Second Order Uncertainty and Prospect Theory

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Abstract

Prospect Theory has been highly influential; however its experimental paradigm lacks higher orders of uncertainty. To introduce this, participants are asked to imagine themselves facing a choice between two bags containing 100,000 blue or red balls in unknown proportions. A red ball wins £500. Participants are shown samples from each bag; e.g., 5 balls from Bag 1 (3 red) and 100 balls from Bag 2 (55 red). The bags can be represented by distributions with Bag 1 having a higher mean probability estimate (60% vs 55%), but more variance (second order uncertainty) in that estimate. By varying observed frequencies and gain vs loss formats, we seek to determine if classic findings remain when higher order uncertainties are present. Results consistent with the four-fold pattern are seen for gains (uncertainty seeking at low probability values, uncertainty aversion at high probability values) but for losses, uncertainty aversion is seen at all values.

Keywords: Prospect Theory; Second Order Uncertainty; Ambiguity; Probability; Risk

Introduction

In one of the classic gambling experiments upon which prospect theory (Kahneman & Tversky, 1979) was based, a participant might be asked to choose between the following:

A. 50% chance to win £1,000 (50% chance to win nothing)
B. £450 for sure

While the expected utility for A (0.5*£1000 = £500) is higher than B, a typical finding is that a majority of participants prefer B, and generally, tend to be risk averse in such situations (we will discuss the more nuanced results from this line of research shortly).

While the psychological dynamics revealed by prospect theory have been highly influential, the experimental paradigm it is based upon depicts the highly unusual situation where we know the first order probability (hereafter 'FOP') of each possible outcome of our decision precisely. In reality this almost never happens outside of the contrived casino-like world of coin flips and die rolls. Instead, humans inhabit an environment in which we have uncertainty about uncertainty, also known as second-order uncertainty (Dewitt et al., 2023; Hykeln, 2014; Lipshitz & Strauss, 1997; Klein, 1998; Mousavi & Gigerenzer, 2014, 2017). At the extreme 'Knightian' (Knight, 1921) level of uncertainty, we have no easily quantifiable information at all about the choices we face, such as when choosing a life partner.

We can take a step closer to approximating a more realistic version of the problem by including second order uncertainty (e.g., Kleiter, 2018). If we introduce this into the prospect theory gambles, we would not know for certain the probability of the outcomes for either A or B. For example, imagine you are in the final round of a gameshow, having so far won £1000. In this final round, the host shows you two bags. Each bag contains 100,000 small balls, each of which are blue or red. You are told that if you pick a red ball, you will take home a further £500. If you pick a blue ball, you just take home your original £1000. You only get to pick once, from one bag.

Figure 1. The image of two bags with accompanying figures shown to participants

You do not know the proportion of balls in either bag; however, the host has shown you a number of 'picks' from each bag (Figure 1), replacing the balls and shaking the bag after each. The host has shown you five balls from Bag 1, three of which were red. They have shown you 100 balls from Bag 2, 55 of which were red. Which bag do you choose to pick from?

These numbers provide a classical frequentist FOP estimate of picking a red ball of 0.6 for Bag 1 and 0.55 for Bag 2. However, this estimate is less firm, or has more variance, for Bag 1 (standard deviation = 0.2) than Bag 2 (0.05). Our estimates for the two bags therefore are better represented as distributions than point estimates (unlike in the classic gamble), as can be seen in Figure 2.

In L. K. Samuelson, S. L. Frank, M. Toneva, A. Mackey, & E. Hazeltine (Eds.), *Proceedings of the 46th Annual Conference of the Cognitive Science Society.*

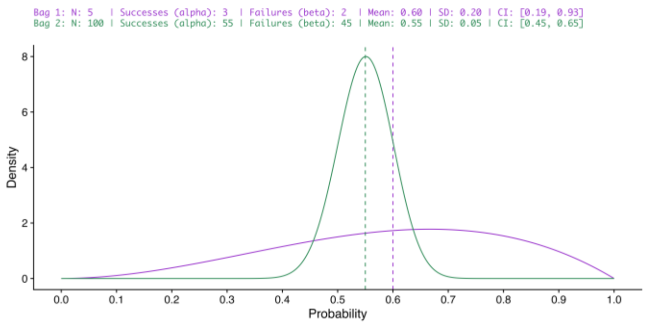

Figure 2. Density distributions of the probability of drawing a red ball for Bag 1 (purple) and Bag 2 (green) as beta distributions. Dotted vertical lines indicate the mean estimate for each bag.

