Helping Students Understand Posterior Probabilities: Research with a Digital Learning Environment on the Monty Hall Dilemma

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Abstract

When initially confronted with the Monty Hall dilemma (MHD), people show a very strong tendency to stick with their initial choice, although switching maximizes winning chances. Previous research demonstrated that certain interventions helped participants to discover and apply the optimal strategy, but generally failed to increase participants' understanding of the MHD solution. An exception on the latter finding is DiBattista's (2011) digital learning environment study, reporting that the majority of participants who used the learning environment learned to understand the MHD solution. However, a major shortcoming was DiBattista's (2011) methodology, which did not allow to infer causal relations and to conclude which (combination of) manipulation(s) was most important for participants' understanding of the MHD solution. The aim of the present study was to fill this research gap by conducting a controlled randomized experiment with an analogous digital learning environment. Participants were high-school students between 16 and 19 years old. The results showed that receiving explanation about the MHD solution was the most important manipulation to improve understanding. Implications for education in (posterior) probability are discussed.

Keywords: Monty Hall dilemma; probability; posterior probability; digital learning environment; experience-based learning; traditional learning

Introduction

The Monty Hall dilemma (MHD) was adapted from the popular TV game show Let's Make a Deal and is known as one of the most counterintuitive posterior probability problems (Friedman, 1998). In the classic version of the MHD, a guest is confronted with three identical doors. One door conceals a valuable prize, usually a car. The two remaining doors conceal worthless prizes such as goats. After the guest initially chooses one door, the host, who is aware of the location of the prize, opens a non-chosen door to show that there is a worthless prize behind it. Next, the guest faces a dilemma when the host asks him to either stay with his initial choice, or to switch to the other unopened door. By applying Bayes' Theorem with the correct prior and marginal likelihoods, it can be derived that switching is the strategy that maximizes the probability to win the valuable prize. More specifically, switching yields a posterior winning probability of 2/3, whereas staying only yields a 1/3 posterior winning probability.

Previous research on the MHD has provided strong evidence for the following four findings. First, there exists a strong sticking tendency: When first confronted with the dilemma, the vast majority of participants choose to stay with the initial choice (Burns & Wieth, 2004; Friedman, 1998; Granberg & Brown, 1995; Granberg & Dorr, 1998). Cross-cultural research revealed that staying percentages range between 79% and 87% (Granberg, 1999). These high percentages indicate how extremely counterintuitive the MHD solution is.

Second, participants have a strong belief that their choice – either staying or switching – does not matter, because they consider both posterior probabilities as being equal (Franco-Watkins, Derks, & Dougherty, 2003; Granberg & Brown, 1995; Stibel, Dror, & Ben-Zeev, 2009). Participants' preference to stay with their initial choice, despite the fact that they judge winning probabilities for staying and switching as equally large, can be explained by the larger amount of regret participants anticipate to experience after a loss due to switching compared with a loss due to staying (Stibel et al., 2009).

Third, previous research has shown that many factors can alter participants' sticking tendency. For example, switching behavior is more likely to occur when more alternatives are included in the problem compared with only three alternatives in the classic MHD (Burns & Wieth, 2004; Franco-Watkins et al., 2003; Saenen, Heyvaert, Grosemans, Van Dooren, & Onghena, 2014; Stibel et al., 2009). Also repeated experience with the problem has a strong impact: When participants are given a series of MHD trials, switching rates increase across trials, showing that people adjust their behavior to increase the gain (e.g., Franco-Watkins et al., 2003; Petrocelli, 2013; Petrocelli & Harris, 2011; Saenen, Van Dooren, & Onghena, 2015a). Importantly, none of these studies, containing the repeated experience with the dilemma, led participants to consistently switch on *all* trials. Thus, optimal behavioral performance was not observed.

Fourth, there exists a dissociation between participants' behavioral MHD performance and their understanding of the problem's solution. Some studies did not only examine participants' behavioral MHD performance, but also asked participants to estimate the posterior probabilities in order to investigate their MHD understanding (Burns & Wieth, 2003, 2004; DiBattista, 2011; Franco-Watkins et al., 2003;

Saenen et al., 2014, 2015a; Saenen, Heyvaert, Van Dooren, & Onghena, 2015b; Stibel et al., 2009). The results showed that although behavioral performance was easily improved by adding particular interventions, correct posterior probability estimates ranged from 0% to 50%. Thus, overall, participants still failed to understand the MHD and its underlying probabilities (Burns & Wieth, 2003, 2004; Franco-Watkins et al., 2003; Saenen et al., 2014, 2015a, 2015b; Stibel et al., 2009).

