Noisy Parameters in Risky Choice

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Abstract

We examine the effect of variability in model parameters on the predictions of expected utility theory and cumulative prospect theory, two of the most influential choice models in decision making research. We find that zero-mean and symmetrically distributed noise in the underlying parameters of these models can systematically distort choice probabilities, leading to false conclusions. Likewise, differences in choice proportions across decision makers might be due to differences in the amount of noise affecting underlying parameters rather than to differences in actual parameter values. Our results suggest that care and caution are needed when trying to infer the underlying preferences of decision makers, or the effects of psychological, biological, economic, and demographic variables on these preferences.

Keywords: Decision making; Random utility; Random preference; Risky choice; Prospect theory

Introduction

Research on risky choice has relied heavily on the use of deterministic models. Perhaps the two most widely used models today are expected utility theory (EUT) (von Neumann & Morgenstern, 1947) and cumulative prospect theory (CPT) (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992). When their functional forms are specified and parameterized, deterministic models can make precise quantitative predictions. However, such models fail to capture an important aspect of choice behavior: namely, that choice is stochastic, and decision makers may respond differently when given exactly the same choice problem on more than one occasion within a short space of time (see Rieskamp et al., 2006 for a discussion).

Modelling stochastic risky choice requires a representation of each individual's preferences as probability distributions. From an early stage in the development of the literature, there were two ways in which this type of modelling was accomplished. One approach involved adding some 'error' specification to each individual's deterministic 'core' preferences (e.g. Luce, 1959). We shall refer to this approach as Fechnerian – a broad term which covers a number of ways in which some form of 'add-on' term might be specified. Another approach allowed the parameters of an individual's preference function to vary from one moment to another, thereby opening up the possibility that if the same choice were presented at different times in the course of an experiment, it might be resolved differently on each occasion (Becker et al. 1963). We shall refer to this way of modelling noise as the random preference or random parameter (RP) approach.

To the extent that preferences can be seen as being constructed, influenced either by contextual and environmental cues, imperfect recall, or momentary fluctuation of attention, it might seem that the RP approach is conceptually more appropriate, as it permits variability in preference through variability in the parameters of the individual's preference functions. However, people may also vary in the way they record their decisions, due to varying degrees of complexity of the stimuli, varying degrees of motivation or engagement, 'interference' from previous decisions within the session or from some other unobservable features of the stimuli or task. Since these factors are largely outside the core structure of people's preferences, they may be better modelled by some Fechnerian noise term that operates in addition to, but independently of, the intrinsic variability in preference (see Birnbaum, 2011, for a discussion).

So it seems plausible that the observed variability in individuals' repeated choices may arise from a number of sources which are not mutually exclusive. Despite this, the great majority of studies inferring preference functions from choice data use specifications which operate as if some form of Fechnerian noise is the only stochastic component of the decision process: very few consider the possibility of variability in terms of interactions between an RP core and Fechnerian factors. Yet if observed choices actually entail multiple sources of variability, there may be serious consequences for theoretical inference if we try to force data into a conceptually inadequate specification.

Risky Decision Models

We can write the gambles in the choices studied in this paper as $X = (x_1, p_1; x_2, p_2)$, so that X can be seen as offering rewards x_1 and x_2 with probabilities p_1 and p_2. Both EUT and CPT describe decision makers' preferences between pairs of such gambles using utility functions. Using a simple power value function formulation for the utility of any payoff x, the utility of a particular gamble under EUT is given as: $U(X|\alpha) = p_1 \cdot x_1^{\alpha} + p_2 \cdot x_2^{\alpha}$. The utility for X according to the power value function formulation of CPT, with Prelec's one-parameter probability weighting function, is similarly: $U(X|\alpha,\gamma) = \pi(p_1) \cdot x_1^{\alpha} + (1- \pi(p_1)) \cdot x_2^{\alpha}$ where $x_1 \geq x_2 \geq 0$ and $\pi(p_1) = e^{-(-\ln p_1)^{\gamma}}$.