Bag 1 therefore has similarities with choice A as the more uncertain option. However, this comes from greater second order uncertainty rather than the use of a non-100% FOP. Where choice A offered a higher possible pay off, but with lower FOP (0.5 rather than 1) than choice B, Bag 1 offers higher FOP than Bag 2 (0.6 vs 0.55) but also greater second order uncertainty. Unlike the classic gamble, the payoff remains constant. When considering Bag 1, participants may be drawn by the possibility that the true proportion of red balls is actually very high (e.g. 0.8) or may be put off by the possibility that it is actually very low (e.g. 0.4). Participants may prefer to take that gamble or may prefer to stick with a relatively well-known proportion (Bag 2).

Table 1. A representation of the 'fourfold' pattern of Prospect Theory.

	Gains	**Losses**
High probability e.g., 95% vs 100% (Certainty effect)	Risk Averse (Seek the certain gain)	Risk Seeking (Avoid the certain loss)
Low probability e.g., 0% vs 5% (Possibility effect)	Risk Seeking (Seek the possible gain)	Risk Averse (Avoid the possible loss)

We are interested in whether classic prospect theory findings translate to this second order realm. In Table 1 the 'fourfold' pattern of Prospect Theory can be seen. Our aim is to present participants with pairs of bags which vary along a number of dimensions. First, in terms of the FOP distance (FOP-d) between the two bags. For example, Bag 1 = 0.6 vs. Bag 2 = 0.55 (FOP-d = 0.05) compared to Bag 1 = 0.6 vs. Bag 2 = 0.5 (FOP-d = 0.1). This will answer the most basic question of how much worse an FOP participants will be willing to accept for the greater second order uncertainty afforded by Bag 2. Second, in the value of both bags across the probability spectrum. For example, Bag 1 = 0.6 vs Bag 2 = 0.55 compared with Bag 1 = 0.15 vs. Bag 2 = 0.1 (FOP-d still = 0.05, but both bags are at low probabilities in the latter). Finally, in terms of loss vs. gain (a red either gains you £500 or loses you £500).

Regarding manipulations two and three, if we see similar results to the fourfold pattern in the second order realm, for gains we will expect more Bag 2 choice (uncertainty aversion) at the higher end of the FOP spectrum and more Bag 1 choice (uncertainty seeking) at the lower end of the FOP spectrum. Conversely, for losses, we will expect more Bag 1 choice (uncertainty seeking) at the higher end of the FOP spectrum and more Bag 2 choice (uncertainty aversion) at the lower end of the FOP spectrum.

Beyond prospect theory, the large literature related to ambiguity and the Ellsberg paradox (1961) are highly relevant to this study. In Ellsberg's 'Two Urn' problem, participants were asked to choose from two urns, one (A) with a known 50:50 ratio of red to black balls, but with no information on urn B at all (it has some unknown mix of 100 red and/or black balls). Participants tend to choose urn A (Curley and Yates, 1989; Einhorn & Hogarth, 1986; Fellner, 1961; Ho et al., 2002; Hogarth & Kunreuther, 1989; MacCrimmon, 1968; Maffioletti & Santori, 2005; Slovic & Tversky, 1974; Viscusi & Chesson, 1999), and this is typically taken as evidence for ambiguity aversion. Ellsberg defined 'ambiguity' as:

"…the nature of one's information concerning the relative likelihood of events... a quality depending on the amount, type, reliability and 'unanimity' of information, and giving rise to one's degree of 'confidence' in an estimation of relative likelihoods." (Ellsbury, 1961, p.657")

While Ellsberg's ambiguity is therefore a broader concept than second order uncertainty, our manipulation varies the 'amount' of information (for Bag 2 we have more information), and due to this, the 'confidence' in an estimation of relative likelihoods (we have more confidence in the estimate for Bag 2).

The term 'ambiguity' has been redefined in a number of ways since Ellsberg's broad definition. For Savage (1954), situations where the distribution was known would not count as ambiguous, and so would not include our paradigm. However, Becker and Brownson (1964) conversely stated that "…for us, ambiguity is defined by any distribution of probabilities other than a point estimate". (Becker & Brownson, 1964, pp. 64) which would include our paradigm. Kahn and Sarin (1988) also equated ambiguity with second-order uncertainty:

"We define ambiguity operationally by second-order uncertainty or, in other words, by a probability distribution for the perceived frequencies" (Kahn & Sarin, 1988, pp. 265).

