 1997. "Agriculture and the Origins of the State in Ancient Egypt." *Explorations in Economic History* 34: 135–154.
- Banerjee, A., and L. Iyer. 2005. "History, Institutions, and Economic Performance: The Legacy of Colonial Land Tenure Systems in India." *American Economic Review* 95 (4): 1190–1213, doi:10.1257/0002828054825574.
- Bayliss-Smith, T. 1974. "Constraints on Population Growth: The Case of the Polynesian Outlier Atolls in the Precontact Period." *Human Ecology* 2 (4): 259–295.
- Blanton, R. E. 2013. "Cooperation and the Moral Economy of the Marketplace." In *Merchants, Markets, and Exchange in the Pre-Columbian World*, edited by K. G. Hirth and J. Pillsbury, 23–48. Dumbarton Oaks Pre-Columbian Symposia and Colloquia. Washington, DC: Dumbarton Oaks.
- Blanton, R. and L. Fargher. 2008. *Collective Action in the Formation of Pre-modern States*. New York: Springer Science & Business Media.
- Boserup, E. 1965. *The Conditions of Agricultural Growth: The Economics of Agrarian Change under Population Pressure*. New York: Aldine.
- Carneiro, R. L. 1970. "A Theory of the Origin of the State." *Science* 169 (3947): 733–738.
- Chayanov, A. V. 1977. "On the Theory of Non-capitalist Economic Systems." In *Peasant Livelihood: Studies in Economic Anthropology and Cultural Ecology*, edited by R. H. Halperin and J. Dow, 257–268. New York: St. Martin's Press.
- Claessen, H. J. M., & P. van de Velde, eds. 2008. *Early State Economics*. 2nd ed. Political and Legal Anthropology Series, Volume 8. New Brunswick, NJ: Transaction Publishers.
- Culbert, T. P. 1988. "Collapse of Classic Maya Civilization." In *Collapse of Ancient States and Civilizations*, edited by N. Yoffee and G. L. Cowgill, 69–101. Tucson, AZ: University of Arizona Press.
- D'Altroy, T. N. 2015. "The Inka Empire." In *Fiscal Regimes and the Political Economy of Premodern States*, edited by A. Monson and W. Scheidel, 31–70. Cambridge: Cambridge University Press.
- D'Altroy, T. N., and T. K. Earle. 1985. "Staple Finance, Wealth Finance, and Storage in the Inka Political Economy." *Current Anthropology* 26 (2): 187–206.
- Dell, M. 2010. "The Persistent Effects of Peru's Mining Mita." *Econometrica* 78 (6): 1863–1903, doi:10.3982/ECTA8121.
- DeMarrais, E., L. J. Castillo, and T. Earle. 1996. "Ideology, Materialization, and Power Strategies." *Current Anthropology* 37 (1): 15–31.
- Demps, K., and B. Winterhalder. 2018. "Every Tradesman Must Also Be a Merchant': Behavioral Ecology and Household-level Production for Barter

- and Trade in Premodern Economies." *Journal of Archaeological Research*: doi:10.1007/s10814-018-9118-6, doi:10.1007/s10814-018-9118-6.
- Drennan, R. D., and C. E. Peterson. 2011. "Challenges for Comparative Study of Early Complex Societies." In *The Comparative Archaeology of Complex Societies*, edited by M. E. Smith, 62–87. Cambridge: Cambridge University Press.
- Earle, T. 2015. "Conclusions: Surplus and the Political Economy in Prehistory." In *Surplus: The Politics of Production and the Strategies of Everyday Life*, edited by C. T. Morehart and K. de Lucia, 307–325. Boulder, CO: University Press of Colorado.
- Earle, T., and M. Spriggs. 2015. "Political Economy in Prehistory: A Marxist Approach to Pacific Sequences." *Current Anthropology* 56 (4): 515–544, doi:10.1086/682284.
- Erickson, C. L. 1987. "The Dating of Raised-field Agriculture in the Lake Titicaca Basin, Peru." In W. M. Denevan, K. Mathewson, & G. Knapp (Eds.), *Pre-Hispanic Agricultural Fields in the Andean Region, Part 1*, edited by W. M. Denevan, K. Mathewson, and G. Knapp, 373–384. Oxford: BAR International Series 359(j).
