

# Minimal Requirements for Productive Compositional Signaling

Thomas Brochhagen (t.s.brochhagen@uva.nl)

Institute for Logic, Language and Computation, Universiteit van Amsterdam  
P.O. Box 94242, 1090 GE Amsterdam, The Netherlands

## Abstract

The ability to form complex linguistic units from simpler ones lies at the center of many explanations of the communicative success and robustness of natural language. A closely related ability is that to generalize knowledge about such constructions to novel ones. The present investigation addresses the question what the minimal conditions for the emergence of such productive compositional communication are. Two features are argued to be required for this: relations between elements and classes over their relations. Using signaling games with reinforcement learning we show that a learning bias involving both aspects can lead to the emergence of such generalizable structure.

**Keywords:** signaling games; generalization; compositionality; reinforcement learning

## Introduction

Compositionality is arguably one of the cornerstones of linguistic productivity, as it allows for the systematic formation of novel complex expressions based on conventionalized, simpler, ones. Thereby, it chiefly contributes to the open-endedness, flexibility and learnability of natural language.

Past investigations have shown that artificial agents equipped with learning mechanisms can establish and effectively employ such structured communicative systems [Brighton, 2002, Smith et al., 2013]. The assumed biases furthermore match the linguistic learning behavior of human subjects in comparable tasks [Kirby et al., to appear].

However, the question what the *minimal* conditions for the emergence of productive compositionality are is still unresolved. The present investigation addresses this question in the context of iterated signaling games with reinforcement learning. Building on Franke [2014], we show that the minimal architecture necessary for productive composition can lead to its emergence if players have a preference for actions that fit the linguistic structure they have previously established.

### Signaling games, learning, and structure.

Lewisian signaling games coupled with adaptive dynamics provide a rich yet simple framework to investigate the emergence of linguistic systems and their features. Following Lewis [1969], a minimal setup of such a game involves two players; a sender and a receiver, each with two signals or acts to choose from, respectively. A game iteration begins with the selection of one of two equiprobable states. The sender observes this state and selects a signal. The receiver, unknowing of the state, receives the signal and selects an act. The outcome of the game is determined by whether or not the act chosen by the receiver is the “correct” one for the state. For example, in a 2-states/signals/acts game, a positive outcome



Figure 1: Perfect signaling conventions in a 2-state/signal/act signaling game.

may involve selecting act  $a_1$  for state  $t_1$  and  $a_2$  for  $t_2$ . Since the receiver has no access to the state and the players’ payoff is determined jointly, players have an incentive to coordinate.

As illustrated in Figure 1, there are two strategies that ensure perfect signaling, i.e. flawless coordination, in such a game. Crucially, a priori, signals are not associated with any meaning; whether the sender picks  $m_1$  to signal one state or the other does not matter as long as the receiver’s act matches the state. Coordination establishes a conventional yet arbitrary mapping from states to acts over signals. In this way, Lewis shows how signals can come to be endowed with conventional meaning.

This also highlights a crucial advantage of signaling games: There is no need to stipulate a relation between states, signals and acts (e.g. in terms of natural salience) nor are any assumptions made about how sophisticated players are. In other words, we can get away with the assumption of naïve players and a “worst-case scenario” where meaning emerges from an arbitrary outset.

More formally, a signaling game consists of a set of states  $T$ , signals  $M$ , and acts  $A$ . Assuming cost-free signaling, the players’ utilities are given by  $U: T \times M \times A \rightarrow \mathbb{R}$

$$U(t_i, m_j, a_k) = \begin{cases} 1 & \text{if } i = k \\ 0 & \text{otherwise} \end{cases}$$

### Reinforcement learning

If change is to be induced over repeated interactions, players need to keep track of their choices and whether these led to communicative success. Here, we adopt Roth-Erev reinforcement learning (RL), which models Herrnstein’s (1970) *matching law*: if a player’s selection yields a positive payoff given a state/act (for sender and receiver, respectively), the player’s predisposition to select it in the future given the same state/signal increases [Roth and Erev, 1995].

To illustrate this for the sender, imagine that for each state there is an urn containing a number of differently colored balls. Each color represents a different signal and the proportion of colors in a given urn represents the signals’ distribution for this state. Each round the sender draws a ball from the urn of the state she is in, and selects a signal according

to the ball’s color. If the outcome of the round is positive, a new ball of the same color is added to the urn together with the one drawn. Otherwise only the ball that was drawn is returned. Analogously, for the receiver signals are urns and each act is associated with a different ball color.