In the large literatures on both prospect theory and ambiguity, we are not aware of an experiment which has used differences in the sample sizes of observations to investigate how participants respond to differences in first order vs second order uncertainty. However, several studies have examined second order uncertainty by providing participants with ranges. Becker & Brownson (1964) did this, presenting participants descriptively with the below range of chances of getting a (desirable) red ball:

	Red Balls	
	Minimum No.	Maximum No.
Urn I......	0	100
Urn II.....	50	50
Urn III....	15	85
Urn IV....	25	75
Urn V.....	40	60

Figure 3. Figure taken from Becker and Brownson (1964) depicting the choices provided to participants

The authors generally found that their participants preferred the urn (II) with only first order uncertainty (guaranteed 0.5 chance) over those with second order uncertainty, preferring smaller ranges within these three (i.e., V > IV > III). They also preferred those over Ellsberg's 'ambiguous' urn (I), suggesting a general preference for less second order uncertainty.

Budescu et al. (2002) also presented participants with ranges for probabilities, and separately, ranges for outcomes, and examined both gains and losses, and across the probability spectrum. They gave participants a range of choices between gambles including, for example, a probability range of 0.1-0.9 of getting $2, a fixed 0.1 probability of getting between $2-18 dollars or a certain return of $1. For gain framing, they reported a preference for what they called 'vagueness', especially at low probability ranges, but a preference for precision for loss framing.

Moving from second order uncertainty up to ambiguity, Abdellaoui et al. (2016) adapted Ellsberg's two urns problem to study whether prospect theory applied for gain and loss framing and found support for the classic concave value function for gains and convex value function for losses but did not test for the presence of the fourfold pattern. Kocher et al. (2017) varied the number of colors of balls in Ellsberg's ambiguous urn (from 10 [0.1 chance] to 2 [0.5 chance]) to study responses to ambiguity across the probability spectrum and found ambiguity aversion for moderate likelihood gains but ambiguity neutrality or seeking behaviour for low likelihood gains and for losses.

Each of these suggests that the classic four-fold pattern of prospect theory may well translate to levels of uncertainty further up than first order. However, a large amount of research translating prospect theory to higher levels of uncertainty has leapt straight from first order only, all the way to full Ellsbergian ambiguity. There is limited research studying how participants respond to degrees of second order uncertainty, and those which do exist (e.g., Becker & Brownson, 1964, Budescu et al., 2002, and Johnson, 2002) have used described ranges, and we can't be sure that participant response to ranges apply to distributions (Dieckmann et al., 2015). Both the latter also used very small (~30) sample sizes of university students. Our paradigm provides a simple and neat way of varying the degree of first and second order uncertainty in a more natural way: rather than simply providing participants with a range with no explanation as to the mechanism behind it, our mechanism is transparent and similar to what could be encountered in any real-life situation where one observes a number of trials to estimate the efficacy of something (e.g., a drug or vaccine). It therefore sits in a grey area between descriptive and experiential approaches (Hertwig et al. 2018), and could be adapted into an experiential paradigm.

Method

Participants

Nine hundred and forty-six participants were recruited from Prolific Academic. Ages ranged from 18 to 74 (*M*=30.7, *SD*=10.2), with 51.4% reporting their gender as female, 48.0% as male, and 6 individuals as 'other'. Generally, the Prolific Academic population has good representation across a range of European countries (39% UK, ~20% other European countries) and the US (31%), but has little representation from Asia, Africa, or South America.

