M3-Situating Embodied Learning: Embedding Gestures in Narratives to learn Mathematical FrActions in a digital tablet environment

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Abstract

Researchers developed Iteration-1 of a digital tablet tutor-game exploring the impact of *narratives* (strong (*S*) vs. weak (*W*)) and *gestural mechanics* (conceptual (*C*) vs. deictic (*D*)) on players' understanding of mathematical fractions. In a two-by-two factorial design, 3rd, 4th, and 5th grade elementary students at an afterschool program in Harlem, NYC (N_{TTL}=72; $\bar{x}_{AGE}$=10.31 years [1.64], 67% female) were randomly assigned to play one of the four tutor-game environments (*SC, SD, WC, WD*). Pre/post scores on formal fractions assessments showed significant learning for all groups. Tutor-log data revealed that students using conceptual gestures were significantly more accurate at estimating and denominating fractions than students using deictic gestures. Observational notes, student exit surveys and clinical interviews corroborated that many students used the tutors' gestures in their explanations of fractions. This collection of data is used to discuss the impact of gesture and narrative on learning fractions and digital-tutor design.

Keywords: mathematics; fractions; embodiment; gestures; situated; narrative; ludology; gaming; design-based research; DBR; data-mining; digital; tablet; tutor.

Introduction

How can cognitive scientist turn ideas into learning opportunities on digital tablets? With many of the tenets of contemporary education emphasizing learning in our experiential cohorts (Dewey, 1938/1963), society (Vygotsky, 1934/1978) and culture (Bruner, 1966), digital tablet simulations are opportunities to situate learners in an environment (Lave, 1988) and utilize the gestural interface as a means for grounding cognition (Barsalou, 2008). Moreover, combining familiar experiences and augmenting them with fantastical ones can motivate learners to explore unknown problems spaces and allow them to consider the choices in their own learning (Cordova & Lepper, 1996; Schwartz & Arena, 2009).

For years, a vast selection of educational titles amounted to little more than interactive worksheets, drills or quizzes, without leveraging cognitive principles and superficially inserting content onto an arbitrary narrative with arbitrary game mechanics that fail to motivate users much less help them learn mathematics (Riconscente, 2011). Fortunately, newer games-like tutors supported by research like *Motion Math* (motionmathgames.com) and *Math Glow* (igeneration.com) are designing activities that leverage the gestural interface of digital tablets.

In the current study, we developed a tutor-game for a digital tablet to investigate how *gestures* and *narratives* impact performance and learning of mathematical fractions. First, will embodying the procedural actions for fracturing objects using different gestures, hence "FrActions", impact students' explicit, implicit and tacit learning and knowledge (Broaders, Cook, Mitchell & Goldin-Meadow, 2007)? Second, can a strong narrative (based on the Emmy award winning PBS Television series, *Cyberchase*) engage learners by helping contextualize the problem space (Graesser, Hauft-Smith, Cohen, & Pyles, 1980) better than a sparsely weak narrative?

Of course, capturing learning is not just about performance. Soderstrom & Bjork (2015) recently addressed how performance assessments often fail to capture student learning. Education, as an applied field, can only capturing student learning if it is sensitive to the context in which it was learned (Brown, 1992). If we take learning as "co-constituted" with the environment (Barab and Squire, 2004), then evaluations of the students, who sit, headphones on, playing a tutor-game on a digital tablet, must come from multiple sources of data. Thus, the current study integrated formal assessments, tutor-data logs, observations, exit surveys and clinical interviews to evaluate 4 versions of a digital tablet math tutor-game: Mobile Movement Mathematics: Iteration 1 (*M3:i1*).

Theoretical Background

Tutor-games provide learners with dynamic experiences that channel their visual, aural and haptic perceptions into their cognitions (Baddeley, 1986; Ricker, AuBuschon & Cowan, 2010). Digital tablets are portals that educators can use to situate learning in contexts for connecting concepts (Barab et al., 2007; Brown, Collins & Duguid, 1989; Lave, 1988; Schwartz & Bransford, 1999) and the gesturally haptic interface is an opportunity to *embody* their conceptual development (Barsalou, 1999; Glenberg, 1999; Lakoff & Johnson, 1980). The affordances (Gibson, 1977) of a tablet's digital ecology enable researchers to craft experiences that have yet their own affordances, giving students freedom to explore while their learning is guided by scaffolding and feedback (Dewey, 1938/1963).

