Intermediate Judgments Inhibit Belief Updating: Zeno's Paradox in Decision Making

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Abstract

Rational agents should update their beliefs in the light of new evidence. Equally, changes in belief should depend only on the quality of the evidence, and not on factors such as the order in which the evidence is acquired, or whether intermediate judgements are requested during evidence acquisition. In contrast we show that requests for intermediate judgments can inhibit belief updating for real decision makers, which represents a new type of decision making fallacy. This behaviour is paradoxical from the point of view of classical Bayesian models, but we show that it is consistent with an a priori, parameter free prediction of a cognitive model based on quantum theory.

Keywords: cognition; decision making; quantum probability.

Introduction

That decision makers should update their beliefs in the light of new evidence is one of the cornerstones of what it means to be rational (Anand, 1993). Equally, for rational agents, the degree of belief change should depend only on the quality of the new evidence acquired and not on other factors such as the order of evidence acquisition or requests for intermediate judgements (Oaksford & Chater, 2009). However it is known that real decision makers do not always conform to the strictest standards of rationality (Tversky and Khaneman, 1974). One notable example of this are order effects, where presenting the same evidence in different orders can have a large influence on the change in belief generated (Trueblood & Busemeyer, 2011).

In this contribution we examine another effect, which is that of requests for intermediate judgments during the process of evidence acquisition. Since requesting such a judgment does not provide a decision maker with any extra relevant information about the problem, making intermediate judgments should have no effect on the process of belief updating for a rational decision maker. Thus at least the simplest classical Bayesian cognitive models should predict no effect of intermediate judgments on the final belief state.

In contrast, in cognitive models where judgment is considered a constructive process, intermediate judgments may well have an effect on belief updating (Schwarz, 2007; White, Pothos & Busemeyer, 2014). One such class of models, which have received much attention recently, are those based on the mathematics of quantum theory (Busemeyer & Bruza, 2011). These models not only predict an effect of intermediate judgments on belief updating, but allow us to make a priori, parameter free predictions of the precise relation between the change in belief and the number of intermediate judgments which can be tested by a suitable experiment. The startling conclusion is that in quantum models, requests for intermediate judgments can strongly inhibit belief updating. In physical systems this singular phenomena is called the quantum Zeno effect, because of the (loose) analogy with Zeno's second paradox; for this reason we term this effect Zeno's paradox in decision making.

In one experiment, we compare the predictions of the quantum model with a simple matched classical Bayesian model and find that the quantum model outperforms the Bayesian one by a large margin. Indeed, the predictions of the Bayesian model are shown to be qualitatively inconsistent with the data. We briefly discuss whether allowing probabilities in a classical model to depend on the memory of a previous judgment can reproduce the quantum behavior.

Experimental Investigation

We performed two identical experiments as a replication exercise. We report only the first experiment here, the conclusions of the second were the same and are reported in full in Yearsley & Pothos (2016).

We recruited 450 experimentally naïve participants, from Amazon Turk. Participants were 49% male and 50% female (1% did not respond to the gender question). Most participants' first language was English (98%) and the average age was 34.8. The experiment lasted approximately 10 minutes; participants were paid $0.50 for their time.

The experiment was implemented in Qualtrics. Our paradigm extends one of Tetlock (1983), which was designed to test for primacy effects in decision making. After some initial screens regarding ethics information and consent, all participants saw the same initial story, regarding Smith, a hypothetical suspect in a murder:

"Mr. Smith has been charged with murder. The victim is Mr. Dixon. Smith and Dixon had shared an apartment for nine months up until the time of Dixon's death. Dixon was found dead in his bed, and there was a bottle of liquor and a half filled glass on his bedside table. The autopsy revealed that Dixon died from an overdose of sleeping pills. The autopsy also revealed that Dixon had taken the pills sometime between midnight and 2am. The prosecution claims that Smith slipped the pills into the glass Dixon was drinking from, while the defense claim that Dixon deliberately took an overdose."