Design

The total data set of 946 participants comprises three parallel studies with the same research question conducted as student projects by authors two, three and four and supervised by the first author at the same time. Participants were assigned to a loss or gain condition, and within these were assigned to a range of conditions which varied the total picks for Bag 1, and the number of reds shown for both bags. Total picks for Bag 2 was always 100. Bag 1 totals studied included three, five, seven and nine. Number of reds in each bag was varied within these totals to study a large range of probability values, producing a large number of different trials. More details about all the exact values studied, as well as materials and data can be found at the online repository *(https://osf.io/qfdzu/?view_only=d2a4ca4f0e714f308e2c4714523ad0ae)*

Materials & Procedure

Participants first consented to take part in the study and provided demographic details. They were then presented with an image like Figure 1, along with the same accompanying explanation that they are in the final round of a gameshow, and have so far won £1000, the number of and results of the picks for each bag, and that they must choose to pick from one of the two bags. Participants were either assigned to a gain condition (Red wins them an extra £500, blue gains nothing) or loss (Red loses them an extra £500, blue loses nothing) and showed the number of picks and number of reds for each bag. They were then asked to provide, across four sliders with values from 0% to 100% their (1) estimate of the proportion of reds and (2) confidence in that estimate, for each bag. On the next page the key information was repeated, including the image of the bags and participants were asked to choose which bag they wished to pick from and then to explain their reasoning for that choice in an open text box. Following this they were debriefed and redirected to the survey site for payment.

Results

Preliminary results

Participants' estimates of first order probability (FOP) were highly related to their correct classical frequentist value for Bag 1 estimates (*r* = .728, *p*<.001) and for Bag 2 estimates (*r* = .803, *p*<.001).

Participants' confidence that their FOP estimate was equal to the true value of the bag was lower for Bag 1 (*M*=51.0, *SE*=1.0) than Bag 2 (*M*=63.0, *SE*=2.1), confirmed by a paired samples t-test: *t*(1, 945) = -13.5, *p*<.001. There was also a correlation between participant's confidence in Bag 1 and Bag 2 (*r* = .55, *p*<.001) suggesting an individual component.

For the following analyses, we computed a variable called FOP-d (first order probability difference) which is calculated by subtracting Bag 1 FOP from Bag 2 FOP i.e., if Bag 2 FOP is 0.6 (e.g. 60/100), and Bag 1 FOP is 0.5 (e.g. 3/6), FOP-d would be +0.1.

Quantitative Results

Gain vs loss framing. We see a clear distinction in bag choice between gain and loss framing. Using binary logistic regression, we find participants were more likely to choose Bag 2 in the loss condition (*n*=499, 66.5%, *SE*=2.1%), than in the gain condition (*n*=447, 38.7%, *SE*=2.3%) (*OR* =.32, Wald = 71.4, p < .001). To control for differences in the values used for loss vs gain, we examined instances where 'FOP-d' = 0 (i.e., where Bag 1 and Bag 2 have the same FOP): the same pattern remained with more Bag 2 choice for loss framing (*n*=81, 75.3%, *SE*=4.8%) than gain framing (*n*=78, 56.4%, *SE*=5.7%) (*OR*=.42, Wald = 6.2, *p*=.013).

Gain framing. In the following analysis we wished to check whether both the first and second order uncertainty differences between the two bags were affecting participant choices. As can be seen in Figure 4, as FOP-d becomes more negative (i.e., where Bag 2's FOP becomes increasingly less than Bag 1, making it a poorer choice) we see greater Bag 1 choice. At FOP-d = 0.0 we see roughly equal choice of Bag 1 vs Bag 2 (56.4% choosing Bag 2, *SE*=5.7%) while at FOP-d = -0.15, we see only 25.4% (*SE*=5.4%) choosing Bag 2. A binary logistic regression was used to test the effect of FOP-d on Bag 2 choice (*OR*=1.06, Wald = 8.3, *p*=.004), showing fewer individuals choosing Bag 2 as its FOP of getting a red (i.e., winning £500) becomes lower compared to Bag 1.

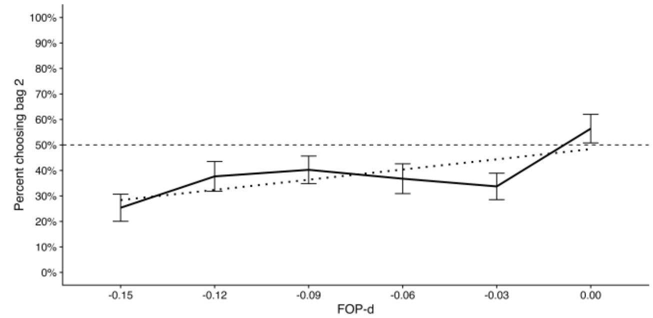

Figure 4. The percentage of participants in the gain framing choosing Bag 2 for a range of FOP-d values (i.e. the difference in FOP between Bag 2 and Bag 1). Error bars indicate one standard error.