Thereby a predisposition for particular combinations emerges from repeated interaction, leading to a strong association between state-signal and signal-act pairs that were successful in the past. This increases a player’s propensity to select the same signal/act in subsequent rounds given the same state/signal. To this end, RL is incorporated in the game by associating each possible state-signal and signal-act pair,  $\langle t, m \rangle \in T \times M$  and  $\langle m, a \rangle \in M \times A$ , with its *accumulated rewards*,  $ar(x, y)$ . After a round,  $U(t_i, m_j, a_k)$  is added to  $ar(t_i, m_j)$  and  $ar(m_j, a_k)$ , the accumulated rewards of sender and receiver for their respective state-signal and signal-act pair. A player’s subsequent choices are thusly informed by

$$P(m_j|t_i) = \frac{ar(t_i, m_j)}{\sum_{m \in M} ar(t_i, m)} \quad P(a_k|m_j) = \frac{ar(m_j, a_k)}{\sum_{a \in A} ar(m_j, a)}$$

The widespread adoption of RL for adaptive signaling games is due to its good fit to the behavior of human subjects in games [Bruner et al., 2014, Erev and Roth, 1998, Roth and Erev, 1995], its simple and well-understood learning mechanism, as well as its convergence properties [Beggs, 2005, Catteeuw and Manderick, 2014].

### Compositional signaling: conditions to be met

In a compositional communicative system semantic co-occurrence coincides with form co-occurrence; if two expressions have something in common, e.g. shared constituents, then the meanings associated with them should also have something in common. As put by the principle of compositionality; the meaning of a complex expression is a function of the meaning of its parts and structure (see e.g. Kamp and Partee 1995). In classic signaling games establishing such a regular form-meaning association is purely a matter of chance.

However, this condition is not sufficient for productivity, as regular associations alone do not enable generalization. This is unsatisfactory insofar as natural language composition rules are acquired and applied not only to individual expressions, but to (syntactic and semantic) classes thereof [Taylor, 1974]. Strictly speaking, compositionality is given in a system where for each possible combination there is a different composition function. That is, compositionality vacuously holds if each function is unique to a combination and its structure. However, what we set out to investigate is compositionality as an integral aspect of and explanatory device for linguistic creativity, productivity and as a solution to the acquisition bottleneck. Crucially, this requires rules to be generalizable (cf. Pagin 2013). That is, to account for productivity, structural commonalities between linguistic elements need to be recognized so that learning how to compose two elements can be generalized over their classes. It would not

do if, for example, each possible adjective-noun composition rule, or even more troublesomely each unique combination on sentential level, had to be learned case by case and mentally stored.

In short, there needs to be a systematic association between simplex elements and the complex elements they are constituents of. Such relations, in turn, need to be generalizable to obtain a productive compositional system akin to that of natural languages. In the following we refer to the regular co-occurrence association condition and the generalization condition as condition I and II, respectively.

### Previous approaches

To clarify the challenges faced by signaling systems that deal with complex terms we begin by considering Barrett’s (2009) *syntactic games*. Their defining characteristic is that there are more states than signals and that more than one signal per round is permitted. For example, a game may consist of four states, two signals and four acts. Here, the key assumption is that of two senders and one receiver. The senders independently observe the state and select one signal each. The receiver observes both signals while registering which sender sent what signal and selects an act based on this information. Crucially, discerning which signal corresponds to what sender allows us to consider the two signals as a single complex one. That is, if two signals  $a$  and  $b$  are at the senders’ disposal, the receiver will receive one of four possible signals,  $aa$ ,  $ab$ ,  $ba$  or  $bb$ , where the left constituent is sent by the first sender and the right one by the second.

However, while exhibiting syntactic structure, these complex signals do not fulfill the above conditions. In particular, condition I requires the meaning of complex expressions to be dependent on that of its constituents (cf. Franke 2014). The expressions that result from syntactic games do not promote such constituent-based associations. For example, the instantiation of  $a$  as a first constituent in  $ab$  is not required to be related to that in a different expression  $aa$ .