Participants were then given a short set of questions regarding some details of what they had just read, in order to check that they were engaging with the task. These questions were intended to reinforce memory of the story details and to check for participants who were not concentrating on the experiment. The small number of participants who failed to correctly answer these questions were excluded from subsequent analysis. Participants were subsequently asked whether they thought Smith was likely to be guilty or innocent, based on the information provided in the vignette, and to provide a brief justification for their response, as a further check that they were adequately concentrating on the task and to reinforce memory for the response. The first response is critical, since all model predictions are based on knowledge of the initial (mental) state. Most participants (95%) initially assumed innocence, and so we excluded participants who initially assumed guilt. Participants then saw a screen reminding them of their response.

Participants were split into six groups. The first group was presented with 12 pieces of evidence suggesting that Smith was guilty (participants were told they would only see evidence presented by the prosecution and not by the defense). Each piece of evidence was designed (and pilot tested) to be individually weak, but cumulatively the effect was quite strong. For example one piece of evidence read "Smith had a previous conviction for assault." After reading all 12 pieces of evidence, participants were again asked whether they thought Smith was guilty or innocent, and again asked to justify their choice. Participants in the other five groups were shown the same evidence in the same way, and asked to make the same final judgment, but were also asked to make intermediate judgments (and justify their responses). These intermediate judgments were worded in the same way as the initial and final ones, and were requested at intervals of either 1, 2, 3, 4 or 6 pieces of evidence. A small number of participants gave justifications for their judgments that suggested they were not properly engaging with the task, and were therefore excluded from the analysis. Together with those who failed to correctly answer the questions about vignette, and those excluded from the present analysis because they initially judged 'Guilty,' this left 425 good participants.

The order of presentation of the evidence was partly randomized. The pieces of evidence were split into four blocks of three pieces of evidence each. The order of the blocks was fixed, but the order of the pieces of evidence within each block was randomized. The reason we randomized evidence order in this way, rather than say simply randomizing the order of presentation of all pieces of evidence, is that there are a total of 12!, or about 480 million, possible orderings of the evidence, so it is impossible to capture a representative sample of the orderings by simple randomization.

After the main part of the experiment, participants were shown a list of the pieces of evidence they had encountered, and were asked to rate the strength of each piece of evidence on a (1-9) scale. (The full list of pieces of evidence is presented is Yearsley & Pothos, (2016.))

Zeno's paradox in quantum decision making

We want to explain why the Zeno effect arises in quantum models of decision making using a simple example. In a realistic decision making scenario, such as the experiment we discussed above, the modeling is necessarily more complex and this can make the origin of the effect harder to see.

Consider a 2D quantum system, with a state space spanned by two orthogonal states I and G, corresponding to the beliefs that Smith is either Innocent or Guilty. Presentation of evidence is represented by a rotation of the state such that an initial state I evolves towards G, with pieces of evidence.

We are interested in the probability that a measurement of the state will reveal I, at each of $N \geq 1$ judgments at $T/N, 2T/N...T$ is (where T is the total number of pieces of evidence) In analogy with the physics case, this may be called the *survival* probability after N judgments. For a typical time independent Hamiltonian we have;

$$Prob('Survival,' N) = Prob\Big(I \text{ at } \frac{T}{N}, I \text{ at } \frac{2T}{N} ... I \text{ at } T | I \text{ at } 0\Big) = \cos^{2N}\left(\frac{\gamma}{N}\right) \quad (1)$$

Here γ is a constant encoding the effect of the evidence in the absence of intermediate judgments. As the number of judgments, N, increases, there is a decreasing probability that the system will change from I to G. As $N \to \infty$, the probability that the system will change state vanishes, even after large number of pieces of evidence. This is the famous QZ effect (Misra & Suarshan, 1977), often described informally as proof that 'a watched pot never boils'.)

The derivation leading to Eq.(1) involves a number of assumptions that will not hold in realistic decision making settings. However we can still predict a weakened QZ effect, as a slowing down (in a specific way) of the evolution of the measured opinion state, even under more realistic conditions. We will outline the argument below, full details are given in Yearsley & Pothos (2016).