In the next analysis, we wished to check whether bag choice varied as the value of the two bags moved along the probability spectrum. Bag 1 FOP can be used as a proxy for this as Bag 2 FOP always 'trailed' Bag 1 FOP. Each 'Bag 1 FOP' point shown in Figure 5 combines trials where FOP-d varied from 0.0 to -0.15. Therefore, this analysis ignores FOP-d and focuses on the effect of the position of both bags along the FOP spectrum on bag choice. A binary logistic regression predicting bag choice from Bag 1 FOP was run (*OR*=1.02, Wald = 17.4, *p*<.001), showing more individuals choosing Bag 2 as Bag 1 FOP increases. This analysis suggests more uncertainty seeking behaviour (Bag 1 choice) at the lower end of the FOP spectrum (<20% of individuals choosing Bag 2 when Bag 1 FOP = 0.14), and more uncertainty averse behaviour (Bag 2 choice) at the higher end of the FOP spectrum (>50% of individuals choosing Bag 2 when Bag 1 FOP = 0.86).

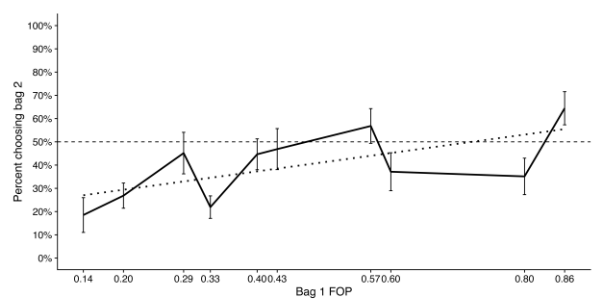

Figure 5. The percentage of participants in the gain framing choosing Bag 2 for a range of Bag 1 FOP values. Error bars indicate one standard error.

To test for both these effects on bag choice simultaneously, a binary logistic regression was run predicting bag choice from both FOP-d (*OR*=1.07, Wald=10.0, *p*=.002) and Bag 1 FOP (*OR*=1.02, Wald=19.0, *p*<.001). In a model additionally adding an interaction term between the two, no interaction was seen (*OR*=1.0, Wald = .21, *p*=.650).

Loss framing. In the following we run the same analyses as the gain condition, in the same order. Within the loss framing conditions, we ran a binary logistic regression to determine the effect of FOP-d on bag choice (*OR*=.98, Wald = 5.0, *p*=.028), with more individuals choosing Bag 2 as its FOP-d becomes more negative compared to Bag 1 (desirable in the loss condition). However, as can also be seen compared to gains, mean bag choice values for the loss condition do not get below 50%: over 50% of participants still choose Bag 2 even when the FOP estimate of getting a red ball (and therefore losing £500) for Bag 2 is 0.15 higher than Bag 1.

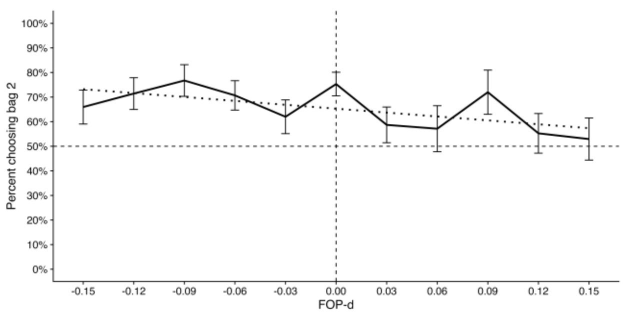

Figure 6. Percentage of participants in the loss framing choosing Bag 2 for a range of FOP-d values (i.e. the difference in FOP between Bag 2 and Bag 1). Error bars indicate one standard error.

Furthermore, we ran a binary logistic regression predicting bag choice from Bag 1 FOP (*OR*=1.0, Wald = 1.4, *p*=.248). We then ran a multiple binary logistic regression model predicting bag choice from both FOP-d (*OR*=.98, Wald = 3.6, *p*= .059) and Bag 1 FOP (*OR*=1.0, Wald = .07, *p*=.798). In a model adding an interaction between the two, the interaction term showed no effect (*OR*=1.0, Wald = 2.5, *p*=.111). Again, as can be seen in Figure 7 there is a majority of individuals choosing Bag 2 at all FOP values.

