Formalizing Risky Choice with a Logistic Model of Fuzzy Trace Theory

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Abstract

We propose a new model of risk preferences that integrates theoretical principles relevant to mental representation, metacognitive monitoring and editing, and individual differences in risk-taking propensity. Our model is based on fuzzy-trace theory, a theory of decision-making under risk. The theory posits that decision-makers use fuzzy gist representations of the meaning of decision information, in parallel with precise verbatim representations of the exact wording of that information. We account for core phenomena in decision theory, such as shifts in risk preference when logically equivalent gambles are described in terms of gains rather than losses—framing effects—and also extend fuzzy-trace theory beyond these phenomena to encompass research on affect and personality.

Keywords: Psychology, Decision making, Mathematical modeling, Gist

Introduction

Fuzzy-trace theory is a leading account of decision-making under risk. In prior work (Broniatowski & Reyna, 2014) we presented a formalization of fuzzy-trace theory that predicted modal responses for risky decisions; however, this prior model did not predict effect sizes. In this paper, we use a parsimonious logistic model of choice to predict effect sizes. We show how risk preferences are determined by combining multiple representations (e.g., Reyna & Brainerd, 2011) of decision options. Our model formalizes the formation of multiple mental representations of risky choice gambles and how the ultimate preference is determined by applying "voting" rules that adjudicate among representations that support differing preferences. The model also incorporates an explicit mechanism for adjusting preferences based on metacognitive monitoring and editing (e.g., Stanovich, West, & Toplak, 2011). That is, people who are high in Need for Cognition (NFC) and cognitive ability (e.g., intelligence) have been shown to edit their preferences when decision problems that are related to one another are presented within-subjects (e.g., Kahneman, 2003; Stanovich & West, 2008). Such individuals are more likely to notice that decision problems are related (e.g., that one decision problem can be derived from another mathematically) and to reconcile their answers to the problems, diminishing framing effects and other cognitive biases. The mix of such individuals in samples of subjects determines the magnitude of within-subjects reduction in framing effects, relative to between-subjects effects. We account for experimental evidence from several classic decision problems and experimental manipulations of these problems (e.g., Kühberger & Tanner, 2010; Peters & Levin, 2008; Tversky & Kahneman, 1981). Standard theories cannot account for all of these effects; indeed, some effects contradict standard predictions (such as those made by cumulative prospect theory (CPT; Tversky & Kahneman, 1992). We apply our formalization to explain how experimental manipulations of decision problems change decision outcomes.

Key Tenets of Fuzzy-Trace Theory

The central tenet of fuzzy-trace theory is that people encode, store, retrieve, and forget memories that are characterized by different levels of detail and meaningfulness. We refer to these levels as "gist" and "verbatim." Research on fuzzy-trace theory has shown that gist and verbatim representations are encoded separately and roughly in parallel (see Reyna, 2012). A gist representation captures the basic *meaning*, or "essence," of a stimulus. In contrast, a verbatim representation of a stimulus captures its surface form (e.g., Clark & Clark, 1977).

Fuzzy Processing: Categorical Decision-Making is Preferred

Another tenet of fuzzy-trace theory is that decision-makers prefer to operate on the simplest gist that can be extracted from information. For numerical information, differences in levels of precision can be thought of in terms of scales of measurement: the simplest level is categorical or nominal because that level is the least fine-grained. Categorical gist entails representing decision outcomes as members of different categories, such as “no money” versus “some money.” This fuzzy-processing preference increases with experience in a domain (e.g., Reyna, Chick, Corbin, & Hsia, 2014; Reyna & Lloyd, 2006). When two decision outcomes fall into different qualitative categories (e.g., no money vs. some money), the gist representation compares these two categories rather than the specific details. Each of these categories is associated with a valence (e.g., money has a positive valence) and the category that is more highly valued (e.g., some money) will be chosen.

