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The Integration of Two Disparate Processes: Punctuated Equilibria Characterize Urbanization Over Time, While Kondratieff Long Waves Characterize Long Term Economic Change

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In 1972 Eldridge and Gould proposed a model of episodic change characterizing the process of speciation. Since that initial publication, significant evidence has accumulated in support of their proposal. It is the position of this paper that a similar pattern, although with very different underlying mechanisms, exists for the process of world system urbanization, and it will be shown that periods of rapid change in the rates of world-system urbanization punctuate periods of stasis. Further, it will be inferred that these periods of punctuation are the result of tipping points being exceeded during the previous periods of stasis. A preliminary math model of this process is presented which shows that the interaction of world system population, carrying capacity, and level of technology are capable of producing such punctuated patterns. These results suggest that this macropattern of urbanization in which stasis alternates with punctuation provides a context in which to consider other world system processes. Chief among those processes are Kondratieff cycles, cycles that have particular pertinence over the last several centuries. These cycles may act as potential ratchets in which the world-system is pushed toward some threshold, i.e. the aforementioned tipping points, that, when crossed, will result in a rapid improvement in the access to energy resources. This improved access to energy resources will among other things result in increased world-system urbanization.

Introduction

Cyclicity is embedded in our physiology, both reproductively and somatically, and this embedding is reflected in our individual behavior. Collectively, minimally at the population level but more elaborately in our social, cultural, and economic

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activities, cyclical behavior manifests itself in a variety of ways. Kondratieff, Jugal, and Kuznets were pioneers in identifying and investigating cyclical behavior in economics, and it is in the reciprocal effect of this behavior on urbanization, specifically the reciprocal context of Kondratieff or K-waves with respect to the macro-pattern of urbanization over historic time and more specifically over the last millennium that will be the focus of this paper. [It should be noted that my underlying philosophy is that there are general principles of urban organization, function, and distribution that are grounded in our biology and by default all biology (West 2011 and Grinin et al 2013). Also with regard to general principles and the empirical evidence supporting those general principles, and specifically with respect to human behavior, the reader should be aware of three books authored singly or jointly by Peter Turchin on historical cycles. Beginning with the publication of Historical Dynamics (Turchin 2003) but continuing with his joint publication of Secular Cycles (Turchin and Nefedov 2009) and most recently, Ages of Discord (Turchin 2016), Turchin has provided both a theoretical framework for the study of historical cycles and, with Sergey Nefedov and singly, has provided empirical evidence supporting his theoretical framework, all done effectively with the use of quantitative data and mathematical modeling.]

The macro-pattern of urbanization over historic time has been a focus of my research for over a decade. The context of this research is at the level of the world-system as initially defined by Wallerstein (2004) and consequently is about system behavior as reflected by urbanization patterns. However, at levels below that of the world-system does this research have significance? I believe it does, especially with respect to understanding system change, as the actual mechanisms of system change originate at these sublevels. Here in order to make this point with greater clarity, this research will rely initially on the paper of Eldridge and Gould on punctuated equilibria [1972]. In tandem with that paper and providing a more general context is the work of the late Per Bak (1996). As will be explained later in detail, these researchers, Eldridge and Gould and Bak, established a model of punctuated change applicable to a range of systems. If one is to take the title of Bak's book, How Nature Works: The Science of Self-organized Criticality, at face value, his model of punctuated change is applicable to all complex adaptive systems. It should be noted here that the Eldridge and Gould paper and their research in general on this topic has a more phenomenological tenor, and that of Bak and his collaborators is more theoretical. In fact, on a happenstance meeting between the two when Bak asked Gould if it wouldn't be nice if there were a theory of punctuated equilibria, Gould answered pointedly, "Punctuated equilibria is a theory!". The point being here that, while Gould unquestionably missed Bak's point, Bak was interested in developing a theory of the mechanisms underlying all modes of punctuated change in complex adaptive systems, not simply in explaining this process as it applies to a more specific process, speciation.

A word of explanation is due here about the term, *levels of organization*, as it is being used in this paper. In biology, this term usually refers to a hierarchy of levels beginning at the top with the earth's biosphere, working down through biomes to ecosystems, then to populations of individuals and leading to the individual, all as units of study. Intra-individual levels then proceed in this subsuming process from organ-systems through organs and tissues to individual cells. Intracellularly, examples of succeeding levels of organization would include organelles, macromolecules, subunits of macromolecules down to the individual atom, and if need be, attention to subatomic particles. In this paper, the world-system as an entity sits at the top of a hierarchy which includes, in descending order, but is not limited to the triumvirate of core, semi-periphery, and periphery. Each of these collections of interacting polities, i.e. the core, semi-periphery, and periphery, have interactions also between these world-system subdivisions. These units of organization are then followed by the individual polity, factions within polities, e.g. political or religious factions, and then to local populations leading finally to the individual. While levels of organization refers to just that, system organization at different and subsuming levels, this term is used with more flexibility here than it is in a biological context and while contextual in nature, this term is itself dependent on the context of use; its required flexibility is a consequence of differing contexts of use.

Given this understanding of the definition of levels of organization, while the context of this paper is at the level (of organization) of the world-system, as was previously mentioned, it will also consider mechanisms generated at lower levels of organization within this larger system, and those mechanisms will influence both the structure and the trajectory of the world-system through time; the pattern of urbanization will be used to represent this trajectory. More specifically, the pattern of urbanization as reflected by the magnitude of the maximum urban area or more appropriately, maximum urban system. The use of maximum urban area/system, C_{max}, can be justified based on the assumption that urban area magnitudes can be reasonably modeled by a Pareto-Zipf frequency distribution, $F = \alpha C^{-\gamma}$ Eq. 1, where F is the frequency of a given urban area size, $\alpha = C_{max}^{\gamma}$, and γ is a fitted constant that characterizes the distribution of urban areas. Further, C_{max} can be related to T, the total population of the world-system and to γ by the equation, $C_{max}^{\gamma} - C_{max} - (\gamma - 1)T = 0$, Eq. 2. [For an explanation of the derivation of this equation, see Harper (2017c).] Data for both C_{max} and T are available for a time span of 5000 years, essentially most of human history from both Modelski (2003) and Chandler (1987).

The fact that data are available over this period of time and that simple math models can be constructed representing key variables associated with the pattern of urbanization does not in any way verify the use of these models over time. The key question is of course: Is the general pattern of urbanization (reasonably) constant over time, i.e. do the models, Equations 1 and 2, presented previously apply historically as well as having contemporary pertinence? In a suite of papers starting in 2012 Bettencourt and colleagues have shown that this is so for general urban scaling models, specifically for area-population relationships, and have also shown that these models of urban scaling can be associated with Zipf distributions (Bettencourt et al 2012). Further and in some detail Bettencourt et al (2013) have shown that urban scaling relationships hold over 3000 years for prehistoric Central Mexico, in the Central Andes prehistorically (Bettencourt et al 2016), for both Medieval (2015) and contemporary Europe (2015), and more generally for ancient societies (2014). I believe that the statistic, C_{max}, has what Krakauer defines as statistical sufficiency (Krakauer 2019), consequently, C_{max} specifically, but total world-system population should be included as well, has been chosen to be the signature variable for the study of historic urbanization patterns in this paper.

More explicitly, this paper will describe the phenomena of punctuated change and stasis as they are reflected in historic macro-patterns of urbanization and suggest mechanisms of societal patterns embedded within punctuated equilibria that (could) lead to both rapid system change from stasis to punctuation and also to system stasis. It will be proposed that these mechanisms are a consequence of both Kondratieff cycles and the formation of hypercycles within K-wave cycles. It is not, however, the intent here to establish a single cause or set of causes for punctuated change but instead to suggest possible, in fact reasonable, causes for punctuated change and stasis, both underlying continual system self-organization.

The phenomenon of system stasis will be addressed next. Eldridge and Gould (1972) suggested that with respect to species characteristics, including but not limited to diagnostic characteristics, that stasis implied some mean condition about which variation would be exhibited, the variation would be the result of both mutation and genetic recombination; stasis was not meant to imply a static morphology, physiology, and most certainly not a gene pool with static composition. So, variation generated by mutation would be matched against a (more or less) stable set of selection pressures with the result being a mean species extending over some period of time. From the perspective of an adaptive landscape, variability would be represented by a scatter of points about some local fitness peak, but a variability that would not drift or be selectively moved from that peak, i.e. a variability that would reflect stasis.

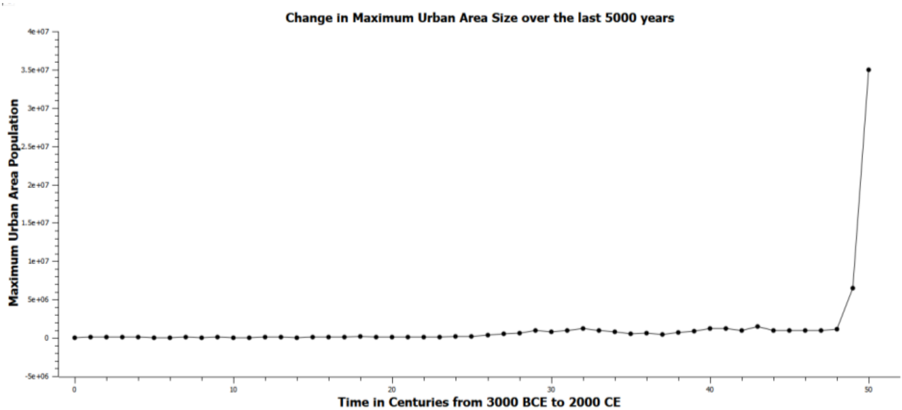

Figure 1. The growth of maximum urban area size exhibits a hyperbolic mode of growth, i.e. growth that is greater than exponential growth.

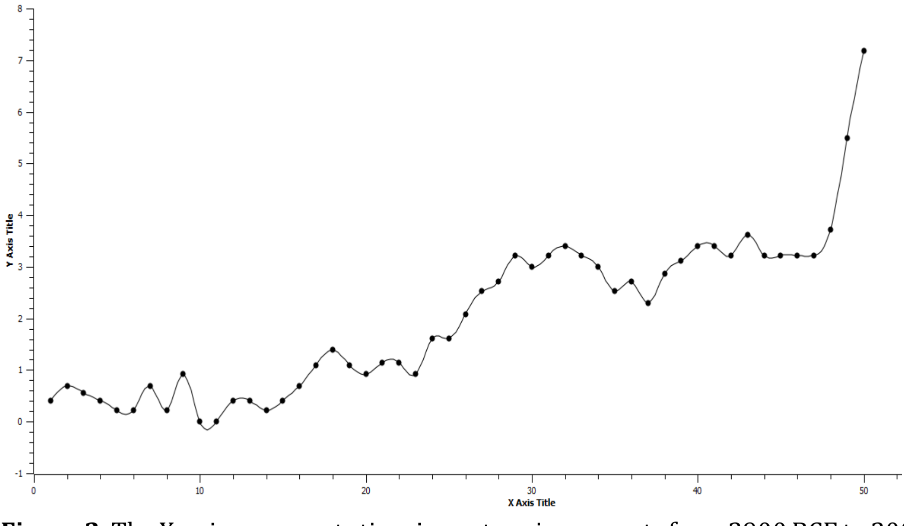

Figure 2. The X-axis represents time in century increments from 2900 BCE to 2000 CE. The Y-axis represents accumulate change in the natural logarithm of the maximum urban area population per century. This plot represents three periods of (relatively) rapid change, possibly phase changes w.r.t. energy acquisition and three plateaus.

