Process Modeling of Qualitative Decision Under Uncertainty

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Abstract

Fuzzy-trace theory assumes that decision-makers process qualitative "gist" representations and quantitative "verbatim" representations in parallel. Here, we develop a formal model of fuzzy-trace theory that explains both processes. The model also integrates effects of individual differences in numeracy, metacognitive monitoring and editing, and sensation seeking. Parameters of the model varied in theoretically meaningful ways with differences in numeracy, monitoring, and sensation seeking, accounting for risk preferences at multiple levels. Relations to current theories and potential extensions are discussed.

Keywords: Decision making; need for cognition; risky choice; framing effect; Allais paradox

Fuzzy-Trace Theory

Risk preferences are fundamental to psychological and economic theory, and to decision neuroscience. We propose a model of risk preferences that integrates theoretical principles relevant to mental representations with individual differences in metacognitive monitoring and risk-taking propensity. Our model is based on fuzzy-trace theory, an account of decision-making under risk, which posits that decision-makers use qualitative, categorical, "gist" representations of the meaning of decision information, in parallel with precise, eidetic or metric, "verbatim" representations of the exact words and numbers in that information (for an overview, see Reyna, 2012). By "mental representation," we mean the manner in which a stimulus is encoded into a subject's memory. Decision makers operate on these representations of the stimulus rather than on the stimulus itself. Specifically, a gist representation captures the basic *meaning*, or "essence," of a stimulus. Furthermore, fuzzy-trace theory posits a hierarchy of gist that is, in the domain of numbers, analogous to scales of measurement (Reyna, 2008; Stevens, 1946). We approximate this hierarchy with three levels of representation: categorical, ordinal, and interval, described below.

The outline of this paper is as follows: first we present a motivating example. Next, we describe our novel formalization, accounting for factors that vary with individual-differences. Finally, we test our model's parsimony and predictions.

Example

Fuzzy-trace theory is motivated by the insight that one's representation of a decision problem can drive decision outcomes. For example, consider a choice between:

1. winning $180 for sure; versus
2. a .90 chance of winning $250 and .10 chance of no money.

One might represent this decision as a simple choice between the following two options:

1. Possibility of some money
2. Possibility of some money and possibility of no money.

Given this representation, most decision makers would favor option 1 because it promises some money without the chance of no money. Alternatively, one could instead represent the choice as follows:

1. More chance of winning less money
2. Less chance of winning more money and possibility of winning no money.

This representation, although more precise, does not allow for a clear decision to be made because most people would prefer winning more money to winning less money, but they would also prefer more chance of winning to less chance of winning. Finally, one may choose a precise representation of the problem whereby one calculates the expected value of each option by multiplying its respective outcomes by their probabilities, as follows:

1. Expected value of $180 (i.e., $180 * 1)
2. Expected value of $225 (i.e., $250 * 0.90 + $0 * 0.10)

This representation seems to favor option 2, since it promises more money on average. Overall, this example illustrates our approach to categorical gist representation.

The Decision Space

We represent the complements in these options as points in a 2-dimensional space, representing all possible combinations of amounts of money (or, generally, some outcome) and probability that a decision-maker could encounter:

Categorical representation of decision space.

The gist representation of the choice above is between:

1. Possibility of some money
2. Possibility of some money and possibility of no money.

These gists are represented in a 2-dimensional space. All points in this space are interpreted according to the part of the diagram in which they are located. These gist representations can overlap. For example, a point that falls into the part of the space marked as "possibility of no money" also falls into the part marked as "possibility of some money." Thus, multiple gist representations are possible for some points.

The extended fuzzy-processing preference.

We model the relationship between categories as a hierarchy in which higher elements are preferred interpretations when compared to lower elements (Broniatowski & Reyna, 2014). Each decision complement is a point in our space. If we determine each complement's gist representation, and then locate that gist in the associated hierarchy, the model stipulates how that decision complement is interpreted.