α captures the shape of the subjective value function for payoffs, with $\alpha < 1$ describing concave value functions that correspond to risk averse preferences under EUT, and $\alpha > 1$ describing convex value functions that correspond to risk seeking. γ captures the shape of the probability weighting function, with $\gamma < 1$ generating an overweighting (underweighting) of small (large) probabilities, and $\gamma > 1$ generating the opposite. Typically, decision makers behave as if they have both $\alpha < 1$ and $\gamma < 1$ when the above gambles are explicitly presented to them.

In binary choice, these models predict that X is chosen over Y whenever $U(X|\alpha,\gamma) > U(Y|\alpha,\gamma)$. As these models are deterministic, they need to be modified in order to capture probabilistic choice data. The most common approach to doing this has been to assume that the utilities for the two gambles are each subject to Fechnerian noise ε, with $E[\varepsilon] = 0$, so that X is chosen over Y whenever $[U(X|\alpha,\gamma) + \varepsilon_X] - [U(Y|\alpha,\gamma) + \varepsilon_Y] > 0$. Supposing further that ε_X and ε_Y are independent of each other, we can define $\varepsilon = \varepsilon_X - \varepsilon_Y$. Then the probability of choosing X is the probability that $U(X|\alpha,\gamma) - U(Y|\alpha,\gamma) + \varepsilon > 0$. When ε is distributed according to the Gumbel distribution, this leads to the logit choice rule specified by Luce (1959): $\Pr[X \text{ chosen}] = f(\theta \cdot U(X|\alpha,\gamma) - \theta \cdot U(Y|\alpha,\gamma))$, where f is the logistic function. This error specification, when applied by itself, predicts that the modal choice will always be the option with the higher utility according to the deterministic core.

Now let us consider separately the effect of noise in the decision maker's parameters, as represented in the RP approach. With a simple additive formulation for parameter noise, the above equations can be rewritten with $\alpha = \alpha^* + \eta_\alpha$ and $\gamma = \gamma^* + \eta_\gamma$, where η_α and η_γ are noise terms with $E[\eta_\alpha] = E[\eta_\gamma] = 0$ (Becker et al., 1963; Loomes & Sugden, 1995). η_α and η_γ are liable to vary from trial to trial, and thus α and γ, and subsequently $U(X|\alpha,\gamma)$ and $U(Y|\alpha,\gamma)$, also tend to vary from trial to trial. However, since the expected values of α and γ are $E[\alpha] = \alpha^*$ and $E[\gamma] = \gamma^*$, α^* and γ^* characterize the central tendency of a decision maker's underlying preferences. If variability came only from sources represented by noisy parameters, the probability of X being chosen would be determined by the proportion of cases where $U(X|\alpha,\gamma) - U(Y|\alpha,\gamma) > 0$ over the range of feasible values of η_α and η_γ, weighted by their probabilities.

However, we now suppose that RP variability is combined with other sources of variability captured by a Fechnerian specification of the kind proposed by Luce (1959), we have, for each α and γ pair: $\Pr[X \text{ chosen}] = \Pr[U(X|\alpha,\gamma) - U(Y|\alpha,\gamma) + \varepsilon > 0]$. Now on those occasions where the α and γ drawn from the distributions of parameters are such that $U(X|\alpha,\gamma) - U(Y|\alpha,\gamma)$ is small, there is a relatively high chance (though still less than 0.5, of course) that Fechnerian variability will result in choosing the option with the lower $U(.)$, whereas there is a smaller chance of that happening when RP variability produces a larger $U(X|\alpha,\gamma) - U(Y|\alpha,\gamma)$ difference.

A combination of Fechnerian and RP specifications offer a more adequate account of choice data involving dominated gambles than each of these approaches alone. Models with only standard Fechnerian noise predict much higher frequencies of violations of transparent dominance than are generally observed, while RP-only models predict that dominance is never violated at all, contrary to the evidence (Loomes, 2005; see also Busemeyer & Townsend, 1993). Scholars have also found that allowing for both types of noise is necessary to provide a good quantitative account of behavior (Blavatskyy & Pogrebna, 2010; Loomes, 2005).

Effects of Noisy Parameters

Fechnerian noise and random parameters are necessary to characterize probabilistic choice. Yet despite this, much empirical decision research neglects the effect of RP when deriving predictions from deterministic models. Such neglect is no doubt due to convenience. Luce's choice rule has an analytical representation, which greatly facilitates model fits and related analyses.