Two assumptions need to be relaxed. First, realistic measurements are not perfectly reliable. For each measurement, there is a small probability that a participant will incorrectly provide a response not matching his/her cognitive state. This is problematic when several identical measurements are made, since error rates may compound. Imperfect measurements require the use of positive-operator valued measures (POVMs), instead of projection operators. Instead of freezing as $N \to \infty$, some evolution may still occur, but it will depend only on details of the imperfect measurements (Anastopoulos & Savvidou, 2006). The effect of imperfect judgments is encoded by a simple POVM operator with one free parameter, ε. The parameter $0 \leq \varepsilon \leq 1$ reflects how error-less measurements are. For example, if a participant considers Smith innocent, then the probability of responding innocent is only $1-\varepsilon$, leaving a probability to respond guilty of ε.

Second, evolution of cognitive variables is better modeled by a time dependent unitary evolution to capture the fact that the weight given by participants to a piece of evidence depends on its position in the sequence of evidence, implying a primacy or recency effect. We must also take account of the fact that not all pieces of evidence will be regarded as equally strong by participants.

A form for the time dependent unitary evolution general enough for our purposes is

$$U(t_m, t_n) = \exp(-i\sigma_x B(t_m, t_n)), \tag{2}$$

where σ_x is one of the Pauli matrices and

$$B(t_m, t_n) = \alpha \sum_{i=m+1}^{n} a_i e^{-\beta(i-m-1)^2} \tag{3}$$

with α and β real numbers.

The function $B(t_m, t_n)$ specifies the angle a participant's cognitive state is rotated through when presented with pieces of evidence t_m through t_n. Here the a_i represent the strengths of the individual pieces of evidence, as measured in isolation. Thus the first piece of evidence in a sequence is given a weight a_1 the second is given weight $a_2 e^{-\beta}$, and so on. Depending on the value of β this form for $B(t_m, t_n)$ can encode a primacy ($\beta > 0$) or a recency ($\beta < 0$) effect. Using the above, we can show that:

$$\begin{aligned} Prob(I \text{ at } t | I \text{ at } 0) = &(1-\varepsilon)^2 \cos^2(B(0,t)) \\ &+ \varepsilon(1-\varepsilon)\sin^2(B(0,t)) \end{aligned} \tag{4}$$

Eq(4) allows us to determine ε and B(0,t), from empirical classical data on the probability of judging Smith's innocence, assuming innocence initially, and varying the number of pieces of evidence presented (without intermediate judgments). We can also use Eq(4), together with some assumptions about the way judgments change the cognitive state classically, to construct a Bayesian model of the same decision making process. We will do this below, but note that in the case of no intermediate judgments the QT and Bayesian models will coincide. This means that we can use data obtained in the absence of any intermediate judgments to fix all the parameters in both the QT and Bayesian models. Our central predictions, of the specific way in which intermediate judgments affect opinion change, will therefore be parameter free.

The Quantum Model

We now state the prediction of a QZ effect in this decision making setting, the full derivation is presented in Yearsley & Pothos (2016). The result is that a participant deciding Smith's innocence will be less likely to change his/her initial opinion as the number of intermediate judgments increases. Specifically, the expression for the survival probability is:

$$\begin{aligned} Prob^Q('survival', N) &= Prob\Big(I \text{ at } \frac{T}{N}, I \text{ at } \frac{2T}{N}, \ldots | I \text{ at } T\Big) \\ &= (1-\varepsilon)^{N+1} \sum_{i=0}^{N-1} \cos^2\left(B\left(\frac{iT}{N}, \frac{(i+1)T}{N}\right)\right) \\ &+ \varepsilon(1-\varepsilon)^N \sin^2\left(B\left(\frac{(N-1)T}{N}, T\right)\right) \\ &\times \sum_{i=0}^{N-2} \cos^2\left(B\left(\frac{iT}{N}, \frac{(i+1)T}{N}\right)\right) + O(\varepsilon^2) \end{aligned} \tag{5}$$

The first term in this expression corresponds to the probability that the cognitive state is always consistent with innocent, and all the judgments reflect this. The second term corresponds to possibility that the state does change between the second to last and final judgments, but the participant nevertheless responds 'innocent' due to the imperfect measurements. Further terms would correspond to more judgments not matching the cognitive state, or the state changing back from innocent to guilty, these terms are negligible compared to those included in Eq.(5). If $\varepsilon = 0$, $\beta = 0$ and the a_i's are equal then Eq(5) reduces to Eq(1).