DiBattista (2011) developed a digital learning environment aimed at tackling people's general inability to understand to MHD solution. Its characteristics were specially designed to increase people's understanding of the MHD solution and can be described as follows. First, in the 'playing' part, one could complete as many trials as one wanted of both a 3-door and 20-door MHD. After each trial, feedback about the number of trials one had won and lost, conditional on the behavior (i.e., staying or switching), was updated. Second, in the 'simulation' part, one could ask the computer to generate *N* trials of both a 3-door and 20-door MHD and choose the desired type of behavior (i.e., always staying, always switching, or alternating between staying and switching). In this part, constantly updated feedback was also provided. Third, in the 'explanation' part, one could access explanations for both the 3-door and 20-door MHD solution.

In DiBattista's (2011) study, participants solved the classic MHD in paper-and-pencil format as a pretest measure. Participants were asked to indicate the optimal behavioral response in order to win the prize (staying, switches, or it makes no difference) and to explain in detail their reasoning behind their chosen response. Next, the participants were motivated to use the digital learning environment with unlimited access for a period of five weeks. Hereafter, the participants completed a 6-door variant of the MHD as a posttest measure. Another four weeks later (i.e., nine weeks after the pretest), participants again completed the classic MHD – identical to the pretest – as a follow-up measure. The results of DiBattista's (2011) pretest-posttest study revealed that at the pretest, only 4.5% of the participants correctly indicated switching as the optimal behavioral response and none of them could give a satisfactory explanation for why switching was beneficial. For the posttest and follow-up measure, the answers of participants who accessed the digital learning environment at least once were compared with the answers of those who never accessed it. The results were impressive: At the posttest, participants who accessed the digital learning environment gave the optimal behavioral response and a satisfactory explanation statistically significantly more often compared with participants who never accessed it (77.5% and 61.2% vs. 41.4% and 13.8% respectively). At the follow-up, no statistically significant difference was found on how often the optimal behavioral response was given between participants who accessed the digital learning environment and those who never accessed it (89.4% and 87.5% respectively). However, participants who accessed the digital learning environment statistically significantly more often gave a satisfactory explanation for why switching was beneficial, compared to those who never accessed it (62.7% and 6.2% respectively). No other empirical study so far ever reported percentages of participants understanding the MHD solution as high as 61.2% and 62.7%.

A major shortcoming to DiBattista's (2011) study is that the characteristics that were designed to promote the understanding of the MHD solution were not systematically manipulated between (or within) participants. Next, the study was not conducted in a controlled environment, and its use was not experimentally manipulated. Moreover, participants' use of the digital learning environment was self-selected and thus not randomly assigned. Thus, it is impossible to infer causal relations and to conclude which (combination of) manipulation(s) was most important to improve participants' understanding of the MHD solution.

To investigate that question, we developed our own MHD digital learning environment, analogous to the one developed by DiBattista (2011), and conducted various controlled randomized experiments. This paper presents the results of our first experiment, in which we examined the effects of both repeated experience with the MHD and explanation. The choice for the inclusion of repeated experience was made because there is already a lot of research literature available on this manipulation (e.g., Franco-Watkins et al., 2003; Petrocelli, 2013; Petrocelli & Harris, 2011; Saenen et al., 2015a), which makes it easy to compare our results. The choice for the inclusion of explanation about the MHD solution as a manipulation was made because of the practical relevance for education (see discussion section).

Methods

Participants and Design

Two-hundred and thirteen Flemish high-school students participated in the experiment. Seventy-eight of them were excluded from the data analyses because of prior familiarity with the MHD. As a result, our final sample consisted of 135 participants (80 females, 55 males; age range: 16-19 years, M_{age} = 16.92, SD_{age} = 0.54).