This neglect may also reflect the intuition that unsystematic variability in parameter values, independent of Fechnerian noise, has no systematic effect on choice, so that modal choices can be used to make qualitative inferences about how underlying preferences rank the available options, regardless of the randomness in underlying parameters. This paper tests this intuition and finds that it is incorrect. Unlike the Fechnerian noise term ε, the parameter noise terms η_α and η_γ affect utility non-linearly. Even if they have a zero mean and are symmetric, changing their variance can alter both absolute choice probabilities and the ordering of relative choice probabilities. Thus it is possible that we observe X being chosen more frequently than Y, with $\Pr[X \text{ chosen}] = \Pr[U(X|\alpha,\gamma) - U(Y|\alpha,\gamma) + \varepsilon > 0] > 0.5$, but the central tendency of the decision maker's underlying preferences more frequently favours Y over X, with $U(Y|\alpha^*,\gamma^*) > U(X|\alpha^*,\gamma^*)$ and $\Pr[U(Y|\alpha,\gamma) > U(X|\alpha,\gamma)] > 0.5$.

Risk Attitudes

As an illustration of this, let us now consider the choice between a risky gamble X offering a 50% chance of obtaining \$10 and a 50% chance of obtaining \$0, and its safe expected value equivalent Y offering \$5 with certainty. We assume that a decision maker's central tendency is described by the power form of EUT and that his choices display both Fechnerian and RP noise. Keeping things simple, we suppose that the Fechnerian noise is modelled via a Luce choice function (with $\theta = 1$) while the RP component involves $\alpha^* = 0.9$ with η_α being distributed uniformly in the interval [-0.5, 0.5]. Supposing $\alpha^* < 1$ implies that the decision maker's underlying preferences more often than not entail risk aversion. However, when each of the possible realizations of $[U(X|\alpha,\gamma), U(Y|\alpha,\gamma)]$ are embedded in the Luce formulation, the above assumptions give $\Pr[X \text{ chosen}] = 0.53 > 0.5$. Thus, despite underlying preferences predominantly supporting Y, the decision maker chooses X more frequently than Y.

This happens due to the nonlinearity of utility difference in α. In the Luce choice rule assumed above, the probability of choosing X is an increasing function of $f(\alpha) = U(X|\alpha,1) - U(Y|\alpha,1) = 0.5 \cdot 10^\alpha - 5^\alpha$. f is convex in α for the range of α we are considering. This implies that the expected value of f is greater than f applied to the expected value of α, i.e. $E[U(X|\alpha,1) - U(Y|\alpha,1)] > U(X|\alpha^*,1) - U(Y|\ \alpha^*,1)$. In this case we obtain $E[U(X|\alpha,1) - U(Y|\alpha,1)] > 0$, resulting in a higher choice probability of X. This is despite $E[\alpha] = \alpha^* < 1$, and that subsequently $U(X|\alpha^*,1) - U(Y|\alpha^*,1) < 0$.

The point is expanded upon in Figure 1, where we plot the probability of choosing X = (\$10, 0.5; \$0, 0.5) over Y = (\$5, 1) according to power function EUT with only Fechnerian noise (using a Luce rule with θ = 1) and compare that with a specification where RP (with η_α distributed uniformly in the interval [-0.5,0.5]) interacts with Fechnerian noise. The Fechnerian-only model entails Pr[X chosen] less than, equal to or greater than 0.5 according to whether α^* is less than, equal to or greater than 1, as shown by the solid line in Figure 1. However, this is not the case when α is noisy. As shown by the broken line in Figure 1, there are a range of values of α^* between 0.87 and 1 where Pr[X chosen] > 0.5. Over this range, the decision maker's expected modal choice suggests risk seeking whereas the central tendency of his preferences, as represented by α^*, represents risk aversion or risk neutrality.