Ordinal Comparisons

Fuzzy-trace theory predicts that subjects interpret decision outcomes on a continuum of detail ranging from categorical gist (e.g., win some money), on one end, to verbatim detail (e.g., win $200) on the other. More precise but nevertheless qualitative representations are generated simultaneously, such as ordinal (i.e., relative) representations (e.g., small vs. large amount of money). Levels of distinction that are intermediate between categorical and verbatim become evident when two decision options’ outcomes fall into the same category, and, thus, cannot be discriminated. For example, if one medical treatment is described as having a 20% chance of death and another treatment as having a 10% chance of death, both treatments can be categorized as having “some” risk of death (e.g., Reyna, 2008). To discriminate between treatment options, a more fine-grained ordinal distinction needs to be made: the first treatment has a high risk relative to the second treatment.

Interval Comparisons

When categorical and ordinal comparisons lead to an indeterminate decision outcome, even more precise representations such as comparing interval-level values become evident. For example, the classical expected value (i.e., the product of outcomes and probabilities) is an interval representation, which we predict that subjects encode. Using interval-level numbers, the expected value of a decision option with a 1.0 probability of winning $180 is 1.0 multiplied by $180. In contrast, another option with a .90 probability of winning $250 has an expected value of $250 multiplied by .90 (plus .10 times $0).

Values: Decisions made by Comparing Valenced Affects

The final tenet of fuzzy-trace theory that we review is that decisions are made on the basis of simple valenced (i.e., positive or negative) affect (e.g., Peters & Levin 2008). Thus, once options are represented in a categorical, ordinal, or interval fashion, the more positively valenced option is chosen (e.g., winning money is preferred; saving lives is preferred). Consider the decision below:

1. Winning $180
2. .90 chance of winning $250 and .10 chance of no money.

The categorical gist representation is:

1. Some money
2. Some chance of some money and some chance of no money.

Given the affective value that some money is preferred to no money (i.e., money has a positive valence), the categorical gist would favor option 1. In contrast, the ordinal representation is indifferent between these two options because more money ($250) is preferred to less money ($180), but less money ($180) is preferred to no money (vote is 0). Finally, the interval, or verbatim, representation would favor option 2 because the expected value of money is $225, greater than the $180 of option 1. The categorical and interval representations in this problem favor different options (the sure vs. risky options) and would therefore compete in the sense that each produces an opposite vote (-1 vs. 1).

Formalizing Fuzzy-Trace Theory

At the categorical level, given a pair of decision options represented by points, ϕ and θ, the decision option corresponding to ϕ is preferred to the decision option corresponding to θ if the associated category is preferred in the domain of values (e.g., "some money with some chance" is preferred to "no money with some chance"). At the ordinal level, a decision option is preferred if its corresponding points are strictly preferred along all dimension of the decision space (e.g., "some money with more chance" is preferred to "some money with less chance") Points in disjoint categories cannot be compared. At the interval level, decisions options are evaluated according to expected values.

An Error Theory for Risky Decision Problems

The model of fuzzy-trace theory outlined thus far is deterministic – each representation provides one vote and the option with the most votes is selected. Here, we account for deviations from this mode. The need for such an error theory in the domain of risky decision-making has long been recognized; for example, Kühberger (1995) remarked on the absence of an error theory for risky framing problems. Consistent with the literature on qualitative discrete choice models, we represent error using a standard multinomial logistic distribution (e.g., Luce, 2005). For decisions with two options, our error is thus distributed according to a standard logistic distribution – a functional form that is commonly used in Signal Detection Theory (e.g., McNicol, 2005) because of its computational tractability, ease of interpretation, and its similarity in shape to the cumulative normal distribution. For our specific application, we model the probability, P, that a subject will choose a given decision outcome in a risky choice gamble by $P(x)=\frac{1}{1+e^{-(a\cdot x+b)}}$ where **x** is a three-element vector containing an entry for each representation (categorical, ordinal, and interval), **a** is a three-element vector containing a decision weight applied to each representation. A dot-product operation is used to combine **a** and **x**, yielding a scalar quantity. Additionally, b is a scalar quantity representing risk-taking propensity. Thus, people who are high in NFC and numeracy reduce conflict between representations by weighting votes from each representation and risk propensity increases (or decreases) the tendency to choose riskier options regardless of representation, NFC, or numeracy (see next sections).