- Fedick, S. L., and A. Ford. 1990. "The Prehistoric Agricultural Landscape of the Central Maya Lowlands: An Examination of Local Variability in a Regional Context." *World Archaeology* 22 (1): 18–33.
- Feinman, G. M., and C. P. Garraty. 2010. "Preindustrial Markets and Marketing: Archaeological Perspectives." *Annual Review of Anthropology* 39: 167–191, doi:10.1146/annurev.anthro.012809.105118.
- Feinman, G. M., and L. M. Nicholas. 2016. "Framing the Rise and Variability of Past Complex Societies." In *Alternate Pathways to Complexity: A Collection of Essays on Architecture, Economics, Power, and Cross-Cultural Analysis in Honor of Richard E. Blanton*, edited by L. F. Fargher and V. Y. Heredia Espinoza, 271–289. Boulder, CO: University Press of Colorado.
- Garraty, C. P., and B. L. Stark, eds. 2010. *Archaeological Approaches to Market Exchange in Ancient Societies*. Boulder, CO: University Press of Colorado.
- Goland, C. 1993. "Field Scattering as Agricultural Risk Management: A Case Study from Cuyo Cuyo, Department of Puno, Peru." *Mountain Research and Development* 13 (4): 317–338.
- Goldstone, J. A. 1991. *Revolution and Rebellion in the Early Modern World*. Berkeley, CA: University of California Press.
- Gutiérrez, G. 2013. "Negotiating Aztec Tributary Demands in the *Tribute Record of Tlapa*." In *Merchants, Markets, and Exchange in the Pre-Columbian World*, edited by K. G. Hirth and J. Pillsbury, 141–167. *Dumbarton Oaks Pre-Columbian Symposia and Colloquia*. Washington, DC: Dumbarton Oaks.

- Harris, M. 1959. "The Economy Has No Surplus?" *American Anthropologist* 61 (2): 185–199.
- Henrich, J., and R. McElreath. 2003. "The Evolution of Cultural Evolution." *Evolutionary Anthropology* 12 (3): 123–135.
- Herskovits, M. J. [1938] 1967. *Dahomey: An Ancient West African Kingdom, Volume 1*. Eanston, IL: Northwestern University Press.
- Hirth, K. G., and J. Pillsbury, eds. 2013. *Merchants, Markets, and Exchange in the Pre-Columbian World*. Dumbarton Oaks Pre-Columbian Symposia and Colloquia. Washington, DC: Dumbarton Oaks Research Library and Collections.
- Hunt, T., and C. Lipo. 2011. *The Statues That Walked: Unraveling the Mystery of Easter Island*. New York: Free Press.
- Isaac, B. L. 2013. "Discussion." I In *Merchants, Markets, and Exchange in the Pre-Columbian World*, edited by K. G. Hirth and J. Pillsbury, 435–448. Dumbarton Oaks Pre-Columbian Symposia and Colloquia. Washington, DC: Dumbarton Oaks.
- Kirch, P. V., G. Asner, O. A. Chadwick, J. Field, T. Ladefoged, C. Lee, C. Puleston, S. Tuljapurkar, and P. M. Vitousek. 2012. "Building and Testing Models of Long-term Agricultural Intensification and Population Dynamics: A Case Study from the Leeward Kohala Field System, Hawai'i." *Ecological Modelling* 227: 18–28.
- Kohler, T. A., D. Cockburn, P. L. Hooper, R. K. Bocinsky, and Z. Kobti. 2012. "The Coevolution of Group Size and Leadership: An Agent-based Public Goods Model for Prehispanic Pueblo Societies." *Advances in Complex Systems* 15 (1/2): 1150007-1150001-1150007-1150029.
- Kohler, T. A., and S. E. van der Leeuw. 2007. "Introduction: Historical Socionatural Systems and Models." In *The Model-Based Archaeology of Socionatural Systems*, edited by T. A. Kohler and S. E. van der Leeuw, 1–12. Santa Fe: School for Advanced Research Press.
- Lake, M. W. 2014. "Trends in Archaeological Simulation." *Journal of Archaeological Method and Theory* 21: 258–287, doi:10.1007/s10816-013-9188-1.