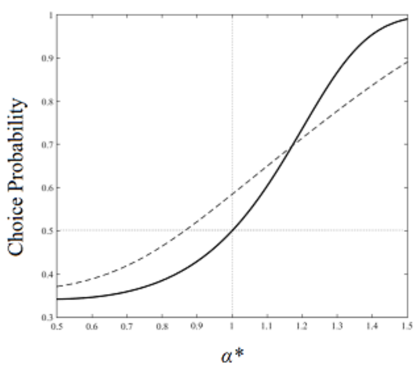


Figure 1. The probability of choosing a risky gamble X over its expected value Y for varying values of α^*, plotted with only Fechnerian noise (solid line) and with both Fechnerian and RP noise (dashed line). Here we can observe a higher choice probability of X over Y for some values of $\alpha^* < 1$ in the presence of RP noise.

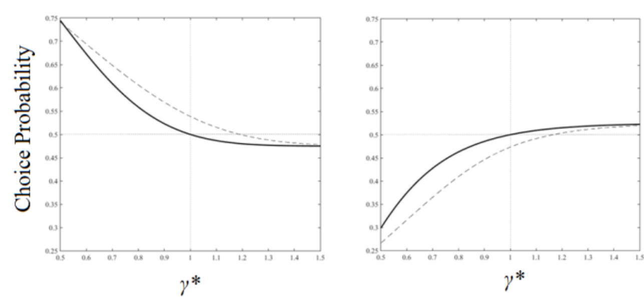
The important point of this illustration is to show that when there is both Fechnerian and RP noise, we cannot make reliable inferences about the decision maker's risk attitude using only modal choice frequencies, even if we assume independence between the different sources of noise. Moreover, as we show in the next two sections, the co-existence of Fechnerian and RP noise in conjunction with EUT core preferences can generate patterns of choice that have been interpreted as providing support for non-EU models such as CPT. In related work (not reported here) we also illustrate the pernicious effect of RP noise on parameter recovery and quantitative model fitting.

Probability Weighting: The Four-Fold Pattern

We now turn to cases where probabilities may be transformed nonlinearly as with CPT using the single parameter Prelec formulation given earlier. When $\gamma < 1$, this function overweights low probabilities and underweights high probabilities. Typically the crossover point is around 0.37, with $\pi(p) > p$ for $p < 0.37$ and $\pi(p) < p$ for $p > 0.37$. Such an 'inverse-S' function is crucial in capturing the famous four-fold pattern of risky choice (Tversky & Kahneman, 1992).

In the positive domain considered here, this pattern corresponds to modal choices favouring a risky gamble that offers a high payoff with a small probability relative to a sure option with the same expected value (which looks like risk-seeking), while at the same time generating modal choices favouring a sure option over a risky gamble with the same expected value that offers a large probability of a slightly higher payoff and a small probability of a considerably lower payoff (which looks like risk aversion). Thus in the choice between a risky gamble X^I offering a 1% chance of obtaining \$10 and a 99% chance of obtaining \$0, and its safe expected value equivalent Y^I offering \$0.10 with certainty, decision makers typically choose X^I. In contrast, in the choice between a risky gamble X^{II} offering a 99% chance of obtaining \$10 and a 1% chance of obtaining \$0, and its safe expected value equivalent Y^{II} offering \$9.90 with certainty, decision makers typically choose Y^{II}.

This pattern cannot be generated by deterministic EUT or EUT with only standard Fechnerian noise. But let us consider a setting with both Fechnerian and RP noise. For simplicity, we fix α = 1 so the utility function is linear, and we allow RP noise only in the γ parameter, with η_γ being distributed uniformly in the interval [-0.5, 0.5]. For Fechnerian noise, we use the Luce function with θ = 1, as in the previous section.



Figures 2a and 2b. The probability of choosing a low-probability risky gamble X^I over its expected value Y^I (left) and the probability of choosing a high-probability risky gamble X^{II} over its expected value Y^{II} (right), for varying values of γ^*. These figures are plotted with only Fechnerian noise (solid line) and with both Fechnerian and RP noise (dashed line).