Participants were randomly assigned to one of four conditions, created by a 2 × 2 between-subjects design. The first independent variable was 'Explanation' and indicated whether or not participants could access the 'explanation part' of the MHD game. The second independent variable was 'Playing' and indicated whether or not participants could access the 'playing part' of the MHD game. This led to the following four conditions: control condition (neither explanation nor playing), 'playing only' condition (playing, but no explanation), 'explanation only' condition (explanation, but no playing), 'playing and explanation' condition (both playing and explanation). Data of respectively 28, 38, 34, and 35 participants were included in the analyses.

The study protocol was approved by the Ethical Committee of the KU Leuven − University of Leuven.

Materials

The following materials were used in the study: a paper-and-pencil questionnaire operating as the pretest measure, another paper-and-pencil questionnaire operating as the posttest measure, and a digital learning environment.

Our pretest questionnaire included the classic MHD, as in DiBattista's (2011) study. Participants were asked to answer three questions. First, participants were asked to indicate the optimal behavioral response (i.e., question 1) by choosing between one of three options: staying, switching, or it does not matter. This behavioral response question was the same as in DiBattista's (2011) study. Unlike DiBattista (2011), we operationalized understanding of the MHD solution by asking participants to estimate the posterior winning probability for both staying (i.e., question 2) and switching (i.e., question 3), instead of letting them explain their reasoning behind the behavioral response answer they gave on question 1.

In our posttest questionnaire, we included the items that DiBattista (2011) used for his posttest and follow-up measure. Thus, our own posttest questionnaire included two items. The first item was a 6-door MHD variant with one prize (cf. DiBattista's (2011) posttest), in which the participant initially selected two doors, the host then opened three other non-winning doors, and the participant then was faced with the dilemma to either stay with his two initially selected doors (winning the prize when located behind one of those two doors), or to switch to the one remaining unopened door. Note that the posterior probabilities of this 6-door MHD variant are equal to the posterior probabilities of the classic MHD: Staying leads to winning the prize in 1/3 of the cases, while switching yields a 2/3 winning probability. The second item of our posttest questionnaire was the classic MHD (cf. DiBattista's (2011) follow-up measure), completely identical to the item of the pretest questionnaire. For both items of our posttest questionnaire, participants were asked to complete the same three questions as in our pretest questionnaire. Summarized, for all MHD items there were three dependent variables. The first dependent variable was the behavioral response, the second one was the posterior winning probability for staying, and the third one was the posterior winning probability for switching.

The MHD digital learning environment we created[1] was analogous to the one developed by DiBattista (2011) and contained the same three major parts: a 'playing' part, a 'simulation' part, and an 'explanation' part. In the 'playing' and 'simulation' parts, feedback about the number of trials one had won or lost, conditional on the behavior (i.e., staying or switching), was constantly updated and provided. In the 'explanation' part, the MHD solution was explained stepwise by providing little information at a time on each screen. Both forward and backward navigation was possible in the 'explanation' part.

When opening the digital learning environment, a description of the classic MHD was always presented first in which all necessary elements were mentioned: the host, the three doors, the random location of the prize, the participant's initial choice, followed by the host opening another door than the one chosen by the participant showing it did not contain the prize, and ultimately the participant's final choice. Next, the same description was given for a 20-door MHD variant in which the host, after the participant made an initial choice, opened 18 other doors that did not contain the prize. After navigating through the descriptions of both the classic MHD and the 20-door MHD variant, the participant got to see a menu bar that listed the specific parts of the digital learning environment the participant could use. Which parts were listed in the menu bar depended on the condition a participant was assigned to. For example, a participant assigned to the 'playing only' condition only saw the 'playing: 3 doors' and the 'playing: 20 doors' parts listed in the menu bar, whereas a participant assigned to the 'explanation only' condition only saw the 'explanation: 3 doors' and the 'explanation: 20 doors' parts. Thus, in contrast to DiBattista's (2011), in our digital learning environment, it was possible to limit participants' access to particular parts of the learning environment. Also in contrast with DiBattista (2011), it was possible to set time limitations on the use of our digital learning environment. These adaptions were made in order to conduct controlled randomized experiments.

Procedure

Participants came to the laboratory in groups of eight for the experiment. Before the start of each experimental session, the experimenter placed an informed consent form and a laptop on eight separate tables. Tables and seats were placed so that no interaction was possible between participants.