To address this issue Franke [2014] introduces a *similarity relation* over the set of states, signals, and acts, defined only within each set to avoid assuming similarity between, say, a state and a signal. This relation is operationalized as constituent identity; two elements stand in a similarity relation if they share constituents. Furthermore, similarity is associated with a parameter  $s, 0 \leq s \leq 1$ . This value represents partial overlap of constituents, whereas fully dissimilar elements receive a value of 0 and identical ones a value of 1. For example,  $a$  is similar to  $ab$ , dissimilar to  $b$ , and identical to  $a$ .

In order for this relation to stimulate compositional structure, RL is extended to not only reinforce the state-signal and signal-act pairs selected in a given successful round. Instead, rewards are also distributed amongst other pairs in proportion to their similarity to the ones played – so-called *spill-over reinforcement learning* (SRL)<sup>1</sup>. For example, suppose a sender

<sup>1</sup>As noted by Franke, SRL does not necessarily require more sophistication from players. Distributing rewards to similar elements

pair  $\langle t, m \rangle$ , of state  $t$  and signal  $m$ , yields a positive outcome. The payoff is then added to all accumulated rewards  $\langle t', m' \rangle$  in proportion to the product of their pair-wise similarity to  $\langle t, m \rangle$ . That is, the amount added to the accumulated rewards of any pair is the product of their similarity to the pair in play and the round’s outcome.

The idea behind these modifications is that iterated success of, for example, signal  $m_a$  in state  $t_0$  and signal  $m_b$  in state  $t_1$  bootstraps and increases the probability of selecting complex signal  $m_{ab}$  in complex state  $t_{01}$ . This is because 0 and 1 are similar to 01, as well as  $a$  and  $b$  to  $ab$ . Thus, each positive outcome of the simplex pairs will also contribute to the rewards of complex elements they are constituents of, increasing the player’s propensity to associate  $m_{ab}$  with  $t_{01}$ .

Not all values of  $s$  achieve the desired increment in constituent-based association. Instead, some lead to ambiguity and an association of simplex-complex pairs. To counterbalance this, Franke introduces a second parameter to reduce rewards of synonymous and ambiguous pairs (a punishment parameter akin to the discount factor of Roth-Erev RL).

Under these assumptions Franke [2014] shows how compositional associations can arise from a constituent-based similarity relation coupled with SRL. Establishing such a structured convention is upper-bounded at a probability of 50%, depending on the parameters chosen.

However, this model does not enable players to generalize the associations they establish, failing to fulfill condition II. Players are not sensitive to the overall architecture of their communicative system. Thus, while their propensity for constituent-based associations increases, structurally analogous elements are not taken into account.

## Model

We propose a variant of SRL that reinforces elements with shared constituents, as well as clusters over elements that exhibit the same structure. Additionally, the relations considered here are more differentiated than Franke’s (2014) similarity relation, making learning sensitive to syntax. That is,  $ab$  is not related in the same way to  $a$  as it is to  $b$ . While not strictly required, this solves the need for a discount parameter.

To recapitulate, condition I requires a consistent association between elements: The semantic contribution of *car* should be constant across complex instances (*red car*, *blue car*, *fast car*, ...).<sup>2</sup> In turn, condition II requires generalizability over associations: If the rule underlying the combination of *red* and *car* is the same as that to form *orange bike*, then learning to compose the former should ease the formation of the latter. Minimally then, a relation between simplex

may be the result of not being able to differentiate between them. In contrast, in classic RL players always fully distinguish them – even if they closely resemble each other. In fact, Roth and Erev [1995] qualify this kind of learning as both “generalization” and “error”.

<sup>2</sup>Given that natural languages exhibit, amongst others, ambiguity and underspecification, this requirement does not hold in its strongest form for them. Nevertheless, requiring a strict form-meaning homomorphism does not seem unreasonable for smaller-scale systems such as the ones considered here.

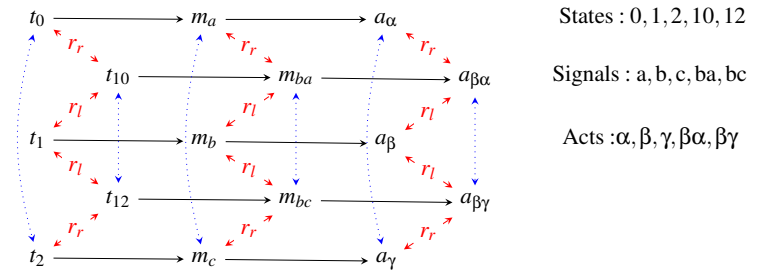


Figure 2: Structure of a possible signaling system. Uninterrupted lines stand for conventions, dashed ones for relations (identity is omitted), and dotted ones link elements with identical fabrics.

and complex elements, as well as a way to compare them in terms of their relations are required to fulfill both conditions.