Figure 2a shows the probability of choosing X^I over Y^I and Figure 2b shows the probability of choosing X^{II} over Y^{II}. As expected, a model with Luce noise but without noisy parameters and with γ^* = 1 entails for both pairs a 0.5 chance of choosing each option. For all $\gamma^* < 1$, the risky option is the modal choice in Figure 2a while the sure amount is the modal choice in Figure 2b. However, when γ itself exhibits symmetric noise and when this source of variability interacts with Fechnerian noise, the effect – as shown by the broken line – is to shift the transformation path up in Figure 2a and down in Figure 2b: that is, the interaction increases the choice probability of X^I over Y^I and for Y^{II} over X^{II} for all $\gamma*$ considered. One implication of this is that at the point where $\alpha = 1$ and $\gamma* = 1$ – that is, in the case where the deterministic model entails a risk neutral expected utility maximizer – the modal choices exhibit the 'mixed attitude to risk' typical of the data and widely viewed as supportive of CPT with $\gamma < 1$. Indeed, there is a range of $\gamma*$ between 1 and 1.15 for which the decision maker's expected modal choice generates a preference for X^I over Y^I and for Y^{II} over X^{II}, a behavioral pattern associated with the overweighting of small probabilities, whereas the central tendency of his preferences represents an underweighting of small probabilities.

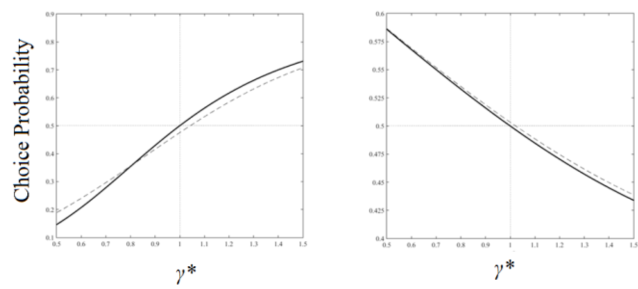
Probability Weighting: Common Ratio Effect

The probability weighting biases assumed by Prospect Theory are also necessary for it to account for the common-ratio effect (Kahneman & Tversky, 1979). The classic common-ratio case involves choices between two pairs of lotteries. One pair offers a gamble $X^{III} = (x, p; 0, 1\text{-}p)$ versus $Y^{III} = (y, 1)$ where p is typically around 0.8 and where y is equal to, or a little below, the expected value of X^{III}. In the example we consider in this section, our scaled-up pair is a choice between a risky gamble X^{III} offering an 80% chance of obtaining $10 and a 20% chance of obtaining $0, and its safe expected value equivalent Y^{III} offering $8 with certainty. Here decision makers typically choose Y^{III}.

The second pair involves scaling down the probabilities of the positive payoffs in the first pair by some factor and correspondingly increasing the probabilities of 0 in both options to give a choice between $X^{IV} = (x, \lambda p; 0, 1\text{-}\lambda p)$ and $Y^{IV} = (y, \lambda; 0, 1\text{-}\lambda)$. Scaling down X^{III} and Y^{III} by a typical factor – letting $\lambda = 0.25$ – gives X^{IV} offering a 20% chance of obtaining $10 and an 80% chance of obtaining $0 versus its relatively safe expected value equivalent Y^{IV}, a 25% chance of obtaining $8 and a 75% chance of obtaining $0. In such scaled-down pairs, decision makers typically choose X^{IV} much more frequently: indeed, it is quite common for a sample of experimental participants to make Y^{III} the modal choice in the first pair but make X^{IV} the modal choice in the second pair. This is inconsistent with EUT, which assumes that individuals' preferences are linear in probabilities. Thus, scaling down the probabilities in this way reduces the EU of each option but leaves the ordering unchanged: a risk averse individual will prefer the *Y* option in both pairs, while a risk seeking individual will consistently choose the *X* options. In a deterministic world of EU maximizers, whatever proportion of the sample chooses X^{III} in the scaled-up pair should also choose X^{IV} in the scaled-down pair.

As with the four-fold pattern described above, the observed change in modal choices can be accommodated by CPT with $\gamma < 1$, leading to the overweighting of low probabilities and the underweighting of high probabilities. This is illustrated by the solid lines in Figures 3a and 3b in which we fix $\alpha = 1$, assume a Luce noise term with $\theta = 1$, and let γ range between 0.5 and 1.5. Here a model with Fechnerian noise but without RP noise and with $\gamma^* < 1$, entails that Y^{III} is the modal choice in Figure 3a while X^{IV} is the modal choice in Figure 3b.