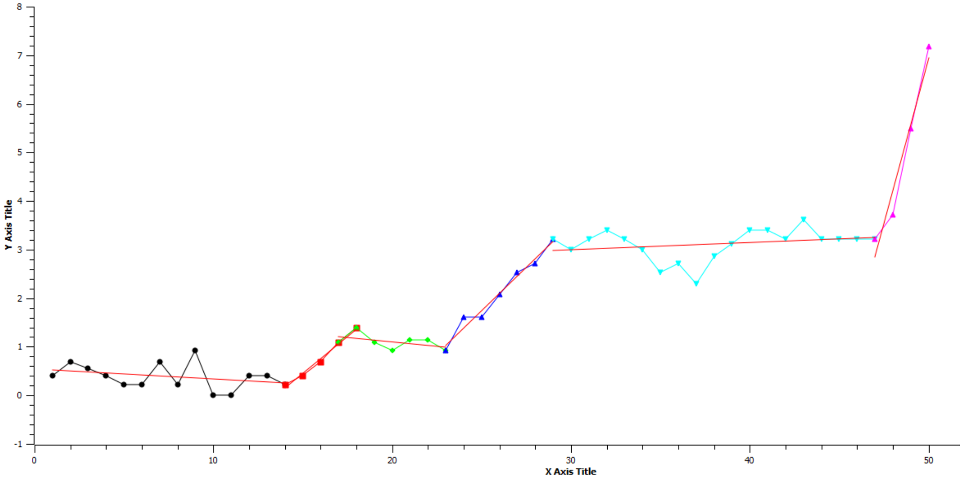

Figure 3. The same as in Fig. 1 but with linear regressions to emphasize the positions of and differences between periods of stasis and periods of punctuation.

With regard to the pattern of urbanization over historic time, the stasis represented is the stasis of the unit, maximum urban area, i.e. the largest city of the time. [Note: It is a fundamental assumption that urban areas of varying sizes have a Pareto-Zipf distribution of the form: $F = \alpha C^{-\gamma}$, where F = class frequency, C = class size, $\alpha = C_{max}^{\gamma}$, and γ is a fitted constant.] When one considers only the raw data for C_{max} population over time, the graph produced is hyperbolic in nature (Korotayev et al 20006) (Figure 1). This is a mode of growth that is characteristic of both human social systems and biological systems in general (Grinin et al 2013). However when the data are log-transformed, in this case using natural logarithms, there are both three periods of punctuated growth and three periods of maximum urban area stasis during this time (See Figures 2 and 3). The plots in Figure 2 and Figure 3 are of the accumulated change in $\ln C_{max}$ and will be explained in detail in the Mathematical Appendix. [It should be noted here that the periods of stasis and punctuation were defined with respect to the variation associated with each regression in as much as that variation is part of the computation of the coefficient of determination. The significance of this relationship has to do with the definition of tipping points given by Lamberson and Page (2012) in which data on either side of a tipping point will exhibit significantly different variations. As the coefficients of determination for stasis regressions differ considerable with respect to those of phases of punctuation and therefore define the differences between these modes of change.] If the periods of stasis are addressed first, their temporal occurrence and extent are as follows: one beginning at 2900 BCE and extending to 1600 BCVE, a second from 1200 BCE and ending at 700 BCE, and a third, by far the period of stasis with the greatest tenure, extending from 100 BCE to 1800 BCE. While each of these three periods can be represented by a linear regression (Harper 2017), each with relatively low R^2 values characteristic of the existence of variability about a mean, it is this last and most extensive period that is of greatest interest here, as Kondratieff cycles can be, with at least some reasonable confidence, extended back through approximately one-quarter to one-half of this time period, i.e. at least to 1400 CE and perhaps to 1000 CE. It should also be noted that each of these periods of stasis is characterized by successively increasing magnitudes of maximum urban area population. For the first period, a figure of 40,000 people is a reasonable approximation, for the second, a figure of 100,000 represents a satisfactory estimate, and for the third period, a figure of 1,000,000 for the population characteristic of this extended period. This last is based on the population of Rome at 100 BCE and the population of London around 1800 CE. It should be mentioned here that Morris (2015) has also recognized an upper limit of maximum urban areas for different periods of history; he cites magnitudes of ten thousand, one hundred thousand, one million, and ten million as his values.

Prior to Ian Morris' recognition of punctuated change in urban area size, there have been other scholars who have suggested punctuated change in social units be they urban areas or polities of scale. Chase-Dunn and Hall (1997) after Taagepera (1979) have proposed a qualitative model in which punctuated change has occurred in the evolutionary sequence from chiefdoms through early states and empires to modern colonial empires. They suggest that the punctuations have been a consequence of "qualitative leaps" that have resulted from the development of new techniques of power; they have done this without suggesting explicit mechanisms for such change. When one views their graphical model, one sees periods of relative stasis characterized by oscillations about some mean. This is directly representative of th e punctuated equilibrium model presented originally by Eldridge and Gould.

An interesting question arises as to why these average maximum values exist at these times. It is the position here that maximum urban area size is a product of the state of the world-system structure and functioning, which is ultimately dependent on the efficiency of energy access by the system in support of its structure and functioning. [Empirical estimates of energy availability for the support of past societies is difficult to measure, but serious beginnings in this area of research have begun. (See Currie et al 2015.)] Please note that the period from 2900 BCE to 1800CE represents 4700 years of human habitation that is directly dependent upon the production (and delivery) of an agrarian surplus; available energy always drives the show, in this case the complex adaptive system which is the world-system show, and the available energy driving the production of an agricultural surplus is by and large solar energy. This brand of energy and its accessibility then represent a world-system limit then that when breached ushers in a period of rapid or punctuated world-system change. These limits are both externally imposed with respect to actual energy availability and internally with respect to the level of system complexity supported by a given level of energy availability. This implies, I believe, a reciprocity between energy access and the mechanisms for accessing that energy. For example, the advent of the steam engine made energy in the form of coal more available some of which could be used to extract more coal; positive feedback resulting in punctuated change was a consequence of the origin and application of the steam engine.

A second question then becomes pertinent: Why, if agricultural surplus is the primary source of converted solar energy driving the processes of urbanization, are there three succeeding and increasing levels of maximum urban area as mentioned previously? If the source or input of some process is the same, then why do outputs at different historic times differ in such a patterned way? The answer, I believe, has to come from the changed organization, technologies, and transport systems of the world-system over that period of time. Simply, but perhaps stated in a facile way, while the quality of agrarian output changed over time, this alone would not supply the needed surplus to support increasing levels of urbanization. Instead, improved technologies and trade routes and the socio-political systems to mediate them and a more integrated world-system would also be required. Improved and extended trade routes along with improved technologies would of course help polities to avoid the Malthusian trap. By this I mean that a given polity being able to access goods (and services) at distance reduces the stress of competition for the same locally and also enhances competitive advantage; possible improved competitive advantages may accrue as well. As a consequence, with these improvements, the energy available to support maximum urban area(s) was increased.

It is also of interest that while the increasing urban area pattern over time is punctuated, the total population of the world-system does not exhibit a similar pattern. As shown initially by von Forster et al (1960) but thoroughly developed and refined by Korotayev et al (2006 a, b, c), the growth of the world-system population is hyperbolic, i.e. it is characterized by greater-than-exponential-growth resulting for most of world-system history in continuous growth resulting in the blow-up regime of the last 200 years starting with under one billion people in total around 1800CE to a current population of well over seven billion and possibly reaching a limit in the second half of this century of between nine and ten billion people. Given this pattern of overall population growth and given that the growth of maximum urban area magnitude is punctuated in form, one has to ask what the state of urbanization is below the level of the maximum urban area?

During stasis, as world-system population increases and there is an upper limit on the size of the maximum urban area, enhanced urbanization at these lower levels has to be where the excess world-system population is housed, and this occurs with improved technologies and improved spread of technologies filtering down from the top; during phases of punctuation the top of the urban area distribution is expanded. Further, the spread of technologies must have resulted in reciprocal feedback with regard to surplus agricultural production and the delivery of that surplus[1].

Each of the previously described periods of stasis is followed by a period of rapid change, change that punctuates the status quo. There are then three such periods of punctuation, one extending from 1600 BCE to 1200 BCE, a second from 700 BCE to 100 BCE, and a final one beginning at 1800 CE and continuing through the present and, with respect to the data at hand, most probably ending sometime in the second half of this century.

Two characteristics set these periods of punctuation apart from their stasis period neighbors; continuous increase in the magnitude of maximum urban areas with essentially no fluctuation, and in agreement with reduced variation, relatively high values for the coefficients of determination of their respective linear regressions, all $R^2 \geq .9$. What this implies is that for these periods of time the world-system in toto behaved differently than during periods of stasis. Specifically, and with respect to the previous comments on accessibility and efficiency of accessibility to energy sources, an event or set of events had to occur prior to the inception of punctuation; tipping points had to be exceeded. It is also of interest that prior to the advent of each punctuation phase the final magnitude of the maximum urban area of the previous period of stasis was at the approximate value at which that period began. So, for the previous period of stasis the initial level of the maximum urban area at 100 BCE was one million, and at the end of this period, 1800 CE, the value was a tenth over the initial value, i.e. 1.1E6. [Critics may call this last statement into question if the data for the period of stasis beginning at 1200 BCE are considered, as the initial value for maximum urban area population at this time is noted as one hundred and sixty thousand, yet the final value at 700 BCE in just one hundred thousand. I consider the actual size of the initial value to be a product of population and ecological overshoot, one of the end results of which was the Late Bronze Age collapse.] During this previous time of stasis the

[1] In his insightful study, The Great Transition (Campbell 2016), in which the author details the climatic, epidemic, and societal changes during this time, Campbell suggests that urbanization occurs at levels below Cmax , specifically at C = 5000, from 1000 CE to approximately 1300 CE calling this urban area size optimal for the time. This would be in keeping with the third period of stasis being attained by approximately 1000 CE. Cmax does fluctuate for about 300 years from populations of one million to one-and-one-half million before achieving a static position of one million post-Black Death through the 17th Century.

population rose form 160 million to 813 million, a slightly greater than five-fold increase, and the initial value of γ in 100 BCE is 1.2757, while the final value for gamma during this period was 1.4112. This means that per level of urban area as defined by a bifurcation process of urban areas, i.e. the first level having one urban area, the second, two, the third four and so forth, the population in each urban area level was approximately 5% greater at 1800 CE than at 100 BCE. Further, the actual maximum urban area population was 10% greater. The implications of these figures are that per level of urbanization, lower levels of urbanization were (much) more highly urbanized in 1800 CE than in 100 BCE. This fact will have significant pertinence later in this paper.

Turning now to the circumstances that give rise to the relatively abrupt transition from stasis to punctuation; these events should be noted as tipping points. With reference to the paper by Lamberson and Page (2012), evidence supporting the existence of tipping points constitutes a difference in variability of some system characteristic across the tipping point and a change in direction of the system embedded process that is being used to characterize or represent the system, in this case the pattern of urbanization over time. Specifically, with reference to the temporal changes, the coefficients of determination of the linear regressions of the punctuated phases when compared to those of the periods of stasis are significantly higher. Further, the slopes of the regression lines themselves differ with those of the stasis periods, varying closely around zero, while those of the punctuated periods are positive, varying from .3040 and .3579 for the first two periods to a uniquely high slope, 1.7301 for the period we are now in but will in all probability be terminating shortly. Both of these forms of evidence are consistent with a tipping point being exceeded during 1600 BCE, 700 BCE, and the 18th Century. However, it should be noted that the context leading to the exceedance of this last tipping point has a deep history (Grinin and Korotayev 2015).