Constructing a matched classical model

The QT model assumes evidence changes the opinion state (determined by Eq(4)), judgments may be imperfect, and judgments are constructive. The third property is the characteristically quantum one, so with the first two elements, we constructed an alternative, Bayesian model for survival probability. It is helpful to denote by I_B the event where a participant believes Smith is innocent, and by I_R the event where a participant responds Smith is innocent, similarly for guilty.

The expression we are interested in is the Bayesian analogue of Eq.(5); the survival probability after T pieces of evidence have been presented, given that N judgments have been made. This is

$$\begin{aligned} &Prob^C('survival', N) = \\ &\quad Prob\Big(I_R \text{ at } T, I_R \text{ at } \frac{(N-1)T}{N}, I_R \text{ at } \frac{T}{N} | I_R \text{ at } 0\Big) \end{aligned} \tag{6}$$

We want to construct this so that it matches the quantum expression in the case of no intermediate judgments (N=1). We will sketch how to do this here, full details are given in Yearsley & Pothos (2016).

As already noted, because Eq(4) does not involve any intermediate judgments it may be interpreted classically. We can therefore read off,

$$\begin{aligned} Prob(I_B \text{ at } t | I_B \text{ at } 0) &= \cos^2(B(t,0)) \\ Prob(G_B \text{ at } t | I_B \text{ at } 0) &= \sin^2(B(t,0)) \\ Prob(I_R \text{ at } t | I_B \text{ at } t) &= (1-\varepsilon) \\ Prob(G_R \text{ at } t | I_B \text{ at } t) &= \varepsilon \\ Prob(G_R \text{ at } t | G_B \text{ at } t) &= (1-\varepsilon) \\ Prob(I_R \text{ at } t | G_B \text{ at } t) &= \varepsilon \end{aligned} \tag{7}$$

The probabilities involving transitions from Guilty cognitive states to Innocent ones are assumed to be 0. Eq.(4) is therefore also our Bayesian survival probability for the case of no intermediate judgments.

To compute the survival probability when there are intermediate judgments made we need to know the appropriate function for the evolution of the state. The natural classical analogue of Eq.(3), $B^C(t_m, t_n)$, is given by

$$B^C(t_m, t_n) = \sum_{i=m+1}^{n} a_i e^{-\beta(i-1)^2} \tag{8}$$

This differs from $B(t_m, t_n)$ only in the fact that the function multiplying the evidence strength depends only on how many pieces of evidence have been presented before it, and not on whether any intermediate judgments have been made. Note that $B^C(0, t_m) = B(0, t_m)$ since the quantum and classical models agree in the absence of intermediate judgments. In particular this means fitting either function to the data in the absence of intermediate judgments produces the same set of parameters, α, β.

In fact we could use the function $B(t_m, t_n)$ in the Bayesian analysis if we desire, despite the fact it is poorly motivated. It turns out that the Bayesian models perform better when using $B^C(t_m, t_n)$, so we will work exclusively with this.

We can use the information above to derive a prediction for the Bayesian survival probability. Doing so involves two assumptions, first that ε is small, and secondly that the probabilities involving transitions from Guilty cognitive states to Innocent ones are negligible. We can then show (Yearsley & Pothos, 2016;

$$\begin{aligned} Prob^C('survival', N) &= (1-\varepsilon)^{N+1} \cos^2(B^C(0,T)) \\ &+ \varepsilon(1-\varepsilon)^N \sin^2\left(B^C\left(\frac{(N-1)T}{N}, T\right)\right) \\ &\times \cos^2\left(B^C\left(0, \frac{(N-1)T}{N}\right)\right) + O(\varepsilon^2) \end{aligned} \tag{9}$$

We are now ready to test the Bayesian and QT predictions in a realistic decision making scenario.