Developing Fractions Tutor-Game (Iteration-1)

Fractions begin with the actions of fracturing. Mathematical thinking is grounded in real-world actions (Lakoff & Núñez, 2001) like sharing an apple. The abilities to search, find, pick the good one, and split it in two equal parts are the roots of humans' *Number Sense* (Dahaene, 1997). They include natural abilities to estimate the magnitude of an object, apportion it and compare it –all skills essential for fracturing. **Figure 1** illustrates the grounded metaphorical schemas of mathematical thinking that contour the situatively embodied curriculum for *M3:i1*.

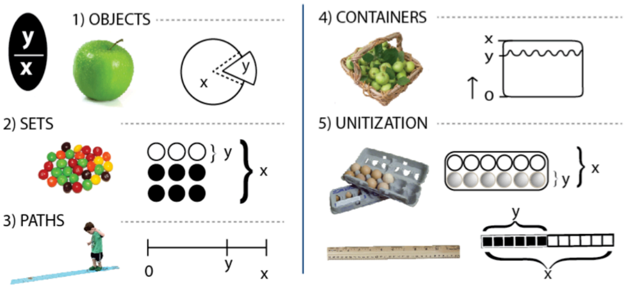

Figure 1. Embodied Experiences of Mathematical Fractions

The tutor-game consists of 5 levels with 5 fractions each. In Part 1, players use gestures to estimate, denominate and numerate fractions by manipulating an enerchi bar (**Fig. 2**, left), a hybrid of a rectangular area model and a number line (Siegler et al., 2010). In Part 2, players determine equivalence between the fractions by ordering them from least to greatest along a horizontal axis, left to right, then verifying their height vertically from bottom to top (**Fig. 2**).

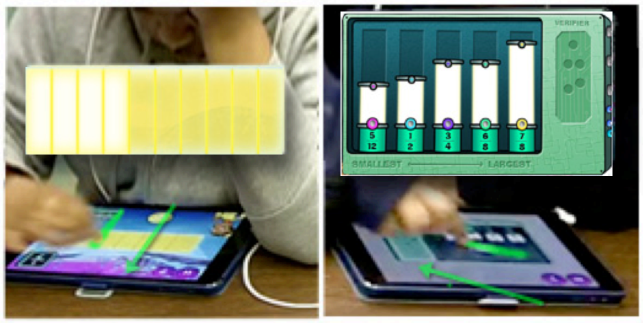

Figure 2. Assets and Gameplay for *Part 1- Object Fracturing "Enerchi Bar"* (Left) and *Part 2 - Ordering Fractions* (5 Enerchi Bars) (Right)

Developing Gestural Mechanics. Gestures are integral components of communication across languages and cultures and species. They represent a robust a means for educators to help learners reactivate (simulate) the perceptual states associated with underlying concepts and any strategies that elucidate understanding (Goldin-Meadow, 1999). For example, Goldin-Meadow, Cook and Mitchell (2009) demonstrated that a pairing gesture (i.e., two fingers to identify two numbers as a pairing) facilitated elementary students strategies for arithmetic problems and demonstrates how gestures as abstractions are still rooted in relation to the body. Alibali and Nathan (2012) documented gestures representing structure, orientation, action and correspondence. For *M3:i1*, we hope the tactile gestural interface of the tablet bridges action and conceptualization.

The gestures used in the current study come from Swart et al., (2014) and are based on McNeills' (1992) taxonomy. Echoing Hostetter's and Alibali's (2008) *Gestures as Simulated Action*, the tutor compares *deictic gestures* (i.e., pointing) that index the environment, to *conceptual gestures* (metaphorical / enactive / symbolic) that embody simulated actions for fractions (**Fig. 3**). Habgood and Ainsworth (2011) reported better learning when there was an intrinsic link between learning content and a game's mechanics. We hypothesize that conceptual gesturers that embody the process of fractions will show better performance and learning than deictic gestures.

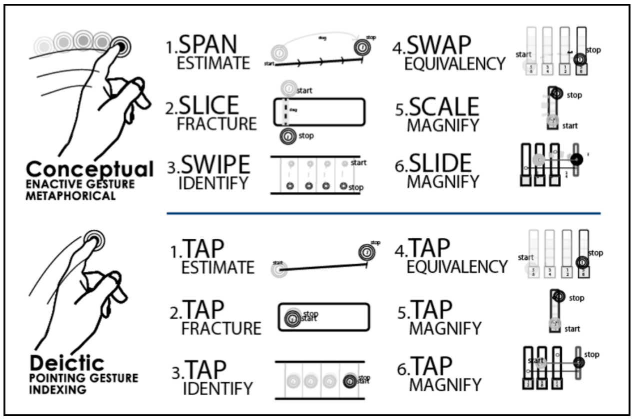

Figure 3. Conceptual Gestures and Deictic Gestures.