Numeracy and Need for Cognition In the domain of decision making, two major individual difference factors associated with metacognitive monitoring and editing have been proposed – numeracy (e.g., Peters et al., 2006) and Need for Cognition (NFC; Cacioppo et al., 1996; Stanovich et al., 2011). Peters and colleagues (2006) defined numeracy as "the ability to process basic probability and numerical concepts," and found that more numerate subjects were less susceptible to attribute framing effects. In the domain of risky decision framing, Peters and Levin (2008) found that more numerate subjects were less likely to show risky choice framing. As these authors argued, these results are consistent with the hypothesis that highly numerate individuals are more likely to notice that decision problems are related (e.g., that the loss decision problem can be derived from the gain version of that problem mathematically) and to reconcile their answers to the problems, diminishing cognitive biases such as framing. We model these effects using the decision weight vector **a**.

Risk-Taking Propensity In addition to individual difference variables, such as metacognitive monitoring (NFC) and editing (numerical computation, facilitated by high numeracy), our model accounts for personality differences associated with risk-taking, including factors related to sensation seeking and impulsivity (e.g., Lauriola et al., 2014). We represent this in our model by a linear additive risk preference, b, which, when positive, is used to indicate a fixed predisposition toward a more risky option. The linear additive nature of this factor is based on evidence presented by Reyna, Estrada et al. (2011) who found evidence supporting independent effects of subjects' sensation seeking.

Testing Our Model

We use our formalization to explain the outcomes of several classic risky choice problems, such as Tversky and Kahneman's (1981) Asian Disease Problem (ADP) and related framing problems. The text of the gain-framed standard ADP is as follows: "Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences of the program are as follows: If Program A is adopted, 200 people will be saved; If Program B is adopted, there is a 1/3 probability that 600 people will be saved and a 2/3 probability that no people will be saved." (Tversky & Kahneman, 1981).

The loss-framed version of the same problem uses the same preamble but presents the decision options as: If Program C is adopted 400 people will die; If Program D is adopted there is a 1/3 probability that nobody will die, and a 2/3 probability that 600 people will die." (Tversky & Kahneman, 1981). Options A and C are typically referred to as the "certain option," whereas options B and D are typically referred to as the "gamble option." The typical result (framing effect) is that most people prefer the certain option in the gain frame, but the risky gamble option in the loss frame. We fit our model to 26 studies of the ADP (Tversky & Kahneman, 1981; Reyna & Brainerd 1991; Tindale, Sheffey, & Scott, 1993; Takemura, 1994; Wang & Johnston, 1995; Highhouse & Yüce, 1996; Wang, 1996; Stanovich & West, 1998; Druckman, 2001a; 2001b; Mandel, 2001; Mayhorn, Fisk, & Whittle, 2002; LeBoeuf & Shafir, 2003; Fischer, Jonas, Frey, & Kastenmueller, 2008; Zhang & Miao 2008; Zhang, Xiao, Ma, & Miao, 2008; Horton, Rand, & Zeckhauser, 2011; Haerem, Kuvaas, Bakken, & Karlsen, 2011; Berinsky, Huber, & Lens, 2012; Stein, 2012; Okder, 2012; Kühberger & Gradl, 2013).

Risk Taking

Each of the studies listed above is associated with a country of origin from which the subjects were recruited. Hofstede (1991) defined a nation's uncertainty avoidance index as "the degree to which the members of a society feel uncomfortable with uncertainty and ambiguity." Thus, national culture is one of several factors that may be associated with risk preference and ambiguity. Hofstede's index is significantly correlated with our risk parameter, b, $r(24)=-0.455$.