We focus on three symmetric relations for each of the sets of states, signals, and acts:<sup>3</sup> identity ( $r_{id}$ ), left-constituency ( $r_l$ ), and right-constituency ( $r_r$ ). Identity holds if two elements are identical, left-constituency if one element is the leftmost constituent of the other and, analogously, right-constituency holds if an element is the rightmost constituent of the other. Except for  $r_{id}$ , two elements are required to be distinct if they stand in a relation.

To capture generalizable associations, we call the set of all relations an element  $x$  stands in its *fabric*,  $F_x = \{r | \exists y. r(x, y)\}$ . This allows for the comparison of not only elements that stand in one of the above relations but also of those that exhibit the same composition pattern. The fabric of a pair is the union of its elements’ fabrics. Figure 2 depicts the relations and fabrics of an exemplary established signaling system.

In terms of learning, rewards are distributed by adding the product of  $U(t_i, m_j, a_k)$  and a spill-over parameter to the accumulated rewards. This parameter  $\pi$  determines the proportion of rewards that spill-over either to elements (i) that stand in a relation to  $\langle t_i, m_j \rangle$  and  $\langle m_j, a_k \rangle$ , or (ii) have the same fabric as these pairs. The former increases, amongst others, the desired  $P(m_{xy} | t_{wz})$  in successful plays with conventionalized  $\langle t_w, m_x \rangle$  or  $\langle t_z, m_y \rangle$ . The latter increases  $P(m_{xy} | t_{wz})$  if some  $\langle t_{uv}, m_{rs} \rangle$  is successful and  $F_{wz} \cup F_{xy} = F_{uv} \cup F_{rs}$ . Thereby compositional structure (condition I) and generalizations over structurally identical elements (condition II) are reinforced.

The requirement for identical fabric unions ensures that compared pairs do not only coincide in terms of their form (e.g. two signals being, respectively, the right constituent of another signal), but also on their state/act relations. In these cases, right- and left-constituency are to be interpreted as semantic commonalities. This assumption crucially presupposes a structured semantic space.

Additionally, to simulate gradual learning, reward increments are proportional to the “degree of conventionalization”

<sup>3</sup>Symmetry is assumed for convenience, as it allows for a reduction of relations introduced and has no impact on the sets considered. A closer adherence to natural language would require asymmetry.

of a given successful pair, represented by its probability conditioned on the first pair element. The idea is that the confidence that an association is applicable to other cases increases with its repeated success. At the beginning of the game players have a low confidence that their choices will succeed, meaning that they will explore different potential pairings. Over time, conventions are established and players lock in on certain strategies. The more ingrained a convention is, the more rewards will spill-over to pairs with either shared constituents or identical structure.

In short, for a sender/receiver pair  $\langle p, q \rangle$ , the product of its utility,  $P(q|p)$ , and  $\pi$  is added to all  $ar(p', q')$ , where it either holds that there is an  $r$  and  $r'$  such that  $r(p, p')$  and  $r'(q, q')$ , or  $F_p \cup F_q = F_{p'} \cup F_{q'}$ . This update function is specified for the sender in Table 1.

While this kind of reinforcement targets the (compositional) associations we are after, players do not have a way to distinguish between multiple pairings that fit these conditions. In turn, many of them lead to partial pooling equilibria, i.e. systems where multiple states/signals are associated with a single signal/act. Or, simply and more generally, non-compositional systems.

## Analysis & simulations

In this model, a setup with related elements will invariably spill over a fraction of the payoff to more than one pair. As a consequence, even games with low  $\pi$ -values will never reach perfect signaling. In principle, this could be counter-balanced by introducing a discount factor to deduce rewards from residual, less frequent, pairs. However, just as with requiring a strict form-meaning homomorphism for compositionality, we focus on a less favorable setting as it decreases the assumptions made on learning and the number of parameters involved.<sup>4</sup>