Results and model fits

Empirical assessment involved two steps. First, without intermediate judgments (ie at the first judgment made after having seen some evidence) the data is classical and simply informs us how opinion changes with evidence. Using Eq(4), we can determine ε and $B(t_1, t_2)$ i.e., the parameter specifying the POVMs for Smith's innocence, guilt and the function specifying the way evidence alters the opinion state (the same parameter values are used in the Bayesian and QT models). Second, we examined whether intermediate judgments produce the QZ effect (slowing down of opinion change, as predicted by the QT model, Eq(5)) or not (in which case the Bayesian model should fit better). The predictions about intermediate judgments from the models were assessed after parameter fixing, the first step; they are *a priori and parameter free*.

In order to determine $B(t_1, t_2)$, we first need to know the a_i's for each piece of evidence. These are the parameters indicating the relative strength of each piece of evidence and they were fixed directly, using the participant ratings for each piece of evidence at the end of the task.

The best fit parameters were obtained by minimizing the sum of the squared deviations between the predictions of Eq(4) and the data. Considering the $t = 3$ data point an outlier, the best fit for Eq(4) is obtained with $\alpha = 0.091, \beta = 0.010$ and $\varepsilon = 0.030$, giving an R^2 of .996 and a BIC of -27.8. (BICs computed following Jarosz & Wiley (2014).) The results of the fitting are shown in Fig(1).

Since the data can be thought of as arising from a binomial distribution the most informative way to display a confidence interval is to imagine using the data to update an initially uniform prior for the value of the survival probabilities, and plot the 95% Highest Density Interval (HDI) of the resultant posterior. Error bars in all figures therefore refer to these HDIs.

For small t, $Prob(I \text{ at } t | I \text{ at } 0)$ is non-linear and (extrapolated) not equal to 1 at $t = 0$. This result justifies our assumption of imperfect measurements. In Fig(1), for large t, $Prob(I \text{ at } t | I \text{ at } 0)$ is close to linear with increasing t. Linearity implies that belief change is proportional to the number of pieces of evidence, which seems an obvious expectation for a rational participant (while the belief state is far from guilty). Note that the best fit value of β is positive, confirming our expectation of diminishing returns (equivalently, there is a primacy effect, regarding evidence strength.)

Now that the model parameters have been fixed for both the QT and Bayesian models, we can use Eq(3) and Eq(4) to compute survival probabilities, for different numbers of intermediate judgments.

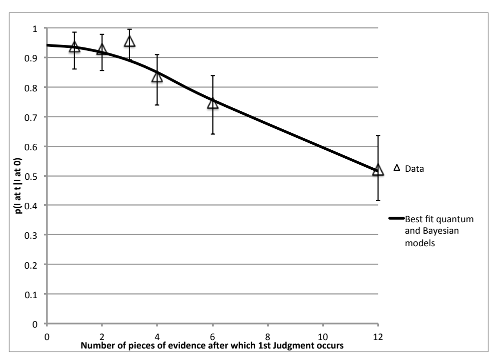

Figure 1: Setting the parameters (opinion change without intermediate judgments): $Prob(I \text{ at } t | I \text{ at } 0)$, for the first judgment a participant made, after having seen different numbers of pieces of evidence. Note the obvious outlier at three pieces of evidence. Data points are the probability computed as an average over participant choices (Number of Participants =64, 71, 70, 73, 71, 76 for each data point) and error bars show the 95% HDI of the posterior.

Empirical results for $Prob('survival', N)$ show a QZ effect, as survival probability increases with small N (Fig(2)). The classical intuition is reduction of survival probability with more intermediate judgments, because of a probability of error at each judgment. The data clearly favor the QT model: the Bayes factor is 3.4×10^5 (Bayes Factor computed following Jarosz & Wiley (2014).) The dip in the survival probability for large N is an effect of the imperfect judgments.

