Connecting learning, memory, and representation in math education

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Keywords: numeracy; math; perception; learning; memory; cognition; models; education

Introduction

In math education the goal is for children not only to master the materials and problems presented, but to understand underlying principles and properties that can be applied broadly to new problems and situations. Teachers in the classroom and policy-makers in Washington thus are both faced with what is essentially a cognitive question: What instructional regimes and practices will produce rapid learning, deep understanding, and broad transfer?

This question has often been approached without connection to cognitive theories of learning, memory, and representation, but the gap has begun to narrow. On one hand, it is now known that domain-general learning mechanisms can acquire quite abstract and structured representations that go beyond the perceptual structure of the environment—a critical requirement for any theory of mathematical knowledge. Conversely studies in math cognition have revealed counter-intuitive behaviors that find ready explanations in cognitive models of learning in other domains. For instance, children, adults, and even math teachers reliably judge some three-sided figures to be better triangles than others, sometimes denying that irregular three-sided figures are in fact triangles.[1] Children transitioning from arithmetic to algebra often generate incorrect solutions to equations because they have learned to ignore the equal sign[2]. Such examples suggest that math learning can be subject to the same factors that govern learning other domains. Yet it remains unclear whether such effects are epiphenomenal, or whether they hint at important common principles underlying concept acquisition across multiple domains.

Our symposium investigates this question by bringing together scientists whose research spans the gap between cognitive and educational science in the domain of mathematical knowledge. **Martha Alibal**i, **Chuck Kalish** and **Tim Rogers** consider how cognitive memory models from non-mathematical domains can shed light on the patterns of transfer shown by children and adults in arithmetic. **Phil Kellman** and **Christine Massey** will show that mathematical competency can improve when children learn to efficiently encode the perceptual structure of equations. **Vladimir Sloutsky** will consider interrelationships between learning of mathematical and object concepts in development. **Jay McClelland** and **Kevin Mickey** will discuss new research investigating the representational prerequisites that might underlie conceptual understanding of trigonometric functions. A short group question period will follow the four talks.

Alibali, Kalish & Rogers: Connecting learning in mathematical and non-mathematical domains.

Different learning tasks can elicit qualitatively different patterns of memory and generalization. In paired-associates, participants who learn to produce "dishtowel" to the probe "locomotive" can correctly generate the reverse pairing (producing "locomotive" given "dishtowel"), but, because pairs are arbitrary, cannot generalize to new probes (e.g. "caboose"). In categorization, participants remember features that aid in predicting the category label and use these to generalize, but fail to learn or exploit other item properties. In property induction, participants learn slowly but remember and generalize all manner of properties.[3] In experiments with adults and children we show analogous phenomena in arithmetic learning. When the graphical elements of an equation are viewed as arbitrary symbols, participants learn individual problems without transfer, as in paired associates. When the quantitative "meaning" of the problem symbols is highlighted, participants acquire a transferable mapping from problem quantities to response quantities, similar to categorization. The extent of transfer in this setting depends, however, on the task: practice retrieving a missing sum transfers to new missing-sum problems, but not to related missing-addend problems. The broadest transfer occurs when participants practice with a mix of problem types, in a setting that emphasizes quantitative relationships among elements—the same properties that produce broad transfer in object concepts. These results suggest a tighter coupling between learning in mathematical and non-mathematical domains than has previously been appreciated.

Kellman & Massey: Perceptual structure and adaptive learning in math education.

While learning of complex structure is often attributed to higher-order processes, we argue that perceptual learning (PL)—experience-driven changes in the process and content of information extraction—plays a much greater role than has previously been appreciated. We consider PL as a crucial component of learning and expertise in mathematics and other complex cognitive domains. Whereas most formal instruction emphasizes declarative and procedural components of learning, learning to extract relevant structure in mathematical problems and representations provides the pattern recognition required for effective use of facts and procedures. We will briefly review research on PL interventions in the form of perceptual/adaptive learning modules (PALMs) that facilitate discovery of structure and recognition of patterns in mathematical domains, including preliminary results from a large efficacy study currently in progress. These efforts illustrate the promise of PL interventions, as shown on tests of mathematical competence. We also examine direct effects of PL interventions on psychophysical endpoints, such as efficient encoding of equations. Results indicate that even relatively brief PALM interventions aimed at improving students' seeing of structure and transformation in algebraic equations leads to reliable changes in basic information extraction. Encoding improvements were shown most strongly by participants who were initially less proficient at algebra. These changes, which were detectable 24 hours after training, provide direct evidence for durable changes in information encoding produced by a PALM targeting a complex mathematical skill.

Sloutsky: What can we learn about mathematical cognition from object category learning?

The primary difference between mathematical and object concepts lies in category structure: the former are rule-based and statistically sparse (i.e., few category-relevant and many irrelevant features) while the latter are statistically dense (i.e., many category-relevant features). Research in object category learning may then elucidate acquisition of math concepts. We review evidence that children distribute attention among multiple stimulus dimensions, making it difficult to learn statistically sparse concepts like those central to mathematics. Consequently children and adults may extract different structures from the same learning experiences. Participants learned a category possessing both (a) a single deterministic rule-like feature and (b) multiple inter-correlated probabilistic features. Whereas 4-5-year-olds used multiple probabilistic features to generalize and were more likely to remember these, adults used the deterministic feature to generalize and were less likely to remember other features. When the deterministic feature was made salient, children were more likely to use it in generalization, but they continued to use and remember all features. Thus, though their response strategy changed, their representation did not. From these findings we argue that perceptually-rich problem instantiations may hinder generalization in math because, like stimuli in our research, they possess one relevant deterministic feature among many irrelevant features. If children naturally acquire dense probabilistic category structures, they may fail to generalize practice problems with sparse structure. We then demonstrate such an impaired transfer in learning of mathematical concepts in young children.

McClelland & Mickey: Building a core conceptual structure for trigonometry.

How can we help students gain a grasp of the basic ideas underlying trigonometric functions? Our approach links to the ideas of Robbie Case, who understood the mental number line as a core conceptual structure for two-digit addition and subtraction upon which one could build an understanding of decimal numbers and fractions.[4] We extend this approach to the 2D coordinate plane, taking the 'unit circle' as a core structure for grounding the extended definitions of the trigonometric functions outside the range of right triangles. In empirical studies with Stanford undergraduates, we have found that (a) students who report using the unit circle do better on an assessment of their understanding of trigonometric identities than those who report using rules or other visualizations; (b) a brief presentation of the core unit circle ideas produces better generalization to identities not explicitly covered in the presentation, relative to a rule-based presentation; but (c) only those performing above chance on a pre-test showed the benefit from the presentation. A second study assessed the unit-circle intervention on a group of high-school seniors, none of whom benefitted. This has led us to construct a structured series of didactic presentations and interleaved activities designed to ensure students have a well-grounded understanding of all of the elements on which the unit circle definitions build. We will report the results of new studies with this interleaved intervention, and will consider the implications of our studies for both education and for understanding the cognitive underpinnings of math knowledge.

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