From an empirical perspective, is there any uniqueness to the actual transitions from stasis to phases of punctuations in the time series being investigated? The answer is, Yes. This position can be characterized by a simple comparative statistic, C_{max}/T. When the value of this ratio is computed over the complete time series of urbanization being investigated, there are only three points at which the value of this ration is .0014 or less, and these three points in time coincide with the beginning of a punctuation phase, i.e. 1600 BCE, 700 BCE, and 1800 CE. There is then something unique with respect to this value of C_{max}/T, its association with the concurrent values of gamma, i.e. respectively 1.5491, 1.5248, and 1.4112, and the distribution of urban area magnitudes promoting this transition. This association will be (one of) the foci of future research.

A quick note is necessary about the resolution of the data being used. The data themselves, i.e. world-system population sizes over time and the magnitude of maximum urban areas over time, come from three sources, Modelsky (2003), Chandler (1987), and the U.S. Census Bureau Historical Estimates of World Population and are taken at century intervals. Consequently, events influencing those two classes of data occurring within century intervals are noted after the fact by changes in either or both sets of data. As an example, the figure of 1.1 million used for the maximum area size in 1800 CE is slightly greater than the initial figure of one million in 100 BCE. This could most reasonably be attributed to the overt initiation of the Industrial Revolution during the middle of the 18th Century, so that in fact the actual phase change from stasis to punctuation occurred prior to 1800 CE. Even so, because of the use of century time periods as unitary, all that can be represented are the values of maximum urban areas and world-system population sizes at those temporal positions.

A Brief Description of K-waves.

While it is important to establish where K-waves fit in to the previously described picture of urbanization, more immediately it is also important to give a brief description of K-waves themselves and also to describe their relationship to the other two classes of economic wave behavior identified by Jugal (Korotayev and Grinin 2015) and Kuznets(Korotayev and Grinin 2015).

To begin, K-waves were first identified by Nikolai Kondratieff (2014) in the middle 1920's. He was able to show the regularity of oscillations in the value of various commodities in capitalist systems, much to the disgruntlement of Stalin, a disgruntlement ending in Kondratieff's execution a decade or so later. K-waves are characterized by a periodicity of fifty to sixty years, and historically have been shown by Modelski (2012) to extend backwards in time just prior to the beginning of the last millennium. Further, each K-wave can be shown to be associated with singular paradigm changing advances. Modelski sees these advances as being technological, social, or fiscal for a given K-wave initiation, e.g. in the 10th Century during the Song Dynasty a K-wave motivated by the advent of printing was followed by one motivated by the establishment of a national market, then into the 11th Century a new wave started due to paradigm shifts in finance, then another by changes in maritime trade, then the establishment of Champaign Fairs and so forth. [This description will not be concerned with the amplitude of these waves.] So, it would seem then that K-waves feed on one another, each creating circumstances in which the first K-wave decays but motivates the changes necessary to initiate the next K-wave. This is a system self-organizing process in some respects not unlike ecological succession, another self-organizing process, in which each stage of succession sets up conditions favoring the next stage but antithetical to its own existence (Horn, 1971). [Note: This metaphor can only be extended so far, as technological climax appears to create tipping points, which, when exceeded, lead for instance to the acquisition of more efficient energy access mechanisms from current energy sources, e.g. improved trade networks, or to the exploitation of new energy sources, e.g. the exploitation of fossil fuels, stored solar energy, rather than direct solar energy. This is in contrast to ecological climaxes which represent relative ecosystem stability for a period of time which may then be followed by a slow decline, but time dependent with respect to other biosphere processes and physical intrusions. It is worth noting that the 10% Rule, a rule-of-thumb for the efficiency of transfer of useable energy from a lower trophic levels to a higher ones, unlike in human dominated systems seems to imply an energy limit which has yet to be exceeded.]

There are other aspects to the dimensions of K-waves that are important to note. The periodicity of a K-wave itself can be divided into upswing phases and downswing phases (Kondratieff as noted in Korotayev and Grinin 2012). Each subphase constitutes about fifty per cent of the total period of a single wave, however, in general upswings are slightly longer in duration. Further, it was recognized by Kondratieff himself that downswings were (or could be) characterized by invention, while upswings represented times when those inventions could be applied. This is a case then of periods of invention alternating with periods of innovation, i.e. paradigm shifts occur during downswings while their application occurs in the following upswing, that later being considered then as variations on the same theme. This context of upswing and downswing being associated with respectively innovation and invention and in which there is reciprocity between direction and event, is clearly a mechanism of self-organization.

Kondratieff cycles are not the only cycles at play in the integrated world-system, i.e. at play in the spheres of society, culture, and economics. Juglar (As noted in Grinin et al 2016) recognized economic cycles of a much shorter period as far back as the 1880's, and Kuznets (1958) identified cycles of an intermediate periodicity. These modes of cycles along with those of Kondratieff have been combined in an elegantly simple but effective clock model by Berry and Dean (2012 and elsewhere). They have shown that these three types of cycles can be represented as a nested set in which Kuznets cycles occur three times within a single K-wave, and Jugal cycles repeat twice within the limits of a single Kuznets cycle, i.e. they occur six times within a given K-wave. Berry and Dean go on to note that associated with the morphologies of each wave are predictable (collective) economic behaviors, including but not limited to the incipient generation of K-waves.

Both Modelski (2015) and Berry and Dean (2015) suggest that the initiation of further waves is associated with an integration process in which innovations are bundled together, as for example: "…the bunching of basic innovations that launch technological revolutions that in turn create leading industrial or commercial sectors." (Modelski, 2012), and "Long waves are driven by clusters of innovations that introduce new techno-economic systems and give rise to waves of economic growth and social change." (Berry and Dean, 2012). Here it is not only important to recognize the impetus both (sets of) authors place on innovation, used here in its most general sense, as a stimulus for wave initiation, but it is also important to note that the notion of collectivity, i.e. *bunchings* or *clusters*, of techno-financial-social innovations was prime in motivating these changes. In other words, it is the collections, the clusters, the bunchings themselves that become or may become unitary within the wave they are initiating. The importance of this collective integration will become apparent further on.

A Brief Description of the General Process of Historic Change to Include but not be Limited to Punctuated Change

The remainder of this paper will include a math model of punctuated change as that change is related to (generic) population size, carrying capacity, and technological development. The outputs of this model will then be converted into common units of entropy, i.e. information. Following this, the context for such changes will be considered and qualitative models based on the pattern of Kondratieff Waves, the relationship that the form of those waves have to both invention and innovation, and ultimately the formation of hypercycles as 'a bootstrapping process, will be presented.

Models of the Process of Punctuated Equilibria: Verbal, Agent-Based, and Equation-Based

Three different models of abrupt system change will be briefly treated here. Two models are introduced to show precedence for this type of change in complex systems; the first, that of Eldridge and Gould (1972), is verbal and qualitative; the second by Per Bak (1996) is broadly based, quantitative, and applicable to all complex physical systems and complex adaptive systems. The first two have history; the third is a relatively new model (Harper 2017), and is different in tenor but not in the general intent of explaining punctuated change. It is based on equations, metaphors of mental models as are the other two, but deterministic in focus. In each model, whether verbal and qualitative, general and quantitative, or precise and quantitative, the behavior of complex systems over time is shown to be non-monotonic and in fact punctuated.[2]

[2] Please note that these first two models are presented as examples of punctuated change and are not per se meant to be applicable to an explanation of the data on historic urbanization patterns presented in this paper.

When Eldridge and Gould proposed their theory of punctuated equilibria as it applied to speciation and how that process would be represented in the fossil record, they relied on the process of allopatric speciation in tandem with the establishment of genetic equilibria to produce a punctuated fossil record. New and selectively favored genetic combinations, due to all the standard mechanisms provided by Neo-Darwinian theory, would arise in peripheral, semi-isolated populations and via gene flow would spread (rapidly) through the rest of the species population. This rapid spread per se would not be represented in the fossil record, but the beginning and ending species morphologies that border the process of rapid change would; hence, the punctuation of stasis as represented in the fossil record. With respect to a more general model of punctuated change, Bak and others, e.g. Bak and Sneppen (1993), produced a general model, a computer model in which changes below some system threshold established by the model would occur until that threshold was breached, and then the system would quickly evolve to a new threshold limited stasis.

The equation-based model presented here (Harper 2017) relates population, i.e. total system population, making no distinction between urban and rural populations, environmental constraints, i.e. Malthusian constraints, and technology (in the most general sense) using a set of three differential equations as follows;

$$dN/dt = \mathrm{r}N(K - N), \qquad \text{Eq. 3,}$$

$$dK/dt = (T - N)/K, \qquad \text{Eq. 4}$$

$$dT/dt = T/(K - N), \qquad \text{Eq. 5}$$

where N = population, r = rate of growth, K = carrying capacity, and T = a measure of technology.

While the model above presents a graphical picture of the sort of punctuated change exhibited by world-system urbanisms over time, It does so without reference to clearly defined units. So, for instance, $dT/dt = T/(K - N)$, represents a unique relationship between three variables, N, K, and T, what actually do these three variables represent? How can each be defined in terms of the other, or is there some common unitary currency to which all three variables may be related? It is not difficult to specify variable signature to both N, population size, and K, carrying capacity; they can both be represented by the individual, N simply being the total number of individuals in the population, and K being the upper limit to the number of individuals in that population at any given time and imposed by contemporary environmental constraint regardless of how that environment might be defined. However, T. the measure of technology, is more difficult to represent. Since technology is labor saving in nature, then perhaps a simple conversion represented by the following similarity will do: $T \sim aN$? This is an elementary and linear relationship in which a given level of technology represents an added fraction of the population capable of performing more labor, or, conversely, a portion of a given population no longer needed to perform the same labor. [As an aside, it should be noted that since technology is a contributing factor to the complex adaptive system that we are all a part of, changes in technology, while labor saving per se, may also bring about the emergence of wholly new phenomena and consequently be not only labor saving in the traditional sense but also be a medium for revealing and colonizing new intellectual territory.]

It would seem that the problem of unitary definition and conversion has been solved, however, with a little math it is clear that the equality of each of the three differential equations being set to zero, and therefore to equilibrium position, does not allow each equation to be equated to the other. Here there might be two alternative pathways to take, experimenting with a variety of different conversion models for T = f(N, K), Eq. 6, especially non-linear ones, thus complicating the untary context considerably, or devising a conversion factor, the mathematician's version of a carpenter's shim, in order to make the equations work. However, there is also an entirely different alternative approach to consider, one involving the relationship between each of the equations and a common physical currency, that of information. This relationship will be described in the following paragraphs.