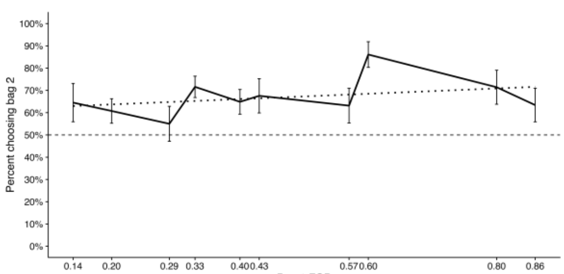

Figure 7. Percentage of participants in the loss framing choosing Bag 2 for a range of Bag 1 FOP values. Error bars indicate one standard error.

Qualitative data. Following their bag choice, participants were asked to explain their choice in an open text box. While there is not enough space in this paper to present a full qualitative analysis, we think it may be illustrative here to at least see some participants demonstrating the classic thinking of the ‘fourfold’ pattern with second order uncertainty, all in the gain condition. First here are P113 and P568 showing second order uncertainty seeking behaviour at low probabilities:

P113 “I choose Bag 1 because I am quite certain that in Bag 2 there is approximately 31% of taking out the red ball. However, in Bag 1, since there is such a small sample (7 of 100,000 balls), I don't know if the percentage of red balls is higher or lower than in Bag 2, so I take a chance with this bag waiting for it to have a higher percentage”

P568 “I am pretty confident that the proportion of red balls in Bag 2 is close to 20% while I have no certainty regarding Bag 1. Given that 20% is a low chance to win, so versus this certainty to lose I prefer to try my luck with Bag 1”

We can see here that both participants reference their greater confidence in the (low) FOP of Bag 2, and explicitly say they therefore want to gamble on Bag 1 as it may have a much higher true FOP. Next, here are P64 and P763 showing second order uncertainty averse behaviour at higher probabilities:

P64: “Seems to be roughly 50 50 in each but there’s a bigger sample from bag two which I think means it’s more likely to be 50 50, bag one might have worse odds.”

P763 “For Bag 1, although there are only 3 balls and 2 have been blue, I prefer to choose the bag number 2 because it is where the matched amount of blue and red balls has been demonstrated, in which it could have about 50% of choosing blue, instead in Bag 1, nobody knows if only 10% contain blue balls or even less and that 2 blue balls have been taken out luckily.”

Here we see the participants again reference their greater confidence in the bag two value, but see this as a positive out of concern that Bag 1’s true FOP may be much lower.

Discussion

In the present study we have adapted the classic prospect theory style gambles to the second order uncertainty realm and have investigated whether key principles such as differing choice behaviour between gain and loss framing, and across the probability spectrum, transfer to this realm. Generally, we have found that they do, but with some key differences. Most notably, gain framing seems to mirror classic prospect theory results, with more (second order) uncertainty seeking behaviour at lower probabilities, and (second order) uncertainty aversion at higher probabilities. However, within loss framing we see a more blanket (second order) uncertainty aversion akin to that seen in the ambiguity aversion literature. This ‘mixed’ result is interesting given that second order uncertainty sits somewhere ‘between’ the FOP only paradigm of Prospect Theory and the full ambiguity paradigm of Ellsberg.

We have also introduced a new paradigm to the literature and have only scratched the surface of the space that could be mapped, but participants seem to show good understanding. For both losses and gains, participant estimates of FOP for both bags were fairly accurate. Participants also recognized that the estimate for Bag 2 was more reliable than for Bag 1, as indicated by their higher confidence ratings in their estimates of the Bag 2 FOP. Participants also responded in a sensible manner to differences in the first order probability between the two bags (FOP-d). For gains, where a higher FOP is desirable, participants chose Bag 1 and Bag 2 in roughly equal proportions at FOP-d = 0 but increasingly chose Bag 1 as the FOP-d became more negative (Bag 2's FOP became less desirable compared to Bag 1's FOP). For losses, where a lower FOP is desirable, participants were generally more inclined to choose Bag 2 at all values, including when FOP-d = 0, but chose Bag 2 in higher numbers as FOP for Bag 2 became more negative (more desirable). Generally, these results suggest that many of our participants were sensitive to both first and second order uncertainty, making their decision based on a balancing of these two factors as well as whether they were in a gain or loss situation.