But now suppose we allow both Fechnerian noise and RP noise to co-exist and interact. As above, to allow for noise in the γ parameter, we suppose η_γ is distributed uniformly in the interval [-0.5, 0.5]. Under these assumptions, we have a shift in the choice probabilities so that the interaction increases the choice probability of Y^{III} over X^{III} and of X^{IV} over Y^{IV} for $\gamma*$ in the neighbourhood of 1. Again, one implication of this is that at the point where $\alpha = 1$ and $\gamma* = 1$ – that is, in the case where the deterministic model entails a risk neutral expected utility maximizer – the modal choices exhibit the reversal in choice probability observed in behavioral experiments, with a preference for Y^{III} in the scaled-up pair, but a preference for X^{IV} in the scaled-down pair. Thus we see that even though the central tendency values of the parameters entail not just EUT but expected value maximization, modal choices display a mixture of risk aversion and risk seeking in a manner resembling CPT with $\gamma < 1$. Indeed, as above, there is a range of $\gamma*$ between 1 and 1.02 for which the central tendency of the decision maker's underlying preferences, as represented by $\gamma*$, represents an underweighting of small probabilities, whereas the behavioral pattern generated by these $\gamma*$ is commonly associated with the overweighting of small probabilities.



Figures 3a and 3b. The probability of choosing a scaled-up risky gamble X^{III} over its expected value Y^{III} (left) and the probability of choosing a scaled-down risky gamble X^{IV} over its expected value Y^{VI} (right), for varying values of γ^*. These probabilities are plotted with only Fechnerian noise (solid line) and with both Fechnerian and RP noise (dashed line).

Discussion

Differences between Decision Makers

We have seen that it is unsafe to infer an individual's underlying preferences from modal choice patterns, if we expect both RP noise and Fechnerian noise to play a role in the choice process. What is true for individuals may also be true for studies which draw conclusions about differences in preferences between different groups of people based either on differences in choice proportions or else on the basis of 'representative agent' assumptions. Between them, the disciplines of psychology, neuroscience, marketing, and economics have produced a large number of studies examining the relationship between risk preference and a wide variety of demographic, social, biological, cognitive, emotional and neural variables. Much of this work makes the implicit or explicit assumption that differences in choice probabilities between different groups reflect differences in underlying utility/value functions and/or probability weighting preferences.

For example, based on choice proportions, men are considered to be more risk seeking than women (Charness & Gneezy, 2012); Chinese are considered more risk seeking than Americans (Hsee & Weber, 1999); the nucleus accumbens is seen as influencing risk seeking choices whereas the anterior insula is seen as influencing riskless choices (Kuhnen & Knutson, 2005); high incentives are associated with more risk aversion than low incentives (Holt & Laury, 2002); and decision makers under high time pressure are seen as being more risk averse than decision makers under low time pressure (Zur & Breznitz, 1981). Likewise stress is seen as affecting the amount of probability weighting in gains and losses (Porcelli & Delgado, 2009), the degree of striatal activity is assumed to influence the overweighting of small probabilities (Hsu et al., 2009), framing the decision as involving precaution is assumed to lead to the overweighting of small and medium-sized probabilities (Kusev et al., 2009), and decision feedback is considered to lead to objective probability weighting (Jessup et al., 2008). Finally, based on one of the most striking findings in contemporary decision making research, it is often assumed that decision makers tend to weigh probabilities differently when gamble payoffs and probabilities are *described* compared to when these payoffs and probabilities are *experienced* (Hertwig, 2015).

However, as we have shown, differences in choice proportions may be due not to differences in central tendency parameter values but rather to differences in the amount of variability in those underlying parameters. To illustrate, let us return to Figure 1. The horizontal axis in that Figure represents varying values of α^* under EUT and the vertical axis represents the choice probability for the gamble X corresponding with those different values of α^*. The two lines reflect varying levels of parameter noise in the model. Suppose that a male decision maker chooses X with frequency 0.52 while a female decision maker chooses X with frequency 0.48. If the degree of parameter noise were the same for both individuals, such a difference could reasonably be attributed to differences in underlying α^*, with the male having an increased propensity for risk seeking. But if the male's underlying preferences involve more parameter noise (the dotted line) than the female's (the solid line), then the opposite would be the case: the male's α^* would be approximately 0.9 as compared with the female's α^* of about 0.95. The same holds for inferences regarding probability weighting, as in Figure 2a and 2b.