In personal communication with Claudio Maccone (2019) regarding the above problem, he suggested equating each differential equation to its information content, information content being a universal currency by which comparison using the following equation: $H = -\int f(x) \ln f(x) dx$, which is the continuous form of the Shannon-Weaver entropy equation: $H = -\sum p_i \ln p_i$, where in the former equation, f(x) represents a given function, in the case of this paper the three differential equations, and in the latter equation p_i represents the frequency of a given entity, e.g. this equation is used in the study of biodiversity to represent the level of biodiversity in a given collection of species in which p_i represents the frequency of any given species in that collection. Therefore, each of the three differential equations may be represented by their entropy content as follows:

$dN/dt = rN(K - N)$ has an entropy content defined by:

$$H_N = -\int rN(K - N) \ln rN(K - N) dN \quad \text{Eq. 7,}$$

$dK/dt = (T - N)/K$ has an entropy content defined by:

$$H_K = -\int (T - N)/K(\ln(T - N)/K) dK \quad \text{Eq. 8}$$

$dT/dt = T/(K - N)$ is entropically defined by:

$$H_T = -\int T/(K - N) \ln T/(K - N) dT \quad \text{Eq. 9.}$$

It is the fact that these conversions are possible and are capable of producing functional results that underpin the use of the initial three equations. The integrated forms of H_N, H_K, and H_T will be given in an appendix, their representation graphically will be presented in a further section of this paper but dealt with in detail in a future publication.

The initial three-equation model produces the following output:

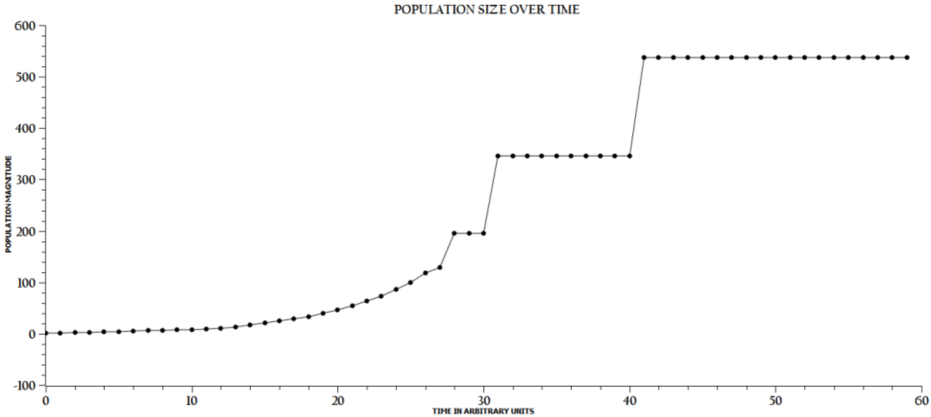

Figure 4. The X-axis represents time, and the Y-axis represents population magnitude, both in unspecified units.

As can be noted in the graph in Figure 3 above, the growth of the population is not continuous but is punctuated; this is due both to the interrelationships specified by the model and by the magnitude of dt specified by the STELLA program. In any case, the equations of the model and the visible representation of the model output suggests that, even though input can be, in fact is, continuous, the output is discontinuous. What are the time courses of all three variables? The graph to follow reveals this.

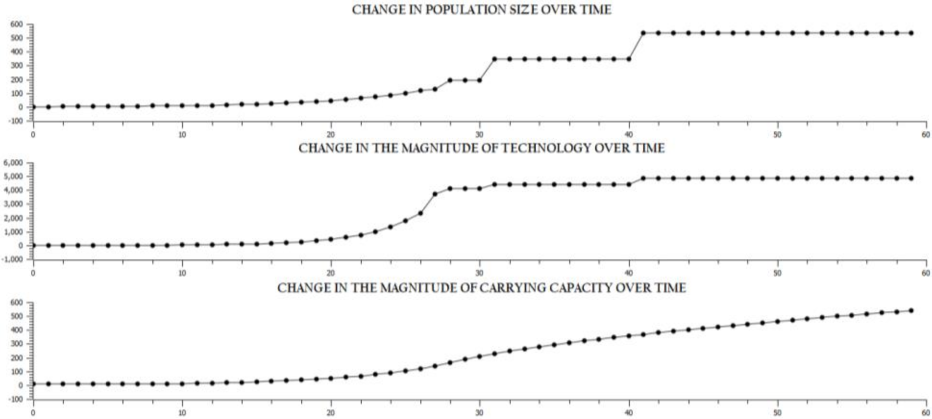

Figure 5. As in the previous graph, the X-axis represents time. However, the Y-axes are scaled for each of the three variables.

Of interest in this graph is that only two of the three variables displayed are discontinuous, carrying capacity, while exhibiting rapid change, does so in a more or less continuous way, and it is the interplay between population and technology that allow for more or less continuous elevation of carrying capacity. Also of interest is the fact that this graph appears to represent a combination of both Malthusian and Boserupian effects, and while some scholars (See Kristensson et al 2016) suggest that during the evolution of human culture Malthusian limits were over taken by Boserupian environmental expansion, the above graph suggests that both are at play and are effectively integrated processes. This suggestion clearly requires further research.

Since the focus of this paper is not the interstices of any of these models, a detailed discussion of their significance will not be included here. However, as a final note on modeling, it is important for the reader to realize that none of these models is *the* model, but rather all of these models provide at least some utility in understanding the processes and significance of punctuation. This in turn will lay the basis for an understanding of the relationship between Kondratieff cycles and the punctuated pattern of urbanization represented previously.

The Graphical Representations of H_N, H_K, and H_T

In this section, graphical representation is given to the data collected on H_N, H_K, and H_T, both singularly and in comparison with the graphs of dN/dt, dK/dt, and dT/dt. It is the intent here to discuss pattern and synchronicity of pattern only but not to delve into underlying mechanisms of the relationships either with respect to the occurrence of sudden change in graphical slope nor in the relationship between a given sudden change in entropy versus the punctuated change exhibited by dN/dt and dT/dt. With this in mind, the task will now be to describe and explain the entropy graphs and their relationships with punctuated change. This will be done in the sequence $H_N \rightarrow H_K \rightarrow H_T$. In each case after a description and explanation is given of the specific entropy graph, a comparison will be made with the appropriate differential equation, e.g. H_N with dN/dt. Also, all three entropy graphs will be displayed collectively and described as a suite of graphs. To begin.

Figure 6 represents the change in entropy, H_N, over the same time span as the integrated form of dN/dt. It exhibits no phases of stasis and punctuation but on comparison with that graph (See Figure 7.) does exhibit rapid changes in slope and in sign of slope; two to be exact. After the initial abrupt change at t (time) = 30, the mode of change until the next point of sudden change, t = 40, is sublinear, and the mode of change following the second tip is also sublinear. If this graph is compared to the graph given by the model, see Figure 7, it can be seen that each change in slope is synchronous with a phase of punctuation. There is also an initial set of punctuation and stasis that do not appear to have a corresponding slope change. This is thought to be due to the resolution of the graph.

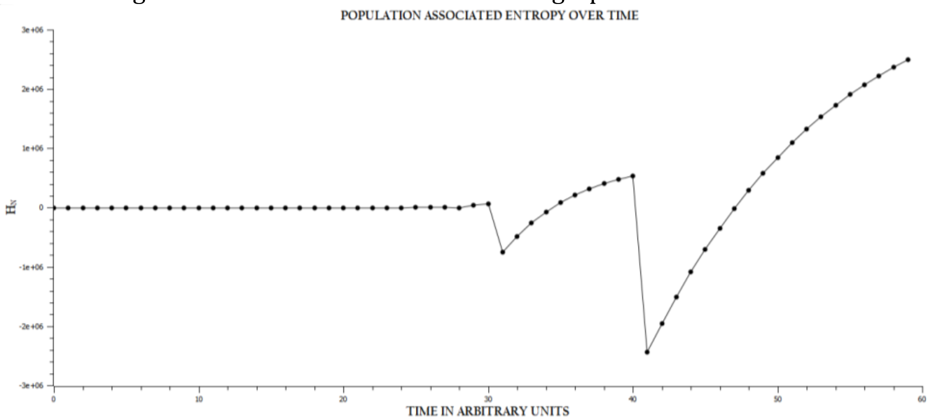

Figure 6. A representation of H_N over time. Note the points of rapid change in both slope and the sign of slope.

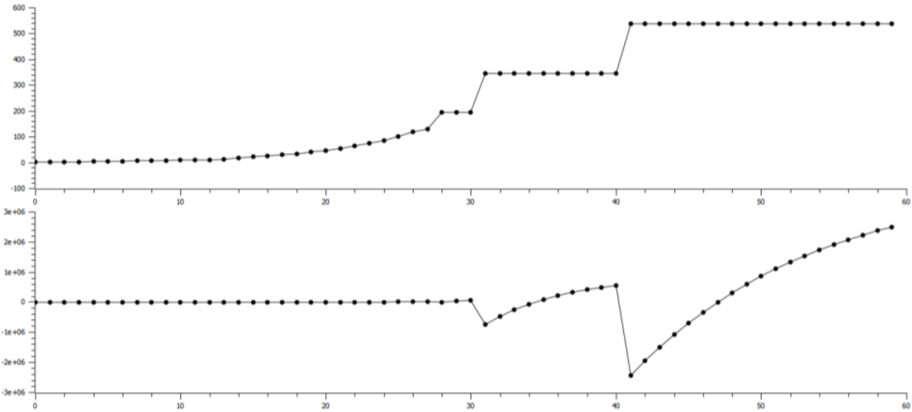

Figure 7. This two panel graph represent the punctuated graph of population as produced by the initial model in comparison with the graph of H_N. Note the synchronous occurrence of phases of punctuation in the top curve with that of the points of rapid change in the bottom curve.

Note that the top graph is the punctuated pattern of population, while the bottom graph represents H_N. The Y-axis is scaled the same as in previous graphs. The rapid changes in the graph of H_N are synchronous with the phases of punctuation in the top graph.

With regard to H_K, Figure 8, quite a different form is exhibited. Initially, and for a period of fifteen or sixteen time steps, the value of H_K remains relatively constant and not much beyond $H_K \sim 0$. Then there is a precipitous drop to a negative range of $H_K \sim -6500$ at $t = 28$, followed by a slow and punctuated rise through $t = 60$. With regard to the behavior of the whole model, and with regard to

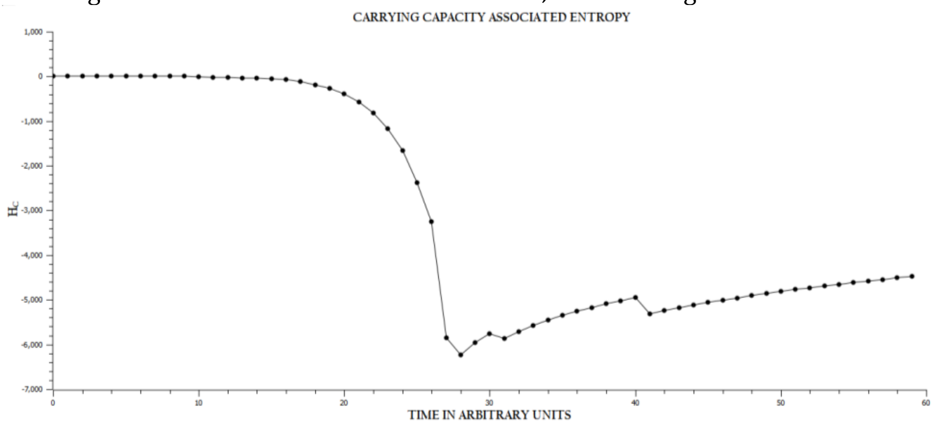

Figure 8. This curve represents the graph of H_K over time. Note the precipitous drop in entropy at t = 28 and the slow rise of H_K following that drop.

the fact that carrying capacity is represented, it is interesting that such a period of negative entropy should develop. It is also interesting that through the time range of the model, t = 0 – 60, after the initial drop the increase in entropy is moderately positive and from t = 28 until t = 60, the positive change in entropy is only, $\Delta H_K \sim$ 2000 to the positive. Note that there are point of rapid slope change and change in slope sign at t = 40 and t = 50.