Mapping problem information to categorical mental representation. We assume that the categorical distinction between "some" of a quantity and "none" of a quantity is primitive. Thus, our model assumes that all points are mapped to one of these two categories. Throughout this paper, we assume that stimuli map to either "some" or "none" in both the domains of outcomes chance for scientific parsimony.

Formalizing the extended fuzzy-processing preference. We introduced the extended fuzzy-processing preference to enable us to differentiate between overlapping gists. According to the extended fuzzy-processing preference, subjects will prefer to interpret a decision option as within the subcategory containing the fewest points– i.e., the highest element within the associated hierarchy. We formalize the extended version of the fuzzy processing preference by specifying that one always prefers the interpretation associated with the category in the decision space with the lowest dimension (similar to Feldman's, 1997, "maximum codimension rule"). In other words, given a point in our space and a set of possible gist interpretations for that point, a subject will always prefer the interpretation that is highest in the associated hierarchy.

Importantly, a preferred interpretation, or mental representation, is not always the same as a preferred outcome. For example, a decision-maker may prefer to interpret of ".10 chance of no money" as "possibility of no money" when compared to "possibility of some money." Given a choice between two options, interpreted respectively as "possibility that of some money" and "possibility of no money," most decision-makers would choose the former option. Thus, once mental representations are chosen, we must define a preference ordering over the decision options with these interpretations.

Values map mental representations to preference. Decision makers choose between options based on which has the high-valued affect. The affect assigned to a given option is a function of how that option is represented. For example, "no money is possible" is a preferred interpretation for the point ($0, .10, a .10 chance of no money) when compared to "some money is possible." However, a prospect that is interpreted as "some money is possible" has a high valence when compared to one that is interpreted as "no money is possible." Thus, a decision-maker would choose the option with the high valence. In order to formalize this prediction, we again use a partial order – i.e., every pair of elements within the category hierarchy may be less than, greater than, equal to, or unrelated to one another in the domain of values. Full mathematical details of this partial order are presented in the paper by Broniatowski & Reyna (2014).

Mapping problem information to ordinal mental representations. Fuzzy-trace theory predicts that decision-makers use ordinal (e.g., "more" vs. "less") in parallel with categorical and interval representations. When mapping problem information to ordinal mental representation, "more" is always in the direction away from zero and "less" is always in the direction toward zero. Importantly, points may only be compared at the ordinal level if they exist within a common category. For example, one may compare "no money with .10 chance" to "$180 for sure" because both may be represented as "possibility if some money" (even if this is not the preferred interpretation for either option, it is an admissible interpretation for both). Since 0 is less than 180 and .10 is less than certainty, the corresponding ordinal representations are "less_1 money with less_2 chance" and "more_1 money with more_2 chance."

Mapping ordinal mental representations to preference. Ordinal decision-making assumes that each dimension in the decision space has a preferred direction. When comparing two decision options, if the ordinal representation of one option is preferred along all dimensions of the decision space, and is strictly preferred along at least one dimension, then that decision option is preferred overall. For example, "more money with more chance" is preferred when compared to "less money with less chance." Otherwise, a decision cannot be made and the ordinal representation is indifferent, such as when "more_1 money with less_2 chance" is compared to "less_1 money with more_2 chance".

Formalizing How Each Representation Chooses Among Decision Options

A "gist hierarchy" is a set of mental representations ranging in precision from a categorical gist representation to an interval verbatim representation, and sets of rules for making decisions that are unique to each of these representations. At the categorical level, each point is represented according to the extended fuzzy-processing preference. At the ordinal level, a point is chosen if it is weakly preferred along all dimensions and strongly preferred along at least one such dimension. Points in disjoint categories cannot be compared. At the interval level, decisions options are evaluated according to their expected values (i.e., the sum of the value of each outcome multiplied by its probability) – the simplest interval representation (i.e., it assumes no decision weights).

Combining Information Across Representations

We address conflicts between representations in our model by assuming that each representation casts a "vote" for its preferred decision options. For example, given a choice between two decision options, each of the categorical, ordinal, and interval representations "votes" (-1 for the first option, +1 for the second option, or 0 if indifferent) for a preferred option according to its own particular representational logic. A sum across these votes (i.e., a weighted sum, as explained below) determines the final decision. We chose summation because it is the simplest combination rule for this sort of aggregation.