It might be argued that there is no reason to suppose that males' parameter values are noisier than females' values. But until comparisons of choice frequencies were made, there was no strong *a priori* reason to suppose that gender systematically affected risk preferences. Why should it be preferences that are affected by gender (or age or time pressure, etc.) rather than the variability in parameter noise? Of course, this is not to imply that the conclusions drawn in the above papers are necessarily wrong. Nonetheless, many of those conclusions rely critically on the claim that it is preferences that are driving observed behavioral differences. To our knowledge none of this work explicitly considers the possibility that the changes in choice proportions observed across decision makers may be attributed to noise rather than to underlying preference. Indeed, in some of the settings considered above, a change to the amount of noise displayed by decision makers might be a more compelling explanation for observed behavioral differences, as compared with a change in the underlying parameter values.

Other Domains

Our analysis has focused upon binary risky choice, a domain in which within-person variability of choice has been widely observed. However, the potential confounding effects of interactions between sources of noise are not limited to risky choice: such effects are liable to distort inferences regarding underlying preferences in all non-linear utility-based models.

Consider, for example, the exponential discounting model (Frederick et al., 2002), which is commonly used to model choices between rewards occurring at differing periods of time. One of the most frequent criticisms of this model is that it cannot account for an increased preference for a proximate reward over a delayed reward as the lengths of the delay diminish by some common amount. For example a decision maker may prefer \$10 in five weeks to \$5 in four weeks, but also prefer \$5 immediately to \$10 in one week. This is typically attributed to hyperbolic discounting and present-bias. However, the findings in this paper suggest it may be possible to explain these reversals using only Fechnerian and RP noise without assumptions of present bias. If this is the case then it means that observed differences in these types of decisions that have been ascribed to demographic, biological, neural, cognitive, emotion, social, and task-based factors, may not necessarily reflect the impact of those factors on discount rates but might to some extent reflect differences in the effects of noise. Similar results may also hold for multi-attribute choice, altruistic choice, and strategic decision making.

Beyond Deterministic Models

Models such as expected utility theory and cumulative prospect theory are deterministic, and their predictions depend heavily on experimenter assumptions regarding their stochastic specifications. Many researchers have already argued that it is as important to choose the right type of noise as it is to choose the right core model, in order to accurately fit choice data (see e.g. Loomes, 2005). It has even been shown that the relationship between these deterministic models and their stochastic implementations is so great that it is possible to alter the relative fits of these models by altering assumptions regarding their underlying sources of noise (Blavatskyy & Pogrebna, 2010).

In this paper we take this point further: the predictions of deterministic utility-based models are so dependent on their stochastic specifications that psychologically desirable assumptions about these specifications (such as noise in preferences) can alter the modal choice predictions associated with these models. These effects pose a strong challenge for utility models of preferential choice. How useful are models like EUT and CPT if their key predictions can be reversed by introducing some noise into the deliberation process?

This problem is ultimately endemic to deterministic models of choice, and cannot be remedied by the application of more rigorous methodological tools. In our opinion it suggests that theoretical research on decision making should attempt to move beyond these types of models when attempting to describe choice. There have already been a number of advances in modelling the cognitive basis of the stochastic choice process (see Rieskamp et al., 2006 and Oppenheimer & Kelso, 2015). Cognitive models of stochastic choice make explicit assumptions about how noise enters into deliberation, and how it interacts with preference, choice, and even decision time and confidence. In allowing stochasticity to play a central role in choice, these models are naturally able to capture a large range of behavioral effects that currently lie outside the descriptive scope of deterministic models. Indeed some of these models even try to explain key decision making anomalies using only unsystematic noise, rather than specific restrictions on value functions or probability weighting (Bhatia, 2014). Most importantly, however, the predictions of cognitive stochastic choice models are clearly defined. Additional assumptions about the sources of variability in choice are not necessary, and thus do not have the potential to reverse the key predictions of these models. Future research should consider using these types of psychologically-grounded stochastic choice models to understand the behavior of decision makers.

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