There is an alternative test of the QT vs Bayesian models. We can employ Eq(5) and Eq(9) to compute survival probabilities for the condition where there is a judgment after every piece of evidence (number of pieces of evidence presented T, and number of judgments N, vary, but T/N fixed to 1). Again, the data clearly favor the QT model (Fig(3)). The Bayes Factor in this case is 8.2×10^9.

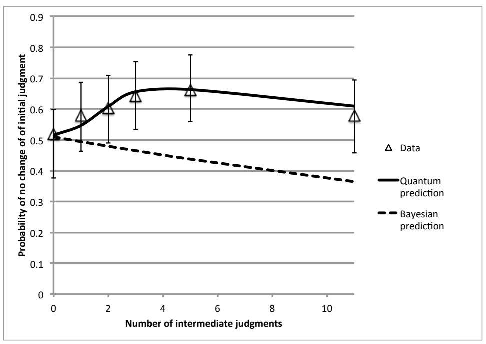

Figure 2: Evaluating the models: Survival probability for N intermediate judgments, for the QT, Bayesian models, against empirical results ; Data points are the probability computed as an average over participant choices (Number of Participants =76, 71, 73, 70, 71, 64) and error bars show the 95% HDI of the posterior.

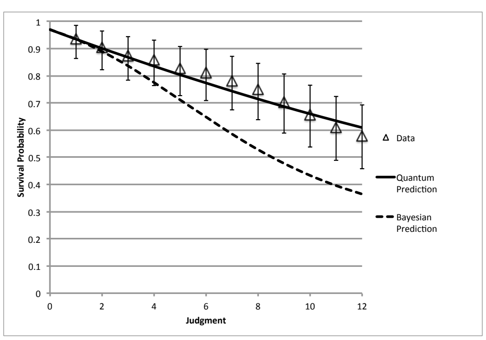

Figure 3: Evaluating the models: Survival probability after each judgment, for the condition with 12 judgments. Data points are the probability computed as an average over participant choices (Number of participants =64 for all data points) and error bars show the 95% HDI of the posterior.

In summary, the data show a clear QZ effect, with survival probability generally increasing with the number of intermediate judgements. Furthermore the behavior is in excellent quantitative agreement with the predictions of the QT model.

Classical Models with Memory Effects

We can use the behavior of the QT model to learn about the structure of any classical model which aims to reproduce these results. Such a model could be constructed by introducing extra variables not directly measured, but that have an effect on the judgment probabilities. This would be post hoc from the point of view of our experimental set up, but it might be possible to motivate the existence and behavior of these extra variables from other considerations. One natural possibility is the memory of any previous judgments taken. This might be motivated by arguing decision makers wish to appear consistent in their judgments, so there is a bias towards aligning a judgment with any previous one[1]. Note that although such a model may be constructed in a classical way, it is still manifestly non-Bayesian, since belief updating depends on factors other than the evidence seen.

We can use the predictions of the QT model to explore how such a model would behave. It is useful to look at the idealized case, Eq(1) and focus on the case of a single intermediate judgment. With T pieces of evidence and no intermediate judgment the survival probability is,

$$Prob(I \text{ at } T | I \text{ at } 0) = \cos^2(T). \tag{10}$$

and this must hold for the classical and quantum models. With a single intermediate judgment the QT probability is,

$$\begin{aligned} Prob^Q\Big(I \text{ at } T, I \text{ at } \frac{T}{2} | I \text{ at } 0\Big) &= \cos^4\left(\frac{T}{2}\right) \\ &= Prob^Q\Big(I \text{ at } T | I \text{ at } \frac{T}{2}\Big) Prob^Q\Big(I \text{ at } \frac{T}{2} | I \text{ at } 0\Big) \\ &= \Big[Prob^Q\Big(I \text{ at } \frac{T}{2} | I \text{ at } 0\Big)\Big]^2. \end{aligned} \tag{11}$$

In order that any classical model agree with the predictions of the QT one, Eq.(11) would have to hold for this model. Suppose we have a classical model containing an extra variable that plays the role of the 'memory' of the judgment made at $T/2$. Then this memory variable would have to 'screen off' the prior evidence so that transition probabilities between $T/2$ and T are identical to those between 0 and $T/2$ before any evidence has been seen. In other words, to agree with the QT model predictions, a classical model incorporating memory cannot simply have the memory function an additional piece of evidence. Instead the memory must override any evidence seen prior to the judgment.