Upon arriving, the experimenter asked the participants to take place at one of the eight tables on which there was an informed consent form. Participants were randomly assigned to the experimental conditions, with the limitation that in each experimental session two participants were assigned to each of the four different conditions. After completing the informed consent form, participants received the pretest questionnaire. Next, the six participants that were assigned to an experimental condition (i.e., 'playing only' condition, 'explanation only' condition, or 'playing and explanation' condition) were asked to use the digital learning environment for a duration of 20 minutes.

How the participants exactly spent and divided those 20 minutes between the different parts of the digital learning environment that were made available, was up to the participants themselves. After 20 minutes, the digital learning environment automatically stopped working. Next, the participants received the posttest questionnaire from the experimenter. During the 20 minutes that the six

[1] Researchers interested in using our digital learning environment for research and/or educational purposes can contact the authors.

participants assigned to an experimental condition used the digital learning environment, the two participants assigned to the control condition immediately completed the posttest questionnaire. Afterwards, they were asked to use the digital learning environment as well (with unlimited access) so that they would keep quiet during the remaining time of the experimental session. An entire experimental session lasted approximately 50 minutes.

Statistical Analysis

To investigate participants' behavioral responses and understanding of the underlying posterior probabilities of the MHD, we performed a logistic regression analysis with 'Explanation', 'Playing', and the interaction between 'Explanation' and 'Playing' as predictors. The significance level was set at $\alpha = .05$. To follow up on statistically significant effects, post-hoc pairwise comparisons were performed using a Tukey-Kramer (HSD) correction.

Results

Pretest: Classic MHD

For each dependent variable and each of the four conditions, percentages correct answers are presented in Table 1. As can be derived from Table 1, participants performed poorly on both the behavioral response question and the posterior probability estimate questions during the pretest.

The results of the logistic regression analyses (see Table 2) showed no statistically significant differences between the conditions before the intervention started, which is consistent with our random assignment scheme.

Table 1: Percentages correct answers given on the items of both the pretest and posttest questionnaire.

	Condition			
Dependent variable	Control	Explanation only	Playing only	Explanation and playing
Pretest: Classic MHD				
Optimal behavior	25.0	26.5	10.5	20.0
P (win | stay)	17.9	11.8	10.5	11.4
P (win | switch)	10.7	11.8	7.9	5.7
Posttest: Classic MHD				
Optimal behavior	25.0	88.2	68.4	94.3
P (win | stay)	17.9	84.8	39.5	79.4
P (win | switch)	10.7	81.8	21.1	61.8
Posttest: 6-door MHD				
Optimal behavior	28.6	76.5	39.5	62.9
P (win | stay)	28.6	70.6	43.2	66.7
P (win | switch)	46.4	73.5	18.9	54.5

Table 2: Logistic regression analysis results for variables predicting outcomes on the classic MHD pretest questions.

	β	SE β	OR	95% Wald CI	Wald $\chi^2(1)$
Optimal behavior					
Play	0.37	0.57	1.44	[-0.76; 1.49]	0.40
Explain	-0.75	0.68	0.47	[-2.08; 0.57]	1.24
Play*Explain	0.68	0.89	1.97	[-1.08; 2.43]	0.57
P (win | stay)					
Play	-0.03	0.75	0.97	[-1.51; 1.44]	0.00
Explain	0.09	0.75	1.10	[-1.38; 1.56]	0.02
Play*Explain	-0.58	1.04	0.56	[-2.63; 1.46]	0.31
P (win | switch)					
Play	-0.73	0.90	0.48	[-2.50; 1.04]	0.65
Explain	-0.28	0.95	0.75	[-2.14; 1.57]	0.09
Play*Explain	0.39	1.25	1.48	[-2.05; 2.83]	0.10

Note. β = unstandardized β coefficients; SE = standard error; OR = odds ratio; CI = confidence interval.

Posttest: Classic MHD

For the 3-door MHD in the posttest Table 1 clearly shows that in all experimental conditions participants performed better compared with participants assigned to the control condition. Participants assigned to the 'playing and explanation' condition performed best on the optimal behavioral response question, whereas participants assigned to the 'explanation only' condition performed best on both posterior probability questions.