- Lee, C. T., C. Puleston, and S. Tuljapurkar. 2009. "Population and Prehistory III: Food-dependent Demography in Variable Environments." *Theoretical Population Biology* 76 (3): 179–188.
- Lee, C. T., and S. Tuljapurkar. 2008. "Population and Prehistory I: Food-dependent Population Growth in Constant Environments." *Theoretical Population Biology* 73: 473–482.
- Mayer, E. 2013. "In the Realm of the Incas." In *Merchants, Markets, and Exchange in the Pre-Columbian World*, edited by K. G. Hirth and J. Pillsbury, 309–317.

- Dumbarton Oaks Pre-Columbian Symposia and Colloquia. Washington, DC: Dumbarton Oaks.
- McAnany, P. A. 2013. "Artisans, *Ikatz*, and Statecraft: Provisioning Classic Maya Royal Courts." In *Merchants, Markets, and Exchange in the Pre-Columbian World*, edited by K. G. Hirth and J. Pillsbury, 229–253. Dumbarton Oaks Pre-Columbian Symposia and Colloquia. Washington, DC: Dumbarton Oaks.
- McConnell, W. J., J. D. A. Millington, N. J. Reo, M. Alberti, H. Asbjornsen, L. A. Baker, N. Brozović, L. E. Drinkwater, S. A. Drzyzga, J. Fragoso, D. S. Holland, C. A. Jantz, T. A. Kohler, H. D. G. Maschner, M. Monticino, G. Podestá, R. G. Pontius, Jr., C. L. Redman, D. Sailor, G. Urquhart, and J. Liu 2011. "Research on Coupled Human and Natural Systems (CHANS): Approach, Challenges, and Strategies (Meeting Reports)." *Bulletin of the Ecological Society of America (Reports)*: 218–228.
- Middleton, G. D. 2012. "Nothing Lasts Forever: Environmental Discourses on the Collapse of Past Societies." *Journal of Archaeological Research* 20: 257–307, doi:10.1007/s10814-011-9054-1.
- Milanovic, B. 2013. "The Inequality Possibility Frontier: Extensions and New Applications." The World Bank, Development Research Group, Poverty and Inequality Team, Policy Research Working Paper 6449.
- Milanovic, B., P. H. Lindert, and J. G. Williamson. 2011. "Pre-industrial Inequality." *The Economic Journal* 121 (551): 255–272, doi:10.1111/j.1468-0297.2010.02403.x.
- Morehart, C. T., and K. de Lucia, eds. 2015. *Surplus: The Politics of Production and the Strategies of Everyday Life*. Boulder, CO: University Press of Colorado.
- Morgan, C. 2014. "Is It Intensification Yet? Current Archaeological Perspectives on the Evolution of Hunter-gatherer Economies." *Journal of Archaeological Research* 23 (2): 163–213, doi:10.1007/s10814-014-9079-3.
- Oka, R., and C. M. Kusimba. 2008. "The Archaeology of Trading Systems, Part 1: Towards a New Trade Synthesis." *Journal of Archaeological Research* 16 (4): 339–395, doi: 10.1007/s10814-008-9023-5.
- Pearson, H. W. 1957. "The Economy Has No Surplus: Critique of a Theory of Development." In *Trade and Market in the Early Empires: Economies in History and Theory*, edited by K. Polanyi, C. M. Arensberg, and H. W. Pearson, 320–341. New York: The Free Press.
- Peterson, C. E., and R. H. Drennan. 2011. "Patterned Variation in Regional Trajectories of Community Growth." In *The Comparative Archaeology of Complex Societies*, edited by M. E. Smith, 88–137. Cambridge: Cambridge University Press.

- Puleston, C., and S. Tuljapurkar. 2008. "Population and Prehistory II: Space-limited Human Populations in Constant Environments." *Theoretical Population Biology* 74: 147–160.
- Puleston, C., S. Tuljapurkar, and B. Winterhalder. 2014. "The Invisible Cliff: Abrupt Imposition of Malthusian Equilibrium in a Natural-fertility, Agrarian Society." *PLoS One* 9 (1): 1–13, doi:10.1371/journal.pone.0087541.