Within-Subjects Framing

Prior work has determined that subjects reconcile answers to gain and loss versions of problems when both frames are presented within-subjects. That is, the magnitude of framing effects in risky choice problems varies systematically with experimental design. In particular, within-subjects framing effects, where subjects are exposed to both gain and loss framing problems, tend to be smaller than between-subjects effects. Subjects with high NFC tend to edit their preferences more than those with low NFC because they are more likely to notice the common structures underlying these problems (i.e., high NFC subjects display "analytic override;" Kahneman 2003; LeBoeuf & Shafir, 2003; Stanovich et al., 2011). Thus, analytic override occurs when the same subject is exposed to two oppositely framed versions of the same problem. This should be reflected in our model by the presence of scale parameters that are significantly smaller than those found in the standard ADP. The average scale-factor value for several studies (Stanovich & West, 1998; Levin, Gaeth, Schreiber, & Lauriola, 2002; LeBoeuf & Shafir, 2003) in which framing was manipulated within-subjects is 0.47 – smaller than the values for 26 studies in which framing was manipulated between-subjects, $t(62)=3.00$, $p<0.01$.

Explaining Truncation Problems

The concept of gist is central to our theory of how decision-makers perceive options. Manipulations of these gist representations can result in different framing effects, or the absence of an effect altogether. Specifically, by emphasizing or removing certain stimuli from a problem in such a way that its expected value does not change, one might change the gist of a decision option (e.g., one might not mention options with zero expected value). PT and its successor, CPT, predict that these manipulations do not change preferences; thus, these "truncation" experiments were initially performed as critical tests of fuzzy-trace theory by Reyna and Brainerd (1991) and later replicated by others (e.g., Kühberger & Tanner, 2010; Reyna et al., 2014). All of these investigators determined that framing effects did not persist when the zero-complement in the gamble option of the ADP (i.e., "none are saved") was removed (a selective attention effect; Reyna, 2008; Reyna, 2012). These effects do not depend on ambiguity; when all of the information is supplied (but attention is focused selectively in different ways), effects remain the same.

Zero-Complement Truncated Framing Problems The zero-complement truncated gain-framed ADP is worded as follows: If Program A is adopted, 200 people will be saved; If Program B is adopted, there is a 1/3 probability that 600 people will be saved. Here, the "2/3 probability that no people will be saved" part of the gamble has been removed. This version of the ADP has the same expected value as the standard ADP. Both options are interpreted as "Some chance that some live" leading to indifference at the categorical level. The ordinal representation is also indifferent:

a) Fewer live with more probability
b) More live with less probability

Finally, the more precise interval representation is also indifferent.

a) 200 saved = expectation of 200 saved
b) 600 saved with 1/3 probability = expectation of 200 saved

Both options have the same expected value, leading to indifference – i.e., **x**=[0,0,0] – and resulting in the absence of a framing effect as reported by Reyna et al. (2014) and others. The loss-framed version of the problem yields similar results (400 die vs. 600 die with 2/3 probability). We found no significant difference between our model's prediction of no framing effect and the data from several replications of the zero-truncated framing problem (Reyna & Brainerd, 1991; Mandel, 2001; Kühberger, 2010; Reyna et al., 2014).

Non-Zero Complement Truncated ADP The opposite truncation effect, which retains the zero complement, yields a framing effect that is twice as strong as that found in the standard ADP because of the combined contributions of the categorical representation and the ordinal representation, both of which support the certain option (the interval representation is indifferent) – i.e., **x**=[-1,-1,0] – in the gain frame. Similar results obtain for the loss frame. The average scale-factor value for several studies (Reyna & Brainerd, 1991; Kühberger, 2010; Reyna et al., 2014) for this class of framing problem is 1.2, which is exactly twice the values for the corresponding standard framing problems, 0.6 in the same studies. This difference is statistically significant, $t(32)=8.38$, $p<0.001$.

Conclusions

Our mathematical framework builds upon three basic tenets of fuzzy-trace theory – the gist/verbatim distinction (formalized by theoretically-motivated, and empirically-tested, subcategories of representations), the hierarchy of gist (formalized by our extended fuzzy processing preference and associated lattices), and preferences over these gist categories based on valenced affect. Our formalized theory, therefore, explains a wide variety of phenomena, integrating known effects and novel predictions.

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