First, the results of the logistic regression analyses (see Table 3) showed a statistically significant main effect of Explanation on behavioral response, Wald $\chi^2(1) = 6.32$, $p = .012$. The odds ratio equaled 7.61, meaning that it is 7.6 times more probable that a participant indicates switching as the optimal behavioral response when assigned to a condition with explanation compared to a condition without explanation. Second, there were also statistically significant main effects of Explanation on the posterior winning probability when staying and when switching questions, Wald $\chi^2(1) = 10.89$, $p = .001$, OR = 5.91, and Wald $\chi^2(1) = 11.47$, $p = .001$, OR = 6.06, respectively. Those odds ratios mean that it is approximately 6 times more probable that a participant gives a correct posterior winning probability estimation for both staying and switching when assigned to a condition with explanation compared to a condition without explanation. Finally, there was a statistically significant interaction effect on the posterior winning probability when switching question, Wald $\chi^2(1) = 3.87$, $p = .049$, OR = 6.19. To follow up on this interaction effect, post-hoc pairwise comparisons (HSD) revealed that the following comparisons reached statistical significance. Participants in the 'explanation only' condition more often gave the correct answer on the posterior winning probability when switching question compared with participants assigned to the 'playing only' condition, $p < .001$, OR = 11.905, and compared with participants assigned to the control condition, $p < .001$, OR = 3.205. In addition, participants in the 'playing and explanation' condition more often correctly answered the posterior winning probability when switching question compared with participants assigned to the 'playing only' condition, $p < .001$, OR = 5.143, and compared with participants assigned to the control condition, $p < .001$, OR = 1.385.

Posttest: 6-door MHD

For the 6-door MHD variant in the posttest Table 1 demonstrates that especially participants assigned to the 'playing and explanation' condition and the 'explanation only' condition performed better compared with the control condition. Participants assigned to the 'explanation only' condition performed best on all three questions.

The logistic regression analyses (see Table 4) indicated statistically significant main effects of Explanation on behavioral response, Wald $\chi^2(1) = 3.91$, $p = .048$, OR = 2.60, and on the posterior winning probability for switching question, Wald $\chi^2(1) = 8.99$, $p = .003$, OR = 5.14. Thus, it is respectively 2.6 and 5.1 times more probable that a participant indicates switching as the optimal behavioral response and gives the correct answer on the posterior winning probability when switching question when assigned to a condition with explanation compared to a condition without explanation.

Table 3: Logistic regression analysis results for variables predicting outcomes on the classic MHD posttest questions.

	β	SE β	OR	95% Wald CI	Wald $\chi^2(1)$
Optimal behavior					
Play	0.79	0.90	2.20	[-0.98; 2.56]	0.76
Explain	2.03	0.81	7.61	[0.45; 3.61]	6.32^{*}
Play*Explain	1.08	1.06	2.95	[-1.00; 3.16]	1.04
P (win | stay)					
Play	-0.37	0.64	0.69	[-1.64; 0.89]	0.33
Explain	1.78	0.54	5.91	[0.72; 2.83]	10.89^{**}
Play*Explain	1.47	0.88	4.35	[-0.25; 3.19]	2.82
P (win | switch)					
Play	-1.03	0.57	0.36	[-2.15; 0.10]	3.20
Explain	1.80	0.53	6.06	[0.76; 2.84]	11.47^{**}
Play*Explain	1.82	0.93	6.19	[0.01; 3.64]	3.87^{*}

Note. β = unstandardized β coefficients; SE = standard error; OR = odds ratio; CI = confidence interval.
$^{*}p < .05$. $^{**}p < .01$.

Table 4: Logistic regression analysis results for variables predicting outcomes on the 6-door MHD posttest questions.

	β	SE β	OR	95% Wald CI	Wald $\chi^2(1)$
Optimal behavior					
Play	-0.65	0.53	0.52	[-1.70; 0.40]	1.49
Explain	0.95	0.48	2.60	[0.01; 1.90]	3.91^{*}
Play*Explain	1.14	0.76	3.13	[-0.34; 2.62]	2.28
P (win | stay)					
Play	-0.18	0.53	0.83	[-1.22; 0.85]	0.12
Explain	0.97	0.50	2.62	[-0.01; 1.94]	3.78
Play*Explain	0.83	0.75	2.29	[-0.64; 2.30]	1.21
P (win | switch)					
Play	-0.84	0.52	0.43	[-1.86; 0.19]	2.58
Explain	1.64	0.55	5.14	[0.57; 2.71]	8.99^{**}
Play*Explain	-0.47	0.77	0.62	[-1.98; 1.04]	0.38

Note. β = unstandardized β coefficients; SE = standard error; OR = odds ratio; CI = confidence interval.
$^{*}p < .05$. $^{**}p < .01$.