In Figure 9 the upper curve of the integrated form of dK/dt is smooth and continuous with a change an initial change in slope at t ~ 25 and an overall form that is mildly sigmoid; there are no punctuations as noted earlier. However, the lower curve of H_K, as just noted, does exhibit points of rapid slope change. These events

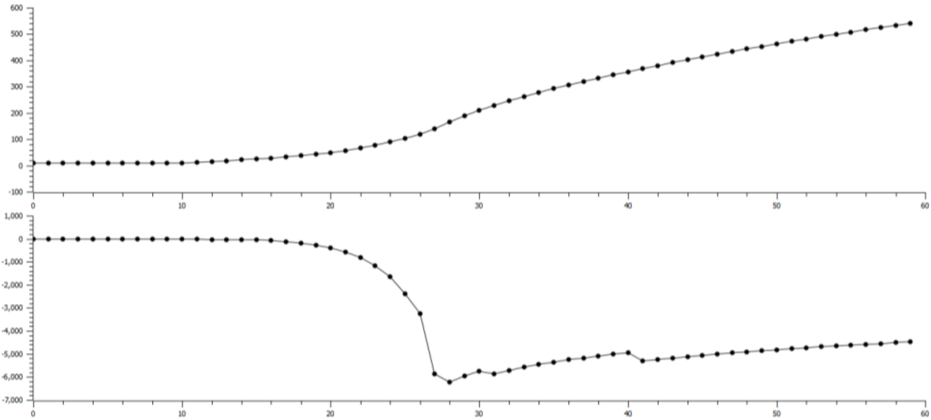

Figure 9. This is a two panel comparison of the integrated form of carrying capacity, the top curve, with the curve of carrying capacity entropy, H_K, the bottom curve. Of interest is the continuous change exhibited by carrying capacity in comparison with the abrupt punctuations of the entropy associated with carrying capacity.

are a consequence of the punctuated behavior of the model as a whole and therefore reflect the effects of both population and technology on carrying capacity.

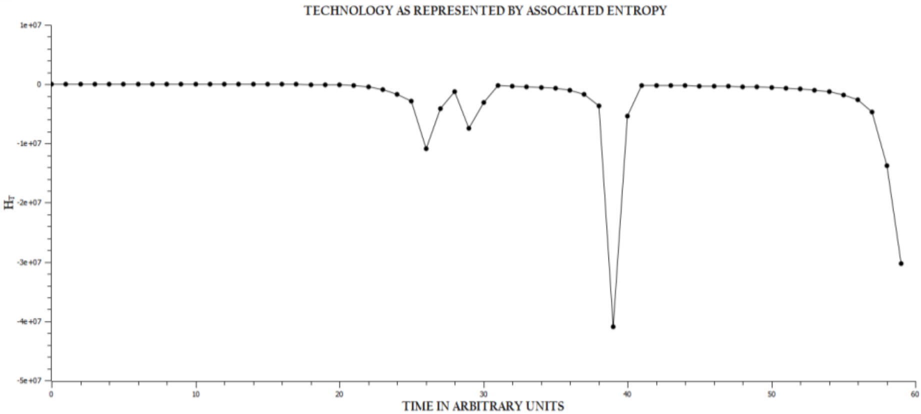

Figure 10. This graph represents the entropy associated with technology over time. There are four troughs of entropy punctuating and otherwise essentially constant and flat plot.

Figure 10 above represents a graph of $\mathbf{H_T}$ **over the same time span as in the above graphs. The pattern of this graph is of overall constancy punctuated by brief periods of rapid negative change.** The minima of those periods of negative change are at t = 26, 29, and 49; offset by one with respect to the initiation of punctuation in both the curve of population and of technology itself. See Figure 11.

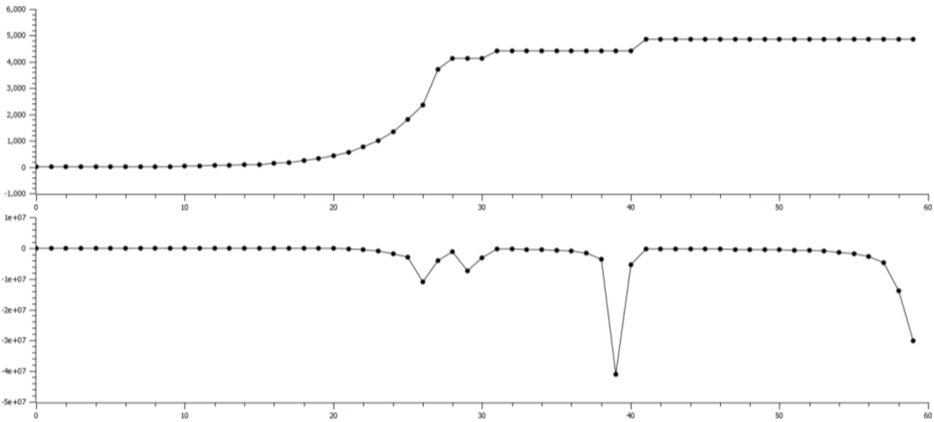

Figure 11. This is a two panel graphical comparison of technology in the top graph with respect to the entropy associated with technology in the bottom graph. The offset by one time step of the troughs in the bottom graph with the initiation of punctuation in the top graphs is significant.

The interesting aspect of Figure 10 is that although there are periods of rapid negative entropy, these periods are brief, and then the system returns, relatively speaking, to its original position. As noted above, these troughs of negative entropy are one time step before the actual punctuated phases of the technology graph. This occurs in both the graphs of population and of technology, and one is tempted to suggest that the negative entropy spikes are preliminary to and therefore a requirement of the phases of punctuation in both the population and technology curves.

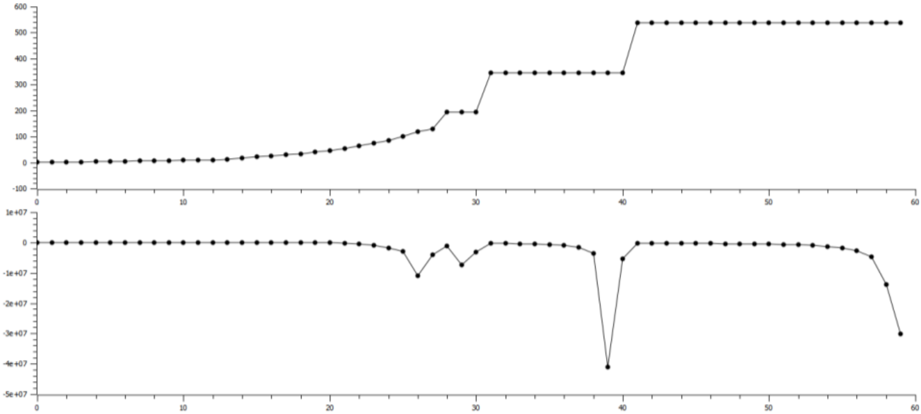

Figure 12. This is a two panel comparison of population change in the top graph with the entropy associated with the change in technology in the bottom graph. Note the offset of the troughs of entropy in the bottom graph with the onset of punctuation phases in the top graph, as in Figure 11.

The Association of K-waves with the Macro-pattern of Urbanization

In what way or ways are K-waves related to the macro-pattern of urbanization, and, further, are K-waves in any way instrumental in the identified tipping points of this pattern, i.e. the points of change in the distribution giving rise to the change from stasis to punctuation evident three times over the extent of this this pattern, i.e. over five thousand years? Korotayev, Zinkina, and Andreev (2016) have also emphasized the importance of understanding the relationships between trends, i.e. millennial trends, and cycles. Since K-waves have a limit to the extent to which they can be extended back in time, at least as labeled entities motivated by specific processes as noted by Modelski (2012), i.e. approximately one thousand years into the past, that places a limit to this analysis of considering only the last one thousand years of urbanization macro-pattern in relation to those K-waves. In fact in this paper only the last approximately eight hundred years will be considered.

The top graph of Figure 12 represents this portion of that macro-pattern, the full extent of which was

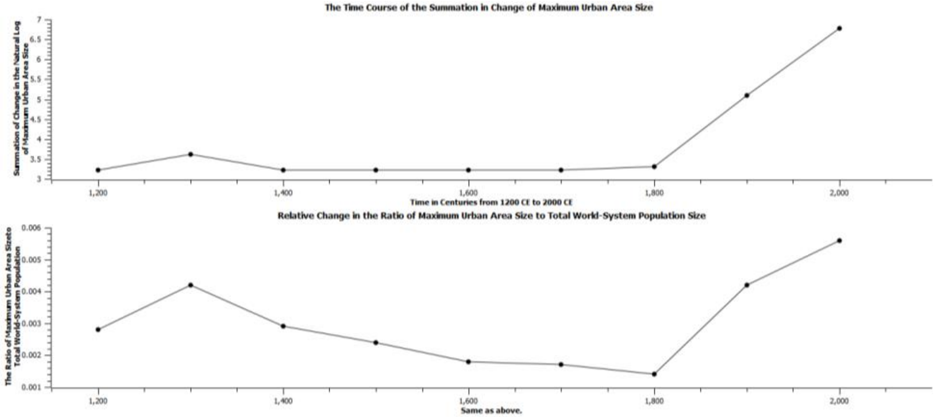

Figure 12. A. The top graph represents the summation in change of the natural logarithm of maximum urban area size over the last eight hundred years. B. The bottom graph represents the trend in the ratio of maximum urban area size to total world-system population over the same period of time.

previously represented in Figure 2. This period begins approximately with the establishment of Champaign Fairs, continues through the expansion of the Black Sea Trade, then the development of Galley fleets, on to the present. This period of time includes two monumental events of historic importance dwarfing most if not all other events of this millennium; specifically, the pandemic spread of Bubonic Plague in the 14th Century and the advent of the Industrial Revolution, as previously noted visible in the 18th Century but with deeper roots.

A further comment is required here regarding the limited extension of K-waves backward in time. It should be noted that the incipient world-system as defined by Abu-Lughod (1989) had a variety of economic systems endemic to each of the eight sub-regions of that incipient system. She further suggests that while cyclic economic activity was apparent, that cyclicity appears not to have been synchronous throughout the system. It is for this contributing reason that the K-wave pattern has only been extended back to the beginning of the 13th Century. [This decision in no way implies that such economic oscillations, i.e. K-waves, did not extend back further in time, only that the economic context for such oscillations is on reasonably secure grounds for the previous 800 years.]

The top graph, A, shows a rise in the change or maximum urban area size at 1300 CE, then a decline in the next century which is maintained essentially through 1800 CE, a period of four hundred years; the remaining two centuries exhibit a pronounced increase in this trend. The stability of the intervening centuries with respect to change in maximum urban area size is a reflection of the technology available to access energy characterizing the world-system over these centuries; the abrupt increase at 1300 CE may well represent urbanization overshoot, a form of ecological overshoot in the manner described by Catton (1983). [Parenthetically, it is interesting to note that the Black Plague began after this elevation in urbanization. It is also interesting to note that while the Medieval Period both begins and ends with severe environmental/climatic downturns (Hoffmann 2014), the latter being the onset of the Little Ice Age, that climatic downturn did not lower the size of the maximum urban area; it remained at one million.]