Developing Narrative. Developing an effective narrative invests the audience in the continuity of the characters, locations, objects, actions and themes and invests them into the plot's trajectory (Graesser, Singer & Trabasso, 1994). The integration between the microstructure (details) and the macrostructure (abstractions) is especially important when building an interactive narrative. If the details are points of entry to the concepts, then designers must situate players in problem spaces that foster mental model constructions (Johnson-Laird, 1980). Narrative has been shown to help learners formulate coherent scripts into schemas and chunk them into coherent mental models (Black, Turner & Bower, 1979). The players' investment in the narrative will hopefully motivate exploration of the problem space and encourage practicing the procedures for creating and comparing fractions along with opportunities for discovery (Brown, Collins & Duguid, 1989).

Figure 4 shows the two narratives for comparison: (1) a *strong* narrative based on the television series Cyberchase, *Fix the Climatron*, in which we embark with Jackie to Penguia to activate the HERObots by fracturing engerchi bars and defeating the villain Hacker, or (2) *weak* narrative, *Fractioneers!*, the same tutor-game but without characters, settings, story or explicit context. We characterize it as "weak" in lieu of "no" narrative to account for researchers

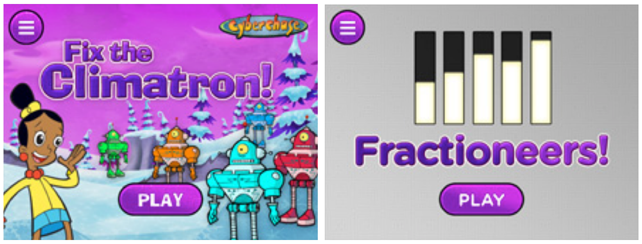

Figure 4. *Strong* (Left) and *Weak* (Right) Narratives.

inability to control for students who might devise their own internal narrative for the tutor-game. We hypothesize that a strong narrative will improve student performance by engaging students motivations and contextualizing the process of fractioning better than a weak narrative.

Study 1 (*M3:i1*)

We devised, designed and developed the gestures, narratives, curriculum, assets, instructions, feedback, and scaffolding for the tutor-game. Four versions of Iteration-1 vary along two dimensions: (1) gestures (*C*onceptual vs. *D*eictic) and (2) narrative (*S*trong vs. *W*eak), resulting in the four conditions *SC*, *SD*, *WC*, and *WD*. Our research goal is not just to determine if gesture and or narrative improve learning, but "why" and "how" they do.

Methods

Participants.

Seventy-two students from grades 3 (n = 24), 4 (n=22) & 5 (n=26) grades (N_{TTL}=72; $\bar{x}_{AGE}$=10.31 years [1.64], 67% female) at an afterschool program in Harlem, New York City obtained parental consent and assented to participate in the program.

Procedure.

Researchers formally tested a total of 72 students in a specially designated classroom proctored by researchers and after-school instructors. In a 2x2 factorial with repeated measures, students were randomly assigned to play one of the 4 tutor environments (*SC*, n=17), (*SD*, n=18), (*WC* n=19), (*WD*, n=18). Each student completed a total of 3 one-hour sessions that included pre-tests, tutor play, post-tests and exit surveys. Students were run in groups of 20 over the course of 3/4 days in a week (5 students per condition) and the program extended over multiple weeks. Portions of tutor play were video recorded, as were students' clinical interviews.

Materials

Direct Pre/Post Test: Parallel Forms A & B (12 items; Cronbach's α = .93) of fraction problems directly from the tutor-game curriculum. Representations of fractions were similar images used in the tutor, and activities included estimation, denomination, numeration and determining equivalence between fractions.

Transfer Pre/Post Test: Parallel Forms A & B (30 items; Cronbach's α = .91) of general fraction assessment that included problems using objects, collections of objects, number lines, numerical fractions in standard contexts, arithmetic, and word problems. Questions included items asking students to estimate, denominate, numerate and determine equivalence between fractions as well as perform arithmetic with fractions.

Exit-Survey: A written form administered to participants upon completion of tutor play and testing. Items included manipulation checks (narratives, gestures), comprehension check, self-efficacy, motivation, engagement, persistence, opinion/preferences and concept learning (15 items; 9 likert; 6 free response)

Clinical-Interview Script: Four multipart questions designed to probe players thoughts on the tutor, the narrative, impact of gesture and concept learning (4 items)

iPad Air & Sony MDR-ZX100 Headphones: Ten sets.