To test these assumptions, games of  $10^3$  to  $10^6$  iterations with  $\pi$ -values of 0, 0.01, 0.02 and 0.03 were simulated. Pilot runs with  $\pi \geq 0.04$  indicate that the amount of rewards distributed amongst pairs introduce too much noise to achieve successful coordination in the analyzed scenarios. All scenarios involved a 6-states/signals/acts game with elements of the form  $x_a, x_b, x_c, x_{ma}, x_{mb}$  and  $x_{mc}$ . That is, players had three simplex and three complex states, signals and acts to pick from. We require from a compositional signaling system that, if a conventional association of  $t_x$  and  $m_y$  is established, then a complex state with  $x$  as its constituent should be associated with a complex signal involving  $y$  as well. Analogous requirements hold for the receiver. For condition II, we test whether establishing such simplex-complex associations improves coordination and increases the proportion of compositional conventions players establish. Given the complexity of the task, we chose a setup where all simplex and complex elements share a respective fabric.

<sup>4</sup>One may argue that forgetting past actions, operationalized as reward decrements, requires less sophistication and is thereby in line with the goal of minimal requirements and simple learning. Thus, in principle, we see no conflict with incorporating a discount factor.

Three scenarios with varied priors were tested, where priors take the form of higher starting values for the accumulated rewards of certain pairs. Thus, players already had some (partial) conventions at hand. Each prior association had its accumulated rewards set to 100. The accumulated rewards of all other pairs were set to 1. Scenario (i) involved three prior conventions of the form  $\langle p_1, q_a \rangle$ ,  $\langle p_2, q_b \rangle$  and  $\langle p_3, q_c \rangle$ . Scenarios (ii) and (iii) involved two priors of the form  $\langle p_1, q_a \rangle$  and  $\langle p_{01}, q_{ma} \rangle$  for scenario (ii), and  $\langle p_1, q_a \rangle$  and  $\langle p_2, q_b \rangle$  for scenario (iii).

Scenario (i) targets the question whether and how well players transition from a simplex 3-term convention to its compositional extension. Scenarios (ii) and (iii) compare how different conventions aid coordination and structural propagation. According to the preceding discussion having a convention for two related simplex-complex pairs should have a greater effect on the structure of the signaling system than the conventional association of two unrelated pairs. Thus, it is expected that scenario (ii) will, in general, yield more compositional signaling systems than scenario (iii). Put differently, success in (i) mainly involves players establishing a convention for complex elements that fit condition I, while a comparison between (ii) and (iii) targets condition II.

We focus on two outcomes: whether a convention is reached and if it is compositional. As mentioned above, flawless coordination is impossible in this model. Therefore, a weaker notion of convention than Lewis' has to be adopted. Sender and receiver are considered to reach a convention if the resulting communicative system achieves successful coordination, on average, more than  $\frac{5}{6}$  of the time ( $> 83.3\%$ ). This corresponds to higher communicative success than that of the best suboptimal equilibrium for a signaling game with six equiprobable states (a partial pooling equilibrium where two states/signals are associated with the same signal/act).

The main result is given by a comparison between  $\pi = 0$  and  $\pi \in \{0.01, 0.02, 0.03\}$  to test whether the proposed model has an effect on the number of conventions and compositional conventions reached. A baseline comparison is given by  $\pi = 0$  as it corresponds to classic RL without the above assumptions. Using Yates'  $\chi^2$  to test for homogeneity on the results obtained after  $10^6$  iterations shows that both groups are homogeneous in scenarios (i) and (ii) in terms of conventions ((i):  $\chi^2 = 0.333, p > 0.01$  and (ii):  $\chi^2 = 4.633, p > 0.01$ ) but differ for compositionality ((i):  $\chi^2 = 26.705, p < 0.01$  and (ii):  $\chi^2 = 16.218, p < 0.01$ ). In contrast, scenario (iii) yielded the opposite (convention:  $\chi^2 = 35.495, p < 0.01$ , compositionality:  $\chi^2 = 5.108, p > 0.01$ ). The specific outcomes of all simulations are summarized in Table 2.