Figure 12 B represents the relationship, a ratio actually, between maximum urban area size and the population of the entire world-system. It is quite clear that while the general shape of both graphs A and B is similar, there is one significant difference, the lack of a plateau from 1400 CE to 1800 CE. This period is not a time of stasis per se with respect to the ratio used but represents a period of decline due to an upper limit to urbanization size but an increasing world-system population. Using X-values of 1, 2, …5 for the designations of the centuries during this time, the slope of a linear regression is: m = -4.2E-4 and has a coefficient of determination of R^2 = .9383. This lower graph then reflects the fact that while there appears to be an upper limit on the maximum urban area size, there is no similar limit on total world-system population size. As previously stated, that excess population must then be absorbed by lower levels of urbanization, the implication of which is that the degree of urbanization must relatively increase at lower levels. Also, there is no specific evidence of population fluctuation at the top of this urban area distribution.

What can be made of this previous observation? K-waves clearly exist over this four hundred year period but appear not be reflected in the data available. First, this data set, the magnitudes of maximum urban area over time, is given in century intervals K-waves have a periodicity of fifty to sixty years, so the temporal resolution of each set is different. Secondly, during this time, the events and processes associated with the K-waves noted by Modelski, at least up until the K-wave associated with the American Plantation, have little to do directly with energy access. The American Plantation K-wave certainly did, as the agricultural surplus produced was (largely) dependent on slave labor, i.e. trapped sunlight in human bodies that was used, exploited, for the benefit of others and to the detriment of the body initially sequestering that sunlight. So, the K-waves associated with the pepper trade or Guinea Gold or spices were not paradigm-shifting with respect to energy access. However, those K-waves certainly did ultimately lay the ground work for the great divergence of the 18th Century, the Industrial Revolution of the West. It is important to note here that total population data with greater resolution than is used here might reveal the impact of K-waves on the world-system, and with this greater resolution, population data at lower levels of urbanization may also be identified as reflecting the effects of K-waves during this time.

While there appears to be no obvious effect, no synchronous population fluctuations, associated with the periodicity characteristic of K-waves during the end

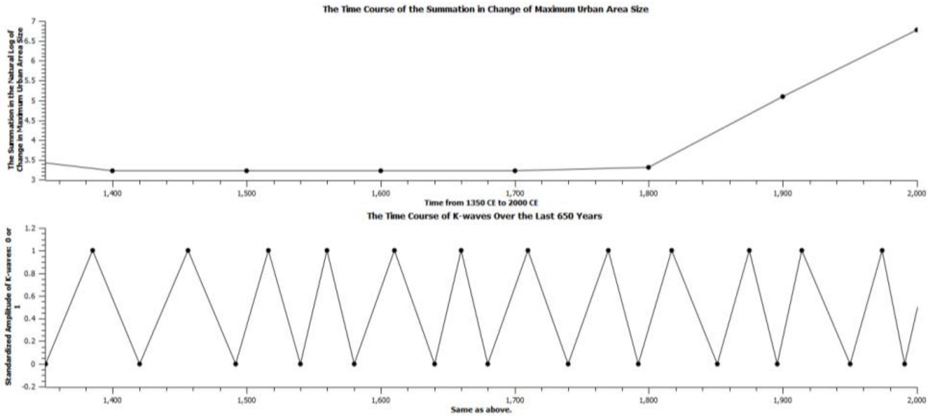

Figure 13. The top panel represents the summative change in the natural log transformed data over the period, 1350 CE to 2000 CE, and the bottom panel represents K-wave periodicity over that same period of time.

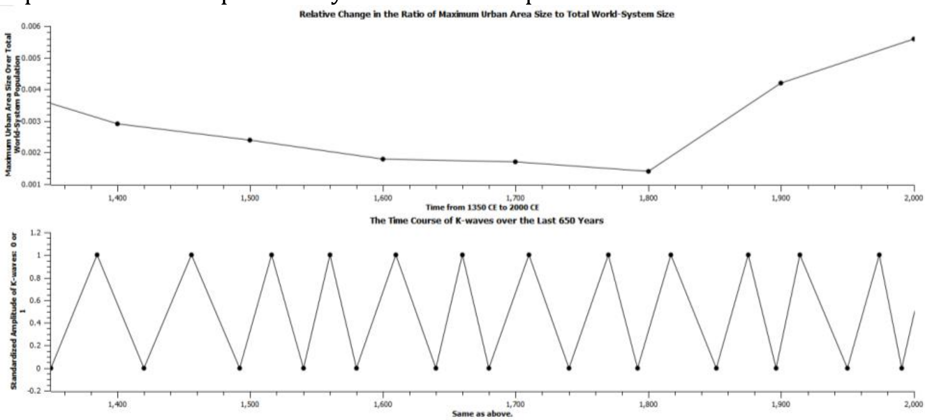

Figure 14. The top panel represents the trend in the ratio of maximum urban area size over total world-system population over the period, 1350 CE to 2000 CE, and the bottom panel represents K-wave periodicity over that same period of time.

of this period of stasis, this time does represent an increase in urbanization and the processes associated with urbanization (Grinin and Korotayev 2015) occurring below the delimited maximum urban area size and does ultimately lead to a graphical point (See Figure 12A and 6B.) that is the visual metaphor for a tipping point, i.e. the low point and end point of this period of stasis and the beginning of a period of punctuation; again, both a change in R^2 and the direction of the maximum urban area pattern reflect a tipping point of system wide magnitude. The K-waves associated with this period of time are, in sequence beginning at 1640, the American Plantation, Amerasian Trade, Cotton and Iron, the development and expansion of the railroads, first in national extent in the U.S. and completed shortly after the Civil War but followed by Russia and others, and by 1850 the advent of a secondary energy source with enormous utility, electricity. Please be aware that the beginning of the age of steam with its reliance on coal occurred at this time and certainly represents an open door, the open door, to the rise of the Industrial Revolution. There is a synergism here between new technology designed to access a fuel source and the use of the fuel source, coal, to access more of that same fuel. This of course represents a potential, a realized potential, to mechanize production in general, the increased output of cotton and iron being key among the end products of this mechanization.

There are two questions to be asked at this point in this paper: First, did the tipping point emerge simply as an unpredictable event in a complex adaptive system, or does this point require a specific, developed context in order to have occurred? Second, is the tip simply a result of specific, individual events occurring, or is there some sort of collective organization of those events required? Remember, crossing each of the major tipping points in world-system history permitted the system to support an even greater level of maximum urban area, implying either more efficient access to the same type of energy or access to a new and better type of energy.

It will be the position taken here that a period of development was required to create the context for the tip leading to the Industrial Revolution and that crossing this tip successfully implies not only invention in the singular, but invention of synergistic systems of inventions with the result that new entities emerged, entities which represent an enhanced level of self-organization and entities that required the input not simply of the benefits of physical, industrial inventions, but of innovative financial structures, new aspects of commerce, reconfigured legal structure, and so on. Further, the interaction of these different entities is such that each enhances the sustainability, the reproduction over time if you will, of another entity, which has the same effect on another entity until the process comes full circle encompassing all the interacting entities. This process will be described in more detail later, but the immediate focus is to describe the development of context for the tip initiating the Industrial Revolution.

It is always nice to follow in the footprints of scholars whose efforts have expedited one's own research. This is the case here, as I will summarize the very informative research of Leonid Grinin and Andrey Korotayev (2015) on the events leading to the divergence of the West from the East, events that are intimately tied to the occurrence of the Industrial Revolution. Grinin and Korotayev (2015) show that there is good reason to accept the Industrail Revolution beginning in the mid-18th Century as the last of three phases of the actual revolution, the initial phase of which began in the second third of the 15th Century but was preceded by a series of inventions which laid the ground work for the long Industrial Revolution.

Inventions such as the mechanical clock, spectacles, ferrous and non-ferrous metallurgy, the compass, the lanteen sail, and this list could be extended, all created a world-system ripe for the expansion of trade, the Age of Discovery, improved devices for warfare, the printing press, and social constructs such as the beginning of the discipline of mechanical engineering, events characterizing the first phase of the Industrial Revolution, a period of time extending to the middle of the 17th Century. The second phase consists of the following one hundred plus years to the middle of the 18th Century and is characterized by fewer inventions but expanded trade, and the significant growth of the non-agrarian population; recall that as the world-system population expanded but the maximum urban area size of approximately one million represented a cap on urbanization at the top. So this population expansion represents that expanded urbanization at lower levels. Also of significance was a more efficient agrarian economy, one requiring less labor and consequently freeing more hands to be involved in non-agrarian labor. It is the third phase that is actually recognized commonly as the Industrial Revolution. The standard industrial inventions will not be mentioned here as they are common knowledge, but this period of time was also characterized by a new economy involving formal exchange of stock and with the capitalist liability of economic overshoot. All of the Industrial Revolution can be thought of as "…the continuous economy of human labor (and energy and animal work) in various fields and forms." (Grinin and Korotayev 2015). In one sense, the Industrial Revolution can be considered as a revolution in labor saving devices. However, Jevon's Paradox (1865) also has to be faced, that labor saving does not mean less work but more work in the same amount of time! This also implies greater expenditure of energy.

The context in which the Industrial Revolution occurred was then an accumulation over time of a variety of inventions and financial, economic, and social interrelations developing between those inventions as well as interdependence of inventions one on the other, and, further, access to mechanisms which fed back on themselves to enhance the rate of system self-organization to a new level. Very clearly, a context of ever increasing complexity is associated with the initiation, the emergence, of the Industrial Revolution. Since the beginning of the (visible) Industrial Revolution is mirrored in the time course of urbanization, nothing surprising here; to what extent, however, might K-waves be involved in the transition from stasis to rapid change? This question will be addressed in the following section.

A System Change to the Next Higher Level of Complexity

Even casual inspection of Figures 13 and 14 reveal no obvious relationship between the pattern of K-waves from 1300 CE to 1800 CE either with respect to the summation of change in the natural log transformed data on maximum urban area size or the ratio of maximum urban area size to total world-system population size, and as has been pointed out previously, this is in large part due both to the lack of resolution of the data on maximum urban areas and any significant data on urban distribution of size below that of maximum urban area. In the case of the data in Figure 12A, the graph of maximum urban area size is a flat line, i.e. one with approximately zero slope and showing no oscillation about the ceiling value of one million, and in the case of the ratio of maximum urban area to total world system population, there is again no indication of oscillation about a mean value. This relationship is graphically represented in Figure 15 where it is shown that while there is some variation about a mean as reflected by the linear regression, that variation is not great, R^2 = .8861, and that it in no way matches the K-wave periodicity reflected in the lower panel. However, based on the data at hand if there is no obvious indication of K-wave effect on urbanization pattern during the most recent period of stasis, what about K-wave effects manifesting themselves during and across the transition to the most recent punctuation phase? Is there any relationship between K-waves and the transition to the current phase of punctuation?