Participant choices also varied across the probability spectrum for gain framing, with more participants choosing Bag 1 at the lower end of the probability spectrum. This result echoes the 'possibility' and 'certainty' effects of prospect theory, at least for gains (Kahneman & Tversky, 1979). At the lower end of the FOP spectrum, participants may see Bag 1 as offering the possibility of a higher true FOP. However, at the higher end of the FOP spectrum, more participants seem to want the higher certainty provided by Bag 2 (i.e., may become averse to the possibility of a lower true FOP that Bag 1 presents). We tentatively demonstrate this in the qualitative data also, illustrating at least that some participants did report thinking in this way.

Even at the highest end of our FOP spectrum for gains, we still only see roughly equal choice between Bag 1 and Bag 2, which is not entirely consistent with the 'certainty' effect, and we see considerable individual differences in preferences in our sample. However, it is important to note that due to having to use small amounts of 'picks' for Bag 1 we were not able to study values very close to either 0% or 100% (e.g., even for 6/7, our most extreme Bag 1 value, FOP = 85.7%). It seems likely, given the trend, that if we were able to get closer to 100%, we would observe even more choice for Bag 2 at higher FOP values. However, this would require using a larger amount of 'picks' for Bag 1, reducing the differences in second order uncertainty between bags and undermining the very thing we want to study, and so it is difficult to see how this could be studied with the current paradigm. While mathematically larger sample sizes for Bag 1 e.g., 20 still produce considerably different second order uncertainty to Bag 2 (100), participants may not be sensitive enough to these differences in sample size (e.g. Tversky & Kahneman, 1971) to observe an effect (they may be confident enough in the Bag 1 value with a sample size of 20 to offset any perceived value of Bag 2),

In contrast to our results for gains, we observe a more blanket 'uncertainty aversion' effect for losses, which was consistent across the FOP spectrum (see Figure 7 where at each point, the percentage choosing Bag 2 is greater than 50%). This is not obviously consistent with the 'four-fold pattern', which observes risk aversion for low FOP for losses, but risk seeking for high FOP. As well as the same caveat that we were not able to study extreme values with this paradigm, this may reflect an interesting interaction between the FOP only realm of the paradigm used to reveal the four-fold pattern and uncertainty / ambiguity aversion in general. As we have mentioned, our study sits somewhere between the pure first order probability realm of Prospect Theory, and the ambiguity realm of Ellsberg. While our results for gains seem fairly consistent with the four-fold pattern, it may be that for losses, general uncertainty / ambiguity aversion (Ellsberg, 1961) is the more powerful effect. This is also actually consistent with other literature, even on risk, which while consistently finding the observed pattern for gains, has been much more mixed for losses (e.g., Bruhin et al., 2010). It does however contrast with the findings of Kocher et al. (2018) who found ambiguity seeking behaviour for losses at low probabilities. Their study used a modified version of Ellsberg's urns. For low probabilities, participants were choosing between an urn with 10 different colors, and no information vs an urn with known proportions (10% desired color). In combination with our study, this continues the trend of more mixed / inconsistent findings for loss framing, compared to the relatively consistent findings for gain framing, and so more research is recommended here, perhaps providing participants with both our bags and various types of Ellsberg urns.

In future research, other Bag 1 and 2 sample sizes should be studied to determine if the current findings translate as well as modifying the reward value to test the convex / concave function findings. Follow-up questions could also be included to determine if participants' saw themselves as seeking or avoiding uncertainty rather than relying on them volunteering that information in an open text box, and the relative value they placed on the ratio of red:blue balls (FOP) as well as the sample size (second order uncertainty) in making their decision. On the whole, participants appear to understand this new paradigm and generally respond to both aspects of the manipulation. It may therefore serve as a useful platform to explore other questions related to first and second order uncertainty. We would be interested to see how participants would behave when faced with a choice between our Bag 2 (with a large number of picks) and option A from prospect theory-type choices (i.e., an FOP-only choice). Would they see Bag 2 as the more uncertain option and behave accordingly? Similarly, how would participants behave when faced with a choice between Bag 1 (with a small number of picks) and Ellsberg's Urn B (no information). Would they see Bag 1 as the less uncertain option here? Generally, it would be interesting to have more studies comparing participant responses to choices at different levels all the way up the risk/uncertainty/ambiguity spectrum.

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