There is a second property less often discussed but also necessary for the QZ effect. Proof that Eq.(1) tends to 1 as the number of judgments increases is based on the following,

$$\begin{aligned} \lim_{N\to\infty} Prob\Big(I \text{ at } \frac{T}{N}, I \text{ at } \frac{2T}{N} ... I \text{ at } T\Big) &= \lim_{N\to\infty} \cos^{2N}\left(\frac{\gamma}{N}\right) \\ &= \lim_{N\to\infty} (1 - \frac{1}{2}(\gamma/N)^2 + O(N^{-4}))^{2N} \\ &= \lim_{N\to\infty} \left[1 - \gamma^2/N + O(N^{-3})\right] = 1 \end{aligned} \tag{12}$$

[1]This possibility was first suggested to us by Gordon Logan.

Crucially this depends on the transition probabilities being non-linear for small amounts of evidence. In other words, the QZ effect holds in part because in the QT model beliefs are 'sticky.' That is, is takes more evidence to change a belief state that is close to a definite state than it does to change a belief that is highly uncertain. We allowed our Bayesian model to mimic this property in the model comparison, but this 'stickiness' of beliefs is in itself a fundamentally non-Bayesian property that must be incorporated into any model aiming to reproduce the QZ effect.

To summarise this section, there is no doubt that one could construct a classical model of belief updating that would mimic the predictions of the QT model (and so fit at least this data and the second set reported in Yearsley & Pothos 2016) by considering the effects of remembering previous judgments. However in addition to any concerns about such a model being post hoc, it would also have to have the two features outlined above; that memories of intermediate judgments screen off evidence, and that beliefs are sticky. It would be interesting to try and construct a classical model wth these properties, although one wonders to what extent this would simply amount to redescrbiing the QT model.

Concluding remarks

Understanding the way beliefs change as a result of accumulating evidence is essential for modeling decision making. Our results suggest that opinion change depends not just on the evidence, but can also be strongly effected by making intermediate judgments, a phenomena we call Zeno's paradox in decision making. This behavior is at odds with Bayesian models of cognition, but agrees quantitatively with the predictions of a model based on QT. Because the QT model was fixed with classical data, this striking prediction follows from a structural feature of quantum theory, the collapse postulate, and not from parameter fixing. Our results show models of decision making need to incorporate influences arising from the process of making judgments.

They also have practical implications. The paradigm we employed, albeit simplified, does have analogies with realistic assessment of evidence; if e.g. juries are expected to reach unbiased conclusions, then the effect of requests for intermediate opinions should be factored in. Likewise, the advent of interactive news web sites (e.g., bbc.co.uk) means that readers can express opinions on news items while reading them, directly and through social media. We raise the possibility that this sort of overlap between acquiring information and expressing an opinion may prevent change in opinion, even in the presence of compelling evidence.

More generally, behaviors paradoxical from Bayesian perspectives have often been interpreted as boundaries in the applicability of probabilistic modeling. Strictly speaking this is not true, since one can always augment Bayesian models with extra variables or interactions, however such models may lack predictive power, or simply be too post hoc. The QT cognition program provides an alternative: perhaps some of these paradoxical findings reveal situations where cognition is better understood using QT, without the need to introduce extra unobserved degrees of freedom. Evidence for the collapse postulate in decision making constitutes a general test of the applicability of QT principles in cognition and adds to the growing body of such demonstrations (Pothos & Busemeyer, 2013). An important future direction will be to understand whether the adaptive arguments used to motivate Bayesian models of cognition (Oaksford & Chater, 2009) need to be adapted to account for phenomena such as the Zeno effect.

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