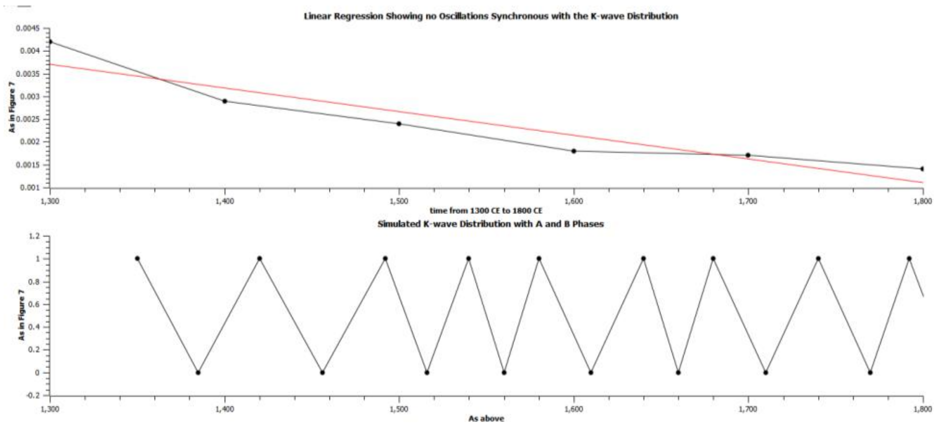

Figure 15. The top panel represents an imposed linear regression on the trend in the ratio of maximum urban area size over total world-system population over the period, 1300 CE to 1800 CE, and the bottom panel represents K-wave periodicity over that same period of time.

In order to investigate the potential K-wave effects on the tipping point to the current phase of punctuation, a phase initially characterized by the occurrence of the Industrial Revolution and now continued in the post-industrial world, an expanded view of any potential synchronization between K-waves and both trends in changes in maximum urban area and the relationship between maximum urban area and total population would appear relevant. Those expanded views are given in Figures 16 and 17. Effectively, both graphs show the same relationship, that the down phase, a Phase B in the terminology of Grinin and Korotayev (2015), occurs prior to the pivotal point representing a tip or prior tip in which the entire system, the world-system, changes course. It should again be noted that it has been recognized by a number of researchers that inventions occur during B phases, i.e. paradigm shifts, and that innovations based on those inventions, in other words the application and further adaption of those inventions, occur during the subsequent A Phase. If both Figures 10 and 11 are inspected, the periodicity of B and A phases is such that it appears that they are positioned appropriately to be associated with

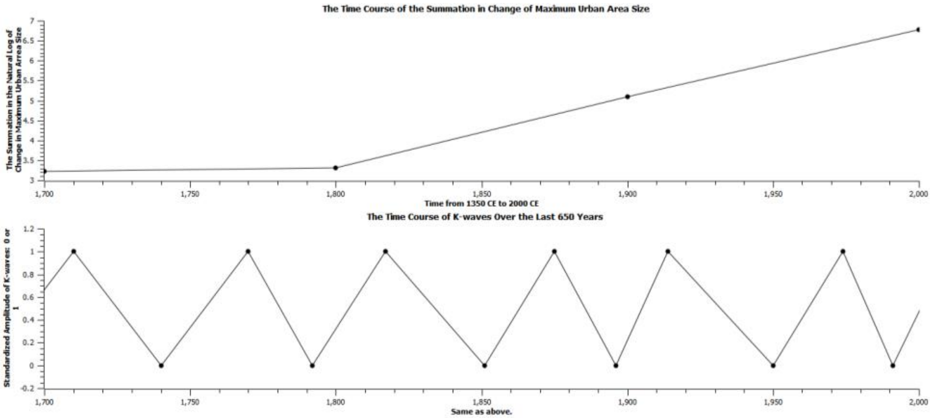

Figure 16. The top panel represents the summative change in the natural log transformed data over the period, 1700 CE to 2000 CE, and the bottom panel represents K-wave periodicity over that same period of time. Note that the down slope and up slope of the consecutive K-waves spanning the abrupt change in slope of the top curve just prior to 1800 CE are consistent in position with the invention-followed-by-innovation sequence recognized by Grinin and Korotayev (2015).

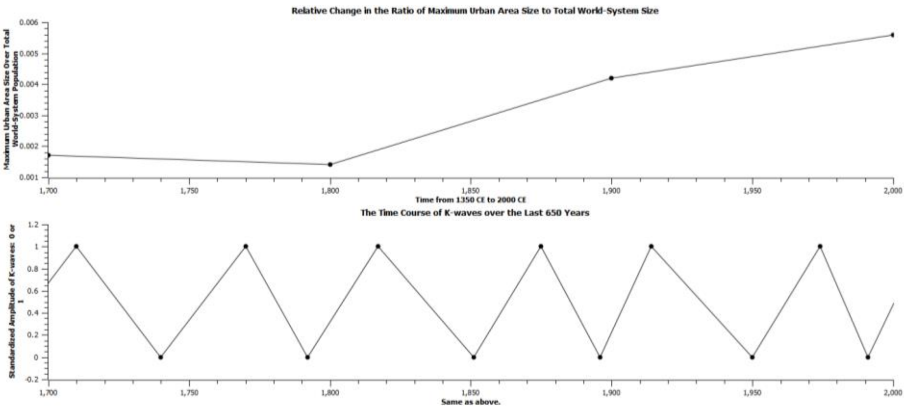

Figure 17. The top panel represents the trend in the ratio of maximum urban area size over total world-system population over the period, 1700 CE to 2000 CE, and the bottom panel represents K-wave periodicity over that same period of time. The top panel represents Note that the down slope and up slope of the consecutive K-waves spanning the abrupt change in slope of the top curve just prior to 1800 CE are consistent in position with the invention-followed-by-innovation sequence recognized by Grinin and Korotayev (2015).

the system pivot to the current punctuation phase. At this point two conditions should be restated, that the context for the emergent tip to the Industrial Revolution has a history and that while single events such as the advent of the Steam Age, the improvement of economic procedures, and the access to new energy sources characterize this occurrence, it is the integration of these and other events into efficient and to some extent singular entities that need to be studied in order to understand the mechanisms underlying this change.

A Potential Mechanism for System Change

The context for system change is one of increasing system complexity in which that complexity has embedded within it events and processes that will then function in concert to create a new direction for the system, one which will be adopted rapidly. Several life processes exhibit this sort of potential for rapid self-organization (or rapid self-***re***organization), ecological succession and in fact the origin of life itself are two such processes. Human history as reflected by urbanization pattern exhibits three periods of rapid change at the whole system scale and at much smaller scales is littered with examples of punctuated change, e.g. the origin of agriculture, the invention of the printing press, the invention of the computer are obvious candidates but are certainly not alone. If the context of the position this paper takes is expanded both temporally and spatially, and then best addressed from the perspective of the new discipline of Big History, it will be seen that (rapid) emergence becomes, no is, part-and-parcel of existence, e.g. the origin of elements, the origin of planets, et al (Spier 2015). All exhibit rapid emergence. Is there real commonness in the mechanisms of rapid change throughout the scale(s) of existence from atoms and below to the complexity of our biosphere and beyond? Per Bak (1996) would say, Yes! The attention of this paper will now turn to one such mechanism, originally proposed by Manfred Eigen and Peter Schuster (1977), co-opted by John Padgett and Walter Powell(2012), and used here as a possible model for the transition from stasis to punctuation that began prior to 1750 and is still occurring today. That mechanism is hypercycle formation.

Hypercycles represent the integration of separate processes into a unitary whole, but with conditions. There are three pertinent here: 1. The condition of autocatalysis. 2. The condition of the behavior of one member of the cycle enhancing the autocatalysis of the next member in the cycle. 3. The cycle itself becomes a unitary entity. Eigen and Schuster (1971) first proposed the hypercycle model as a potential means of bridging the information limit on the existence of complex life. This limit, known as the Eigen Paradox, is that information growth requires the existence of a growth catalyst for which the current reservoir of information is insufficient. [For those needing a more technical and also very possibly more lucid explanation of this information limit that is Eigen's Paradox, please see Smith and Szathmary (1995).] Exceeding this information limit requires not simply the input of information from outside sources but the ability to sequester those outside sources into a functional whole, consequently the two conditions above must be met. For a visual reference of what a hypercycle is Figure 11 below is included (Szostak et al 2016).

Fig 1. A hypercycle.

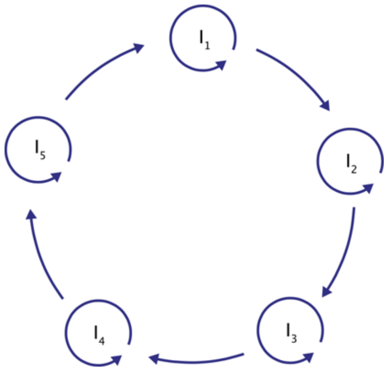

Figure 18. The above figure is given as a visual reference for a hypercycle and includes the three conditions necessary for hypercycle function and integrity: 1. Each element of the cycle is autocatalytic, i.e. it can reproduce or sustain itself through time. 2. The autocatalysis of each element contributes to the autocatalysis of the next member of the cycle. 3. The cycle itself is autonomous.

[There may also be other phenomena motivating the formation of hypercycles. If external or internal shocks occur to the world-system, those shocks may force a synchronization of a series of related processes, and this synchronization may then align those processes in such a way that hypercycle formation occurs. This is known in population ecology as the Moran Effect and has been noted as a potential factor in the synchronization of specifically urbanization and demographic processes and by default, empire function (Chase-Dunn et al 2007). However, how such synchronization occurred in tandem with the harbinger value of C_{max}/T = .0014 is not immediately clear. This parenthetical note then is to make the reader aware of such a possibility only.]

With respect to a simple example of a hypercycle, consider the following theoretical model (Harper In Press):

1. Imagine two rural communities, both requiring an agrarian surplus for existence.
2. There are two strategies that these two communities can adopt.

a. The first requires competitive exclusion, or something like it, in that both communities must among other things produce enough food for at least sustainability and possibly growth, each at the expense of the other; this includes but is not limited to each community producing its own farming equipment. Duplication of equipment and services will ultimately act as a sunk cost limiting the potential for effective competition, sustainability, and growth.
b. The second requires cooperation and as a consequence of that cooperation, specialization and division of labor. Let's say that one community specializes, due to some inherent talent, in crop production, while the other community specializes in producing and maintaining farming equipment. The first community supplies food for all and the second community supplies equipment to produce that food surplus more efficiently. The system of two communities benefits from this cooperation, and it can be shown (Harper In Press), grows more rapidly and will out compete the two individual communities. It must be understood that the two communities must function as an integrated whole, i.e. in a unitary fashion, for this system to work. Any breech of that system integration will cause the system to degrade into its components. Altruism suffers when exposed to the vicissitudes of one-against-all competition, but hypercycle formation depends on it.

What pertinence does this explanation of hypercycles have with respect to the transition from stasis to punctuation? This transition cannot be considered simply as an accumulation of inventions, but rather the integration of those inventions, including not simply physical inventions but enhanced financial, commercial, social and cultural integration to establish a given hypercycle. There are many examples of emergence due to hypercycle formation in the economic world; two will be mentioned here, one historical and one that is contemporary. Both come from Padgett and Powell (2012). The historical example is the formation of merchant banks in Tuscany. This involved the cooption of a variety of already in place mechanisms, e.g. Papal financial practices, physical transfer of goods to Champaign Fairs, and English prohibition of exporting gold, and resulted in the formation of sedentary merchant-banks which were in a position then to expedite loans to states and state rulers with the understanding of responsible repayment funded by taxation. The contemporary example is that of the integration of various institutions in the commercialization of the life sciences, specifically the application of bio-molecular research. In this instance money and ideas were combined with the fusion of university research with venture capitalism along with intellectual property and patent law. The result of this combination may well be a model for much of 21st Century R & D in the life sciences. It is interesting that individual inventions per se may not, in fact will not, succeed without first adapting to and being sustained by the already in place environment out of which they emerged. Consider this quote from Padgett and Powell (2012), "To survive beyond antiquarian curiosity, any organizational innovation must first fit into, and be reproduced by, the autocatalytic flows out of which it emerged." Very clearly, hypercycle integration involves both physical and social interaction, and did (and does) not simply rely on the invention of a collection of labor saving devices.