## Discussion

On the one hand, the results show that a weak bias toward structural analogy and related elements can lead to compositional signaling. A transition to compositional signaling is not guaranteed, but a significant proportion of conventions comply with condition I across scenarios even with this simple learning algorithm. However, this only holds for a limited

$$\text{Update}_s(ar(t', m')) = \begin{cases} ar(t', m') + U(t_i, m_j, a_k) & \text{if } r_{id}(t_i, t') \wedge r_{id}(m_j, m') \\ ar(t', m') + U(t_i, m_j, a_k) \cdot \pi \cdot P(m_j|t_i) & \text{if } r_n(t_i, t') \wedge r_o(m_j, m') \wedge r_n \neq r_{id} \wedge r_o \neq r_{id} \\ ar(t', m') + U(t_i, m_j, a_k) \cdot \pi \cdot P(m_j|t_i) & \text{if } F_{t'} \cup F_{m'} = F_{t_i} \cup F_{m_j} \\ ar(t', m') & \text{otherwise} \end{cases}$$

Table 1: Update function for the sender. The cases correspond to, from top to bottom, Roth-Erev RL with no discount factor, relation/similarity-based reinforcement, structure-based reinforcement, and no reinforcement.

		10 <sup>3</sup>		10 <sup>4</sup>		10 <sup>5</sup>		10 <sup>6</sup>	
		(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
$\pi = 0.00$	Scenario i	33	4	75	13	91	11	92	16
	Scenario ii	12	1	71	3	78	11	81	9
	Scenario iii	16	1	67	3	77	3	86	4
$\pi = 0.01$	Scenario i	41	16	90	34	95	30	100	31
	Scenario ii	1	1	69	13	79	15	83	19
	Scenario iii	3	0	55	8	73	10	89	10
$\pi = 0.02$	Scenario i	34	21	79	44	93	44	95	48
	Scenario ii	0	0	52	25	73	28	81	31
	Scenario iii	0	0	25	20	46	22	64	26
$\pi = 0.03$	Scenario i	16	14	66	55	73	50	73	58
	Scenario ii	0	0	23	20	43	35	51	39
	Scenario iii	0	0	1	1	1	1	2	2

Table 2: Results obtained for three scenarios per parameter showing the amount of (a) conventions, and (b) compositional signaling systems obtained over 100 independent games for each scenario and number of iterations.

range of  $\pi$ -values. The maximum number of compositional conventions reached was 58 for a  $\pi$ -value of 0.03 and 10<sup>6</sup> iterations (scenario i), corresponding to 79% of the conventions being compositional. Interestingly, a number of games came close to this result in remarkably less iterations. For instance, 55 compositional conventions were established under the same condition but in 10<sup>4</sup> rounds. In general, 10<sup>4</sup> iterations were indicative of a parameter’s influence in a scenario.

Higher  $\pi$ -values did not significantly influence the amount of conventions reached in scenarios (i) and (ii). However they did increase the amount of compositional conventions as predicted. Furthermore, a comparison between scenarios (ii) and (iii) showcases that this kind of bias can be detrimental to coordination if there is no prior convention established for (at least two) related simplex and complex pairs. As reported above, the latter resulted in significantly less conventions without an increase in compositional ones. This is to be expected insofar as this kind of reinforcement presumes that there is some structure in place, just as productive rule-application presupposes that there are multiple cases where it could apply. If no structure is in place early exploration of multiple potential rule applications comes at the cost of less coordination.

Both findings align with those reported in O’Connor forthcoming, where generalization is analyzed in evolutionary terms. O’Connor concludes that the advantage of generalization is found in a trade-off between acquisition speed and pre-

cision. The stronger the generalization bias, the faster structure is learned – but at the cost of less accuracy. Generalization is primarily advantageous in large environments where agents have a high chance of encountering stimuli structurally similar to the ones they have already categorized. Furthermore, we conjecture that less precision is not as disruptive for more sophisticated signalers that are able to reason about the input they receive, as human beings do.

On the other hand, a number of caveats apply. First, even low  $\pi$ -values guarantee neither convention nor compositionality. This is a result of players not having a way to eliminate associations made by chance in early stages or via SRL, meaning that exploration of alternatives to conventionalized best responses never fully halts. As discussed by Franke [2014], a margin of error and only a range of successful parameter values is expected given the complexity of the learning task with such simple adaptive dynamics.

Second, the present reward distribution does not differentiate between directly related elements and those linked through fabric identity. In terms of natural language this means that reinforcement treats the link between *car* and *red car* on par with that between *red car* and *orange bike*. However, one may argue that the former two should have a stronger influence on each other than the latter two, just as classic RL reinforces successful actions to a greater degree than SRL does related or similar actions. The differences and commonalities of reinforcement through shared elements

(here; relations) and a shared structure/class (here; fabrics) leave open many issues to be addressed in the future.