An Error Theory for Risky Decision Problems

We represent error using a standard multinomial logistic distribution. For decisions with two options, effect size typically follows a standard logistic distribution. (In principle, our model could be extended to decisions with multiple options using a multinomial logistic distribution.) For our specific application, we model the probability, P, that a subject will choose a given decision outcome in a risky choice gamble by:

$$P(\vec{x}) = \frac{1}{1 + e^{-(\vec{a}\cdot\vec{x}+b)}}.$$

Here, $\vec{x}$ is a three-element vector containing an entry for each representation (categorical, ordinal, and interval), $\vec{a}$ is a three-element vector containing an entry for each corresponding decision weight. We also introduce a factor, b, capturing the risk-taking propensity of a given set of subjects. Thus, we account for conflict between representations by adding weighted votes from each representation.

Factors affecting the decision weight vector. In the domain of decision making, two major individual difference factors associated with metacognitive monitoring and editing have been proposed – numeracy (e.g., Peters et al., 2006; Liberali et al., 2012) and Need for Cognition (NFC; Cacioppo, et al.,1996; Stanovich & West, 2008).

People who are higher in numeracy and/or NFC are more likely to spontaneously convert and compare alternative "framings" of a problem, reducing cognitive biases.

Numeracy. Peters and colleagues (2006) defined numeracy as "the ability to process basic probability and numerical concepts" and found that more numerate subjects were less susceptible to attribute framing effects. In addition, Schley and Peters (2014) found that more numerate individuals treated numbers as more linear when making a risky decision, suggesting that they rely less on categorical gist and more on interval (linear) representations of probabilities and outcomes.

Table 1: Evaluation of Model Fit.

Model	Like-lihood	AIC	BIC
Null	-7491	14982	14987
Saturated	-6570	13491	14049
Analytic Categories	-6672	13409	13510
Single average value for a and b	-6826	13676	13715

Note. AIC = Akaike Information Criterion; BIC = Bayesian Information Criterion

Need for cognition. Prior work suggests that subjects reconcile answers to oppositely framed versions of the same problem when both frames are presented within-subjects, when subjects respond to multiple presentations of the same problem, or when they are exposed to obviously factorial design manipulations within subjects. Kahneman and Frederick (2002) have argued that such designs can lead subjects to focus on the variables that are being manipulated, and to compare different versions of the same underlying problem instead of treating each independently. Thus, the magnitude of framing effects varies systematically with experimental design (e.g., Stanovich & West, 2008).

The tendency to reconcile responses to different versions (or related problems) when they are presented within-subjects is greater for those higher in NFC. Subjects with high NFC tend to edit their choices more than those with low NFC, presumably because they are more likely to notice the common structures underlying these problems (i.e., high NFC subjects display "analytic override;" e.g., LeBoeuf & Shafir, 2003; Stanovich & West, 2008). Furthermore, numeracy and NFC are separate sources of individual differences that are not correlated (Peters et al., 2006; Liberali et al., 2012).

We model the effects of numeracy and NFC using the decision weight vector $\vec{a}$. Furthermore, if we make the simplifying assumption that all of these decision weights are equal, we may replace $\vec{a}$ by a scalar factor, a, which captures the "strength" of a given set of votes. When a is high, subjects will strongly favor one option over another, unless different representations conflict with one another. When a is low, subjects will tend towards indifference.

Risk-taking propensity. In addition, our model incorporates personality differences associated with risk-taking (e.g., Caspi et al., 1997), including factors related to cross-cultural differences (e.g., Du et al., 2002) and sensation seeking or reward-related approach (e.g., Zuckerman, 2007). We represent this in our model by a linear additive risk preference, b, which, when positive, is used to indicate a predisposition toward the higher rewards available in the more risky option in a gamble. The linear additive nature of this factor is based on evidence presented by Reyna and colleagues (2011) who found evidence supporting additive effects (i.e., additive beyond verbatim and gist processing) of subjects' sensation seeking on risk taking.