Flip Video UltraHD Camcorder: 2 camcorders w/ Tripods for video recording tutor play and clinical interviews.

Results

Formal Assessments.

Initial pre-tests revealed no significant differences between groups on any either the direct or transfer assessments. Repeated measures ANOVA revealed that the tutor-game overall is effective at improving learners understanding of fractions with significant learning gains across all conditions for both the direct assessment ($F_{(1,71)}$ = 48.9, p<. 001, η_p^2 = .408 as well as the transfer assessment ($F_{(1,71)}$ = 57.51, p<. 001, η_p^2 = .448). Moreover, a significant positive correlation between the *direct content* and *transfer* assessments (r = .774, n=38, p< .01) show a strong relationship between the tutor content and more general fractions concepts and principles.

Tutor-Game Data Log.

Tutor-game log data revealed significant effects of gesture on tutor play. However, it is important to disclose that the data sets suffered from an imbalance (i.e., number of students per condition) due to a back-end programming error that resulted in a stratified truncation of the data-log. Thus, analysis of the following data sets was done using the non-parametric tests *Mann-Whitney U* and *Wilcoxon-W*[1] to determine significant difference between independent groups.

Estimation. *Estimation error* was lower for conceptual gestures than for deictic users across strong and weak narrative. Means for groups C and D were 23.04 and 24.1; the distributions in the two groups differed significantly (Mann–Whitney U = 248, Wilcoxon W= 477, n_C =26, n_D = 20, p < 0.08). A look at the raw means (**Fig. 5**) also reveals

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[1] Means for *MW-U* and *W-W* statistical tests are transformations of the original means which additionally provided in figures

a possible interaction between gestures and narrative requiring further study.

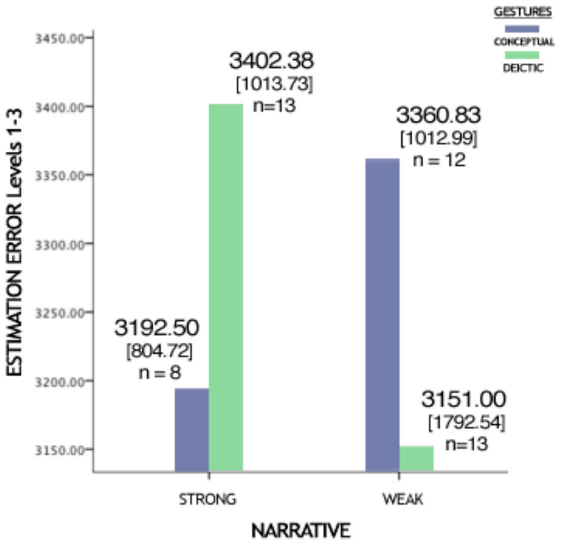

Figure 5. Raw Estimation Error: Levels 1-3

For unit fractions, *estimation errors* were lower for conceptual gestures than deictic gestures and approaching significance, $\bar{x}_C = 23.04$ and $\bar{x}_D = 24.1$, Mann–Whitney U = 231, Wilcoxon W= 462, $n_C = 21$, $n_D = 29$, $p < 0.15$. **Figure 6** shows a similar trend towards interaction between narrative and gesture.

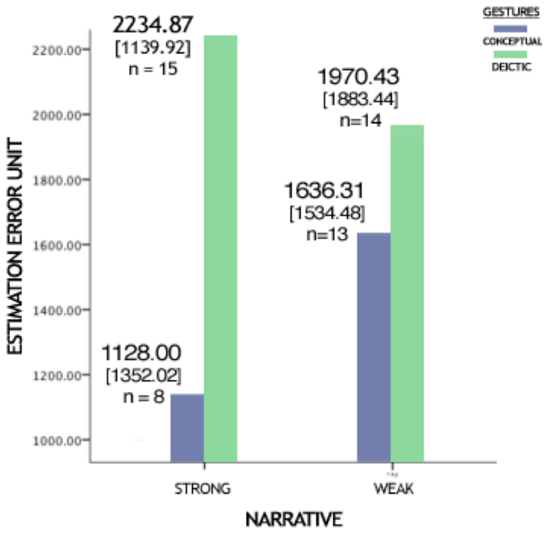

Figure 6. Raw Estimation Error: Unit Fraction

Figure 7 shows *error rates for estimation* (averaged by group) across the entire curriculum (Levels 1 – 5) spike for the fractions (3/3) and (10/10) across the curriculum. Error rates were also higher for fractions (4/5) and (7/8). Looking at trends in the data help pinpoint aspects of the tutor worth further investigation. The increased error rate on these problems could signify content difficulty, usability or conceptual development.