Discussion

Previous research on the MHD demonstrated that although participants' behavioral performance could be enhanced by particular interventions, participants' understanding of the MHD solution did not improve very much (Burns & Wieth, 2003, 2004; Franco-Watkins et al., 2003; Saenen et al., 2014, 2015a, 2015b; Stibel et al., 2009). So far, only DiBattista's (2011) study showed a major increase in participants' understanding of the MHD solution. In his study, participants used an MHD digital learning environment, developed to improve participants' understanding of the problem's solution. The problem with DiBattista's (2011) study, however, is that it could not answer the question *which* (combination of) manipulation(s) of the digital learning environment exactly was most helpful to increase participants' understanding of the MHD solution. This is because his study was not conducted in a controlled environment, the several characteristics of the digital learning environment were not experimentally manipulated, and there was no random assignment of his participants.

With the present study, we aimed to fill (part of) this research gap and to extend DiBattista's (2011) research. To this end, we developed our own digital learning environment – analogous to the one developed by DiBattista (2011) – in which it was possible to limit participants' access to particular parts of the learning environment so that it would be possible to conduct controlled randomized experiments and next, to infer causal relations. The current paper reports on the first experiment we carried out, in which we focused on two out of the three major parts of the digital learning environment: repeated experience with the MHD and explanation about the MHD solution.

First, the results of our experiment are consistent with previous research on the MHD. At the pretest measure, participants in all conditions massively failed to indicate the optimal behavioral response (see Burns & Wieth, 2004; Friedman, 1998; Granberg, 1999; Granberg & Brown, 1995; Granberg & Dorr, 1998) and to give correct posterior winning probability estimates (see Burns & Wieth, 2003, 2004; Franco-Watkins et al., 2003; Saenen et al., 2014, 2015a, 2015b; Stibel et al., 2009). Next, the results of our posttest measure showed that when participants completed a series of MHD trials without receiving further explanation about the MHD solution (i.e., 'playing only' condition), their behavioral response improved, but the majority of participants still did not grasp the underlying posterior probabilities of the problem (see Franco-Watkins et al., 2003; Saenen et al., 2015b).

Second and most important, our study showed which specific manipulation helped participants most in understanding the MHD solution. The results provide strong evidence for the effect of receiving explanation about the MHD solution. Interestingly, being able to experience multiple MHD trials – besides having access to explanation about the MHD solution – did not further increase participants' MHD understanding nor did it affect their understanding in a negative way. This finding is of practical importance for (posterior) probability education. Although experience-based learning may occur in many areas, it appears that repeated experience with the MHD is not enough to help participants reflect about and understand the solution. Explanation about the MHD solution, however, which parallels much more the traditional learning environment (cf. teacher controlled), seems to be more appropriate for teaching students the difficult concept of posterior probabilities.

However, this conclusion should be considered very carefully for several reasons. First, general implications about the use of digital learning environments for (posterior) probability learning are hard to make given the narrow nature of our study and research paradigm. Second, there is a crucial limitation in both DiBattista's (2011) and our own performed study. More specifically, the operationalization of understanding the MHD solution may have been inadequate in both studies. In DiBattista's (2011) study, participants' explanation for why they indicated a particular behavioral response as the optimal one were interpreted as (no) understanding of the MHD solution. Which criteria were used to interpret participants' explanation as either correct or incorrect, is however unclear. Therefore, we operationalized understanding the MHD solution as being able to give correct posterior probability estimates. However, these do not necessarily reflect understanding: Participants might give correct probability judgments accidentally by guessing. Furthermore, the 6-door MHD variant – which we included in order to be able to compare our results with the results of DiBattista (2011) – has the same underlying posterior probabilities as the classic MHD. Therefore, it is impossible to determine whether participants' – who were assigned to a condition in which they received explanation about the solution – correct posterior probability estimates were the result of understanding the MHD solution, or only showed that they had copied the probabilities they just had been reading but still did not understand these posterior probabilities and the problem's solution. Future research could clarify this by including MHD variants in the posttest questionnaire with other optimal behavioral responses (e.g., staying) and other underlying posterior probabilities.

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