A worked example. Consider the decision between a certain gain of \$180 and a 0.90 chance of winning \$250 and a 0.10 chance of no money discussed above. Recall that the categorical representation prefers the certain option (-1), the ordinal representation is indifferent (0), and the interval representation prefers the risky gamble (1).

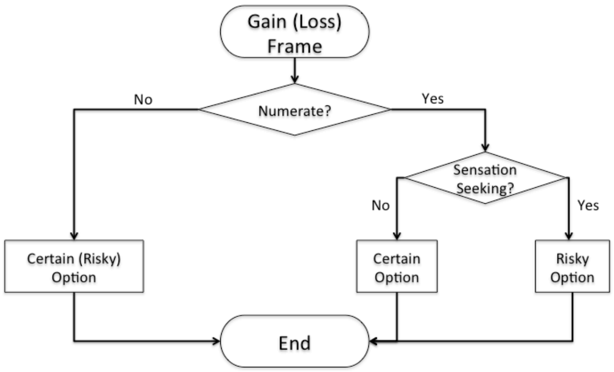

Figure 1: Process Flowchart for Between-Subjects Condition, No Replications, Individual Representation

Thus, $\vec{x}$ = [-1,0,1] in our model. For an experiment that is conducted with frame manipulated between subjects, $\vec{a}$ = [1,1,1]. Thus, $\vec{a} \cdot \vec{x}$ = -1 + 0 + 1 = 0 (indicating that the categorical and interval representations compete). Finally, suppose we estimate our sample's risk propensity from prior data to be b = 0.25, indicating a slight preference for risk. Then, the probability that a randomly chosen subject from our sample will choose the risky gamble option is P= $1/(1+e^{-0.25})$ = 56%, and the probability that a randomly chosen subject will choose the certain option is 44%.

Assessing The Goodness of Fit of our Model

To demonstrate the scientific parsimony of this model, we have adapted a technique used by Busemeyer, Wang, & Shiffrin (2015), wherein we compare our model's fit to a "null" model (in which each decision option is equally likely), a "saturated" model (in which maximum-likelihood parameter values are separately estimated for each replication in our sample), and a model that estimates parameters based upon theoretically-motivated categories: namely mathematical ability (PISA scores) and experimental design (for the a parameter), and stimulus type, nationality, and age (for the b parameter), with parameter values estimated separately within each category using a jackknife estimator. Specifically, given a model, y, the log likelihood function for each experimental replication in our sample of 88 is:

$$\ln[L(y_i)] = n_{1,1}\ln(p_{1,1}) + n_{1,2}\ln(p_{1,2}) + n_{2,1}\ln(p_{2,1}) + n_{2,2}\ln(p_{2,2})$$

where $n_{1,1}$ is the number of people choosing the first decision option in the first problem, $p_{1,1}$ is the predicted proportion of subjects choosing the first decision option in the first problem, given model y, etc., and

$$\ln[L(y)] = \sum_i \ln[L(y_i)]$$

is the total log-likelihood of model y. Given ln[L(y)], we may calculate the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) as follows:

$$AIC = 2k\text{-}2\ln[L(y)]$$

where k is the total number of distinct values for a, b, and x in a given model (e.g., the total number of analytic categories). Similarly,

$$BIC = k\ln(n) - 2\ln[L(y)]$$

where n is the total number of data points in our sample (i.e., 176 data points for 88 pairs of problems, listed in the paper by Broniatowski & Reyna, 2015). Table 1 shows the log-likelihood, AIC, and BIC values for the models that we tested (we tested other models as well, but none surpassed the model with analytic categories). The model containing analytic categories for a and b has the lowest values of both AIC and BIC, and only the saturated (i.e., overfit) model has a higher log likelihood.