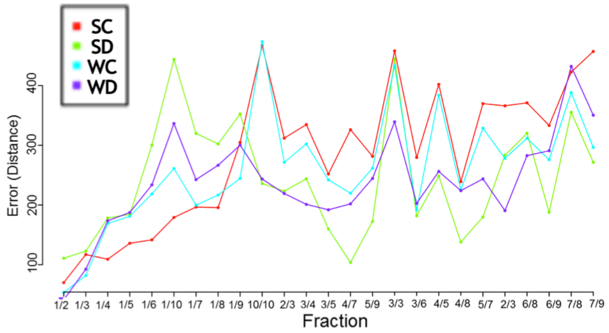

Figure 7. Estimation Error across entire curriculum.

Denomination. Student performances denominating wholes into parts were significantly more accurate for conceptual gestures than for deictic gestures. For levels 1 – 3, students using conceptual gestures denominated (i.e., correct number of divisions) with significantly less error than students using deictic gestures $\bar{x}_C = 18.66$ and $\bar{x}_D = 25.24$, Mann–Whitney U = 164.5, Wilcoxon W= 345.5, $n_C = 19$, $n_D = 25$, $p < 0.10$.

Figure 8 graphs the number of denominations students made in error (i.e., 3 slices of the bar, 4 parts, for a denominator of 3). Visible is the recurring trend for a significant interaction between gesture and narrative. Students were also significantly more accurate denominating unit fractions using conceptual gestures than deictic gestures; $\bar{x}_C = 17.95$ and $\bar{x}_D = 30.97$, Mann–Whitney U = 146, Wilcoxon W= 377, $n_C = 21$, $n_D = 29$, $p < 0.01$.

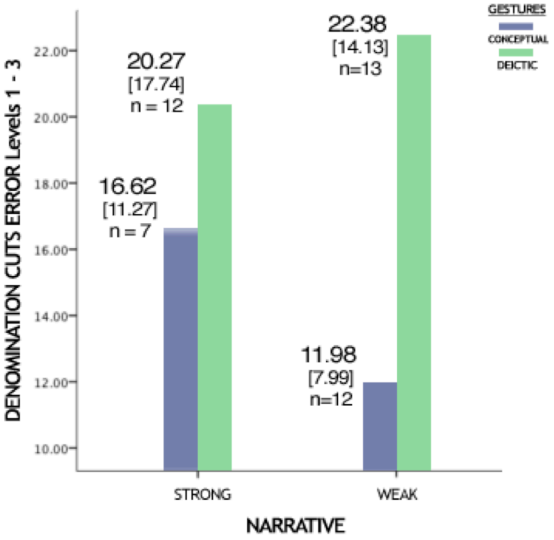

Figure 8. Raw Denomination Cuts: Levels 1-3

Denomination Scaffolding. After 3 attempts to denominate the bar into equal parts, students received automated scaffolding that provided dashed-lines for them to either slice (conceptual) or tap (deictic). For levels 1-3, students using conceptual gestures required significantly less scaffolding (i.e., the tutor-game presented an interactive solution to the user after 3 unsuccessful attempts at denomination) than students using deictic gestures; means

$\bar{x}_C = 18.71$ and $\bar{x}_D = 30.97$, Mann–Whitney U = 165.5, Wilcoxon W= 355.5, $n_C = 19$, $n_D = 25$, $p<0.09$.

Similarly, for unit fractions, students used less scaffolding (i.e., the tutor-game presented an interactive solution to the user after 3 unsuccessful attempts at denomination) than students using deictic gestures; means for groups $\bar{x}_C = 21.76$ and $\bar{x}_D = 28.21$; Mann–Whitney U = 226, Wilcoxon W= 457, $n_C = 21$, $n_D = 29$, $p < 0.117$.

Numeration. There were no significant differences between conditions numerating the fractions.

Interview Data.