## Conclusions

We have shown that even with little sophistication the propagation of rule-like associations between simplex and complex elements can be achieved in signaling games. The assumed architecture followed two conditions argued to be necessary to establish a productive compositional communicative system: a systematic link between related elements and a way to group related elements into classes with the same underlying structure. The former captures the bare requirements for compositionality while the latter enables for a simple form of productivity, where associations of structurally identical elements are reinforced throughout the system. Non-compliance with the first condition means that compositionality cannot be achieved and missing the second means that knowledge about one formation cannot be applied to others. Both are key to the robustness and flexibility of natural language.

An unresolved issue concerns optimal learning conditions in this model. So far, we focused on the overall minimal architecture required for compositionality to emerge. However, we have not explored all options; such as a discount factor, different learning functions and mechanisms (e.g. players with declarative memory following boost and decay activation patterns), more elements, or different structures.

To conclude, in a sense, little is needed for productive compositionality. However, whether or not an initial relation-based association is made is vital for the future development of a communicative system. Without some inherent bias for associations of one kind over the other no structure nor comparisons over it could arise. Determining the nature of this bias is a major open issue to understand why languages are structured the way they are. In turn, this may contribute to our understanding of why non-human communicative systems lack the degree innovation and flexibility found in natural language.

## Acknowledgments

I thank Robert van Rooij, Julian Schlöder, Philip Schulz, and three anonymous reviewers for comments and discussion. This research has been funded by the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement no. 567652 *!ESSENCE: Evolution of Shared Semantics in Computational Environments!*.

## References

- Henry Brighton. Compositional syntax from cultural transmission. *Artificial Life*, 8(1):25–54, 2002.
- Kenny Smith, Monica Tamariz, and Simon Kirby. Linguistic structure is an evolutionary trade-off between simplicity and expressivity. In *Proceedings of the 35th annual conference of the cognitive science society*, pages 1348–1353, 2013.
- Simon Kirby, Monica Tamariz, Hannah Cornish, and Kenny Smith. Compression and communication in the cultural evolution of linguistic structure. *Cognition*, to appear.
- Michael Franke. Creative compositionality from reinforcement learning in signaling games. In Erica A. Cartmill et al., editor, *The Evolution of Language: Proceedings of the 10th International Conference (Evolang 10)*, pages 82–89, Singapore, 2014. World Scientific Press.
- David Lewis. *Convention: A Philosophical Study*. Harvard University Press, Cambridge, 1969.
- Richard J Herrnstein. On the law of effect. *Journal of the experimental analysis of behavior*, 13(2):243–266, 1970.
- Alvin E. Roth and Ido Erev. Learning in extensive-form games: Experimental data and simple dynamic models in the intermediate term. *Games and Economic Behavior*, 8(1):164–212, 1995. 10.1016/s0899-8256(05)80020-x.
- Justin Bruner, Cailin O'Connor, Hannah Rubin, and Simon M. Huttegger. David Lewis in the lab: experimental results on the emergence of meaning. *Synthese*, 2014. 10.1007/s11229-014-0535-x.
- Ido Erev and Alvin E. Roth. Predicting how people play games: Reinforcement learning in experimental games with unique, mixed strategy equilibria. *American economic review*, 88(4):848–881, 1998.
- A.W. Beggs. On the convergence of reinforcement learning. *Journal of Economic Theory*, 122(1):1–36, 2005. 10.1016/j.jet.2004.03.008.
- David Catteeuw and Bernard Manderick. The limits and robustness of reinforcement learning in Lewis signaling games. *Connection Science*, 26(2):161–177, 2014. 10.1080/09540091.2014.885303.
- Hans Kamp and Barbara Partee. Prototype theory and compositionality. *Cognition*, 57:129–191, 1995. 10.1016/0010-0277(94)00659-9.
- Barry P. Taylor. Toward a theory of language acquisition 1. *Language Learning*, 24(1):23–35, 1974. 10.1111/j.1467-1770.1974.tb00233.x.
- Peter Pagin. Compositionality, complexity, and evolution. In Francisco Lacerda, editor, *Symposium on Language Acquisition and Language Evolution*, pages 51–62, Stockholm, 2013.
- Jeffrey A. Barrett. The evolution of coding in signaling games. *Theory and Decision*, 67(2):223–237, 2009. 10.1007/s11238-007-9064-0.
- Cailin O'Connor. Evolving to generalize: Trading precision for speed. *British Journal for the Philosophy of Science*, forthcoming.