Individual Differences Analysis

Estes and Maddox (2005) point out that aggregate data analyses may lead to different conclusions from those reached through individual-level analyses. This concern does not apply to data derived from between-subjects designs, on the other hand, we may use the results of within-subjects designs to perform process level tests of our model. Specifically, we examined three replications of a framing problem for which framing was manipulated within-subjects and individual level frequency data were reported (Frisch, 1993; Stanovich & West, 1998; LeBoeuf & Shafir, 2003). We extracted the number of subjects that were consistent between frames (either choosing the certain gamble or the risk option in both frames), the number of subjects who displayed a framing effect (choosing the certain option in the gain frame and the risky gamble in the loss frame), and the number of subjects who displayed reverse framing (the risky gamble in the gain frame and the certain option in the loss frame. These data were used to test the following process-level accounts of our model.

Between-subjects framing. Figure 1 shows a process-level account of a framing problem for which frame was manipulated between subjects with a single presentation. Here, a subject is exposed to a stimulus containing a given frame. If they are not numerate, they choose the decision option consistent with that frame. If they are numerate, and they are not sensation seeking, they choose the certain option. Otherwise, they choose the risky option. Thus, an individual who chooses the risky option in the gain frame would be numerate and sensation seeking according to our model. Furthermore, we may perform aggregate-level analysis of between-subjects data, which enables us to replace individual choices with probabilities. Specifically, the probability that a subject randomly chosen from our sample would pick the risky option in the gain frame is $p_{gain} = p_{numerate} * p_{ss}$. Similarly, the probability that a subject would pick the certain option in the loss frame is $p_{loss} = (1 - p_{numerate}) + p_{numerate} * p_{ss}$. Furthermore, our model stipulates that $p_{gain} = \frac{1}{1 + e^{-(-a+b)}}$ and

Table 2: 3 Sampled Experimental Replications of Individuals' Decisions When Frame is Manipulated Within-Subjects

Reference	% High NFC			Empirical			Predicted			χ^2	p
	N	MLE	JK	C	F	RF	C	F	RF		
				$\vec{x}$ = [±1,0,0], a=0.89, b=0.22							
Frisch (1993)	99	48	59	63	29	7	71	23	6	2	0.29
Stanovich & West (1998)	292	56	57	202	73	17	205	69	18	0.3	0.87
LeBoeuf & Shafir (2003)	287	61	54	206	60	21	194	74	19	4	0.16

Note. C = Consistent – same decision in both frames; F = Framing – certain option in gain frame & risky gamble in loss frame; RF = Reverse Framing – certain option in loss frame & risky gamble in gain frame; MLE = Maximum Likelihood Estimate; JK = Jackknife estimator generated by averaging across all MLE values except the one corresponding to a given problem

$p_{loss} = \frac{1}{1+e^{-(a+b)}}$. Finally, we may estimate the values of a and b using the by averaging across the maximum likelihood estimates of a and b for all problems with the exception of the problem whose parameters we are trying to estimate. Thus, we have a system of two equations with two unknowns, such that

$$p_{numerate} = 1 + \frac{1}{1+e^{-(-a+b)}} - \frac{1}{1+e^{-(a+b)}} \quad \&$$

$$p_{ss} = p_{numerate} * (1 + e^{b-a})$$

Within-subjects framing. Figure 2 shows a process-level account of a framing problem for which frame was manipulated within subjects.

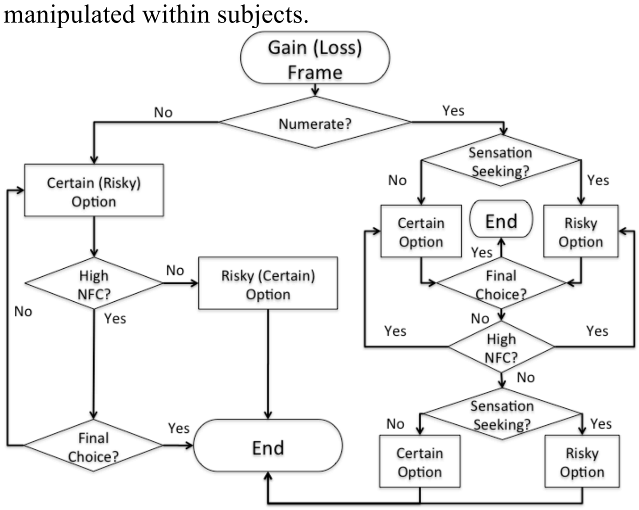

Figure 2: Process Flowchart for Within-Subjects Condition, Individual Representation