Motivation Survey Likert Items. 5-point likert scale items found strong indications that students across all conditions were highly motivated to play ($\bar{x}_M = 4.62$ [.72], enjoyed playing ($\bar{x}_E = 4.59$ [.67]) and would persist in playing more levels ($\bar{x}_P = 4.62$ [.70]). Student's indicated that they liked learning math on the iPad ($\bar{x}_{LM} = 4.44$ [1.00]) even though they found the game moderately difficult ($\bar{x}_D = 3.79$ [1.11]) with a significant medium sized correlation ($r = .442$, N=71, $p < .01$). Also interesting is the significant moderate correlation between students' self-efficacy judgments ($\bar{x}_{SF} = 3.90$ [.94]) and difficulty ($r = .422$, N =71, $p < .01$).

Clinical Interviews. Clinical interviews found students gesturing during their explanations of fractions. **Figures 9-11** are comparing gestures made by students from conditions *SC*, *SD*, *WD*. Each figure overlays multiple frames from the video recordings and adds photo-illustrations in green highlighting the paths and directions of the gestures.

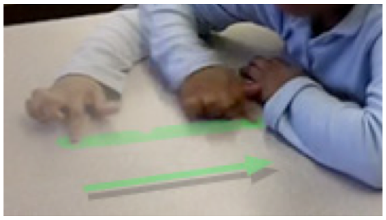

Figure 9. Student *SC* divides by swiping down repeatedly.

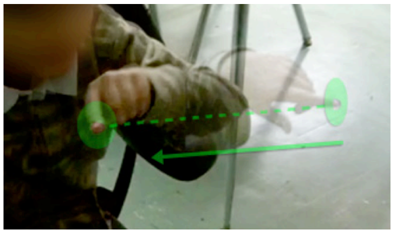

Figure 10. Student *WD* divides from left to right with successive points.

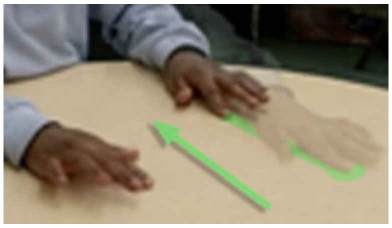

Figure 11. Student *SD* slices down explaining how to denominate (2/5).

In **Figure 9**, *Student SC* explains how to get the denominator of a fraction by splitting it with an enactive metaphorical gesture while the *Student WD* in **Figure 10** uses a deictic gesture to divide the fraction. However, **Figure 11** shows *Student SD* using conceptual gestures to enact the process of dividing a fraction despite having used deictic gestures in the tutor-game. While students in the deictic condition used both deictic and conceptual gestures in their explanations, this crossover was not symmetrical, that is, no students from the conceptual gesture condition employed any deictic gestures in their explanations of fractions. We discuss how these findings might qualify the empirical evidence that supports the usage of conceptual gestures for learning.

Discussion

The empirical evidence supports our hypothesis that *conceptual* gestures (Segal, Tversky, Black, 2014) help students perform better at estimating and denominating fractions than *deictic* gestures. This is not surprising since both estimation and denomination conceptually are grounded in the continuous processes of measurement (Siegler, Thompson and Schneider, 2011) and the deictic gestures are both static and more abstract. The larger question is whether or not students' accurate performances are actually transforming into to a more robust conceptual understanding?

Although *deictic* gestures can metaphorically represent both the indexing of an object or the enacting of a procedure (Kang, Tversky & Black, 2015), students from the deictic condition consistently employed both deictic and conceptual gestures in their explanations. If we take their gestures as indicators of their implicit knowledge or mental models of fractioning (Broaders, Cook, Mitchell & Goldin-Meadow, 2007), their interviews suggest that the natural conceptualizations of the fracturing process is an embodied one, hence students produce gestures that are enactive and metaphorical. For cognitive scientist interested in learning, this supports the existing research demonstrating the influence and efficacy of using gesture in pedagogy (Goldin-Meadow, et al., 2009; Alibali et al, 2012).

For narrative, empirical evidence revealed no significant impact of adding settings, characters or plot to the game-tutor. That few students integrated any elements of the strong narrative into their survey responses or clinical interview explanations of fractions suggests a few possibilities: (1) the narrative was not strong enough to invest users, (2) it could be a seductive detail (Harp & Mayer, 1998), (3) it may interact with gesture and users such that its effectiveness differentiates between users and conditions and requires more study. Nonetheless, the possibility also exists that situating the tasks for constructing fractions into a scaffolded interactive digital environment with feedback was equally impactful for all students (Lesh, 1985; Barab et al., 2007), thus the digital tablet itself outweighed any further impact of contextualizing fractions into a narrative arc.

In *M3:i2*, researchers hope to re-design the narrative, gestures and scaffolds in order to address these issues and clarify their impacts on conceptualization of fractions.

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