Where does this mechanism of hypercycle formation fit into, become integrated as part of, any given K-wave? There are two possible responses here. First, as was mentioned previously, it has been pointed out by a number of scholars, initially by Kondratieff himself but also by Schumpeter and many others (Korotayev and Grinin 2015), that inventions occur on the down side or B Phase of K-waves and their applications or innovations manifest themselves in the following A Phases. If this is the case, then the integration of inventions and innovations would follow, perhaps further on in B Phase position or possibly occurring somewhere on the A Phase side of an on-going K-wave. However, this approach postulates an alteration between invention, i.e. paradigm shift, and integration, while endemic, already in place hypercycles are not considered. Since hypercycles must develop in the presence of already existing hypercycles, then it may be reasonable to also consider analysis based on the parallel processing of both changing context and emergence of new information, whether that new, paradigm changing information takes physical form or processual importance; the analogy of ecological succession to the process and events of stasis-to-punctuation transition then continues to provide a functional model, and the K-wave context of such a shift takes on a more complex meaning.

Conclusion

Collective cyclic behavior of humans has both external and internal causes. Further, that cyclic behavior occurs within the context of other collective behaviors including but not limited to our ability to live in large groups, urban areas, an ability that requires surplus material and energy, energy over most of our history that has been solar in origin but within the last approximately three centuries has been eclipsed by stored solar energy, i.e. fossil fuels, specifically, coal, oil, and gas. It has been the intent of this paper to begin investigating the relationship(s) between the macro-pattern of urbanization over time and the occurrence of economic waves characteristic of capitalist economic systems, waves first identified by Nikolai Kondratieff and now bearing his name as Kondratieff or K-waves.

The macro-pattern if urbanization has been shown to be punctuated in mode, a pattern first associated with data from the fossil record by Eldridge and Gould in 1972 but has been identified as being generally characteristic of change in complex adaptive systems by Per Bak and a number of collaborators. The general mode of change in such systems has been shown to be threshold dependent and exhibits both phases of relative stasis and phases of rapid change to a higher level of system organization (and complexity). The thresholds in question have the nature of tipping points in that this pattern is clearly evident in the macro-pattern of urbanization. However, K-wave oscillations are not immediately obvious in this pattern.

That K-wave oscillations are not immediately apparent in urbanization over time is a consequence of both the resolution of the urbanization data and the temporal depth of that data when compared to the temporal depth of the macro-pattern of urbanization. This has resulted in an analysis of K-wave and urbanization date extending only back to 1350 CE. Even so, much of this time is occupied by urbanization stasis at the top level and again exhibits no oscillation when compared to K-wave periodicity over the same period of time. However, when the ascending and descending phases of this set of K-waves is compared to the transition from stasis to punctuation, a transition that is motivated by and is the Industrial Revolution, the phase settings are appropriately placed so that a K-wave down phase precedes this tipping point and the following ascending phase straddles it. Down-phases of K-waves are times of invention, while up-phases are times of innovation.

It was further noted that the time depth of this set of processes leading to the tipping point giving rise to the Industrial Revolution is considerable and required significant integration of events and processes to create the context for its occurrence. It has been suggested that this integration be modeled along the lines of hypercycle generation, that for this to occur the elements of such a cycle had to be autocatalytic, and that that autocatalysis in some way also contributed to the autocatalysis of the next member in the cycle. A theoretical and simple two-component hypercycle was presented as a model of this process. It was also noted that an overriding necessary and sufficient condition for the existence of a hypercycle was the isolation and protection of the endemic altruism necessary to insure hypercycle existence.

Appendix

A Mathematical Appendix

$$\sum \Delta \ln C_{\max}$$

The use of the statistic, accumulated change in maximum urban area, $\sum\Delta\ln C_{max}$, was done for two reasons; the actual plot of this statistic reveals a (relatively) clear image of both periods of stasis and punctuation, slightly more pronounced than had $\ln C_{max}$ been plotted instead, and this particular metric is based on a comparison with the initial value used, that of the magnitude of the maximum urban area in 3000 BCE. Mathematically, this is shown as follows:

$$\Delta_1\ln C_{max} = \ln \Delta\ln C_{max1} - \Delta\ln C_{max0},$$

and by analogy,

$$\Delta_2\ln C_{max} = \ln \Delta\ln C_{max2} - \Delta\ln C_{max1}.$$

Consequently the expression of each of these differences takes the form,

$$\Delta\ln C_{max} = \ln[C_{max1}/ C_{max0}].$$

As a further consequence, the addition of these differences would then have the form:

$$\sum\Delta\ln C_{max} = \ln[C_{max1}/ C_{max0}] + \ln[C_{max2}/ C_{max1}],$$

which in turn can be represented as:

$$\sum\Delta\ln C_{max} = \ln[(C_{max1}/ C_{max0})(C_{max2}/ C_{max1})].$$

Simple algebraic manipulation will then yield:

$$\sum\Delta\ln C_{max} = \ln[C_{max2}/ C_{max0}].$$

By induction to any subscript greater than 2 this summation can then be represented by:

$$\sum\Delta\ln C_{max} = \ln[C_{maxn}/ C_{max0}],$$

where the subscript, *maxn*, refers to any subscript greater than 2. The data yielded by this method of transformation then represent not simply a time series but a time series in which comparison to a specific standard, the log-transformed initial value of maximum urban area, is embedded.

The Integration of

$$H = -\int f(x)\ln f(x)dx$$

with Respect to

$$dN/dt = rN(|K - N|),\ dK/dt = (T - N)/K,$$

and

$$dT/dt = T/(K - N),$$

where each differential equation is f(x).

$$H_N = (-1)(r/18)[(6N^3 - 9KN^2)\ln(r(|K - N|)N) + 3K^3\ln(|K - N|) - 4N^3 + 6KN^2 + 3K^2N], \qquad \text{Eq. 10.}$$

$$H_K = (-1)(|T - N|)\ln[(T - N)/K]^2(.5), \qquad \text{Eq. 11.}$$

$$H_T = (-1)[T^2/(2(|K - N|)][\ln T/(|K - N|) - .5], \qquad \text{Eq. 12.}$$

The intergrations were performed using software found at: https://www.integral-calculator.com/

Dark ages and urbanization

Environmental stress is a serious, if not the serious global concern of all of us, whether we realize this or not. There are far too many special cases of environmental degradation accumulating, e.g. the draining of the Aral Sea, the bleaching of coral reefs including but not limited to the Great Barrier Reef, the dead zone of the Gulf of Mexico, and the list goes on, to ignore a general trend in the increasing incidence of such eco-disasters for those who actually are aware. Historically, a number of scholars have been concerned with the relationship between environmental degradation and the cultures/societies interacting with and causing that environmental distress.

Chief among these scholars is Sing C. Chew who has produced a trilogy of books on the topic (Chew 1999, 2007, 2008). He notes three extended periods of what he describes as dark ages, the first from 2200 BCE to 1700 BCE, the second from 1200 BCE to 700 BCE, and the third from 300 CE to 900 CE. Each of these three periods of Dark Age has a geographic extent greater than the previous. Chew defines a dark age as having the following characteristics: 1. One or more economic slow-downs accompanied by trade disruptions. 2. Political unrest and breakdowns. 3. De-urbanization. 4. Increased migration. 5. Population loss. He also notes that dark ages are periods of ecological restoration. Clearly, it is the events of urbanization and de-urbanization with respect to these dark ages that are of concern here.

Figure 13 presents the temporal position of Chew's three Dark Ages and does so within the context of changes in urbanization over time as represented by the data presented in Figure 3. Each of the three horizontal bars, one from 2200 BCE to 1700 BCE, the second from 1200 BCE to 700 BCE, and the final one from 300 CE to 900 CE, represent the temporal extent of these Dark Ages. It is clear that each age

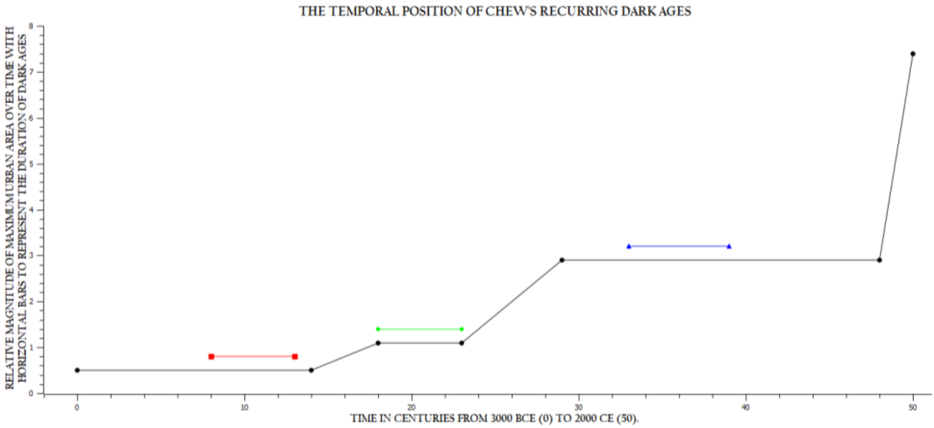

Figure 19. The temporal position of Chew's three recurring Dark Ages is set in context with respect to a skeletonized version of Figure 3. Of note is the fact that these Dark Ages occur only during periods of stasis; implying that periods of punctuation are also periods of eco-degradation.

is associated with a period of stasis. This effectively means both that eco-recovery is occurring at these times and also that the previous periods of punctuated urbanization represent times of eco-degradation.

The third Dark Age period, 300 CE to 900 CE, is worth looking at more closely, as this period of time represents not only a significant period of deurbanization, i.e. from C_{max} = 1,000,000 to C_{max} = 400,000, but also the beginning of re-urbanization from 700 CE at which C_{max} = 400,000 to 900 CE when C_{max} = 900,000. It is interesting that through 1300 CE when C_{max} = 1,500,000, urbanization effectively increases, with one short period, from 1200 CE when C_{max} = 1,000,000, having fallen from C_{max} = 1,200,000 in the previous century. It is also of note that urbanization per se began increasing prior to the end of this third Dark Age.

One further point is worth making, the overall pattern of urbanization presented in this paper suggests that the world-system may well be headed toward a fourth period of stasis during which a dark age may (or will) be embedded. One prescient historian, the late L. S. Stavrianos, has suggested in a book, The Promise of the Coming Dark Age, that in fact humanity will shortly enter into such a period.

Stavrianos suggested that decentralization and a technology designed for multiple tasks, here think about home computers that function as entertainment systems, communication systems, and are encyclopedic in access to general knowledge, will be the hall marks of such a period; he did not, however, foresee the looming robotics/AI tsunami with both predicted beneficial and pathological aspects to it. The world-system population is predicted to top off somewhere between nine and ten billion by the end of this century, and certainly with respect to urbanization levels, the world-system appears again approaching an upper limit of $C_{max} \sim$ 40,000,000; the Tokyo-Yokohama urban complex is currently approximately 35,000,000 as is Chongqing, China. [I base this on the empirical pattern of C_{max} stasis values of 40,000, 100,000, and 1,000,000. Morris (2015) favors a sequence of change by single orders of magnitude. I do not believe that this is possible if a Malthusian limit of nine to ten billion for a total global population remains a reasonable maximum estimate.]

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