Here, subjects are exposed to a stimulus containing a given frame in a manner analogous to a between-subjects framing problem: If they are not numerate, they choose the decision option consistent with that frame. If they are numerate, and they are not sensation seeking, they choose the certain option. Otherwise, they choose the risky option. The individual then makes a second choice on an oppositely-framed version of the same framing problem (if they first saw the loss frame, they now see the gain frame and vice versa). Individuals with high NFC will recognize the similarity of the two problems and remain consistent across frames (LeBoeuf & Shafir, 2003) whereas individuals with low NFC will treat the second problem as if it were independent of the first. As in the between-subjects case, we may replace individual choices with probabilities. This formulation enables us to estimate the proportions of subjects that will be consistent, exhibit framing, or exhibit reverse framing for a given sample where frame is manipulated within-subjects. Specifically, the proportion of subjects in a given sample that is consistent across frames is given by $p_{NFC} + p_{ss}{}^2(1 - p_{NFC})p_{numerate}$, the proportion of subjects that exhibiting framing behavior is given by

$$(1 - p_{NFC})(1 - p_{numerate}) +$$
$$(1 - p_{ss})p_{ss}(1 - p_{NFC})p_{numerate}$$

and the proportion of subjects exhibiting reverse framing is:

$$\mathbf{(1 - p_{ss})p_{ss}(1 - p_{NFC})p_{numerate}}$$

Process-level tests of our model. We compare our model's predictions to individual level behavior (i.e., sequential choices) by estimating values of $p_{numeracy}$, p_{NFC} and p_{ss} for a given experimental sample using the equations above. Specifically, we compare our model's predictions to three replications of a framing problem presented within-subjects that are reported in the literature (Frisch, 1993; LeBoeuf & Shafir, 2003; Stanovich & West, 1998). Specifically, we estimated the baseline proportion of subjects who are numerate from the problems in our sample for which framing was manipulated between-subjects. Specifically, we estimated the value of $\hat{a}$ as the average of maximum-likelihood a values for low-PISA between-subjects framing problems (since all within-subjects problems are also from low-PISA countries) weighted by the total number of subjects (recall that the first choice made in a within-subjects design is analogous to a between-subjects framing problem), excluding the data from Stanovich and West (1998) and LeBoeuf and Shafir (2003) from our average in order to avoid *post hoc* estimation (Busemeyer & Wang, 2000), yielding a value of $\hat{a}$ = 0.89. Similarly, we estimate the value of $\hat{b}$ as the weighted average of all MLE b values of the ADP when presented to comparable (i.e., non-Chinese, who are known to be more risk-taking; Du et al, 2002) college student samples, again excluding the data from Stanovich and West (1998) and LeBoeuf and Shafir (2003), yielding a value of $\hat{b}$=0.22. By extension, $p_{numeracy}$=59% and p_{ss}=58%. Finally, we calculated p_{NFC} to compare the number of subjects choosing consistency, framing, or reverse framing with our model's predictions. The value of p_{NFC} was chosen to minimize the sum of squares between the predicted and actual numbers of subjects within each category. Finally, we averaged across all values of p_{NFC} excluding the value associated with that particular problem (i.e., using a jackknife estimator). Table 2 shows that the data do not differ significantly from our model's predictions in any of these three experiments.

Conclusions

This model is the first to explicitly formalize the key concepts of gist, the gist hierarchy, and qualitative decision-making. Previously, (Broniatowski & Reyna, 2014; 2015) we introduced the mathematical underpinning and error theories underlying our model. Here, we demonstrated that this model is both scientifically parsimonious and robust. The structure of our model also enables it to outperform leading theoretical alternatives, such as Cumulative Prospect Theory (CPT; Tversky & Kahneman, 1992), which make modal, and not precise, predictions. Thus, our formalized theory explains a wide variety of phenomena, integrating known effects and novel predictions.

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