- Puleston, C., and B. Winterhalder. n.d. "Demography, Environment and Human Behavior" (in press). *Handbook of Evolutionary Research in Archaeology*, edited by A. M. Prentiss. New York: Springer.
- Sidky, H. 1996. *Irrigation and State Formation in Hunza: The Anthropology of a Hydraulic Kingdom*. Lanham, MD: University Press of America.
- Smith, E. A. 1987. "On Fitness Maximization, Limited Needs, and Hunter-gatherer Time Allocation." *Ethology and Sociobiology* 8: 73–85.
- Smith, M. E. 2004. "The Archaeology of Ancient State Economies." *Annual Review of Anthropology* 33: 73–102, doi:10.1146/annurev.anthro.33.070203.144016.
- Spencer, C. S. 2010. "Territorial Expansion and Primary State Formation." *Proceedings of the National Academy of Sciences* 107 (16): 7119–7126, doi:10.1073/pnas.1002470107.
- Stanish, C., and L. S. Coben. 2013. "Barter Markets in the Pre-Hispanic Andes." In *Merchants, Markets, and Exchange in the Pre-Columbian World*, edited by K. G. Hirth and J. Pillsbury, 419–434. *Dumbarton Oaks Pre-Columbian Symposia and Colloquia*. Washington, DC: Dumbarton Oaks.
- Steponaitis, V. P. 1981. "Settlement Hierarchies and Political Complexity in Nonmarket Societies: The Formative Period of the Valley of Mexico." *American Anthropologist* 83 (2): 320–363.
- Stoelhorst, J. W., and P. J. Richerson. 2013. "A Naturalistic Theory of Economic Organization." *Journal of Economic Behavior & Organization* 90, Supplement, S45–S56, doi:10.1016/j.jebo.2012.12.012.
- Tisdell, C., and S. Svizzero. 2017. "The Ability in Antiquity of Some Agrarian Societies to Avoid the Malthusian Trap and Develop." *Forum for Social Economics*, doi:10.1080/07360932.2017.1356344.
- Topic, J. R. 2013. "Exchange on the Equatorial Frontier: A Comparison of Ecuador and Northern Peru." In *Merchants, Markets, and Exchange in the Pre-Columbian World*, edited by K. G. Hirth and J. Pillsbury, 335–360. *Dumbarton Oaks Pre-Columbian Symposia and Colloquia*. Washington, DC: Dumbarton Oaks.
- Trigger, B. G. 2003. *Understanding Early Civilizations: A Comparative Study*. Cambridge: Cambridge University Press.
- Turchin, P. 2008. "Arise 'Cliodynamics'." *Nature* 454: 34–35.

Winterhalder and Puleston: The Exchequer's Guide. Cliodynamics 9:2 (2018)

- Turchin, P., and S. Gavrilets. 2009. "Evolution of Complex Hierarchical Societies." *Social Evolution and History* 8 (2): 167–198.
- Turchin, P., S. Gavrilets, and J. A. Goldstone. 2017. "Linking 'Micro' to 'Macro' Models of State Breakdown to Improve Methods for Political Forecasting." *Cliodynamics* 8: 159–181, doi:10.21237/C7clio8237429.
- Wade, L. 2017. "Unearthing Democracy's Roots." *Science* 355 (6330): 1114–1118.
- Webster, D. 1985. "Surplus, Labor, and Stress in Late Classic Maya Society." *Journal of Anthropological Research* 41 (4): 375–399.
- Winterhalder, B. 2002. "Models." In *Darwin and Archaeology: A Handbook of Key Concepts*, edited by J. P. Hart and J. E. Terrell, 201–223. Westport, Connecticut: Bergin & Garvey.
- Winterhalder, B., C. Puleston, and C. Ross. 2015. "Production Risk, Inter-annual Food Storage by Households and Population-level Consequences in Seasonal Prehistoric Agrarian Societies." *Environmental Archaeology* 20 (4): 337–348, doi:10.1179/1749631415Y.0000000025.
- Wood, J. W. 1998. "A Theory of Preindustrial Population Dynamics: Demography, Economy, and Well-being in Malthusian Systems." *Current Anthropology* 39 (1): 99–135.
- Wright, H. T. 2006. "Early State Dynamics as Political Experiment." *Journal of Anthropological Research* 62 (3): 305–319.