

# Even when people are manipulating algebraic equations, they still associate numerical magnitude with space

Tyler Marghetis  
tmarghet@indiana.edu

Robert L. Goldstone  
rgoldsto@indiana.edu

David Landy  
dlandy@indiana.edu

Psychological and Brain Sciences, Indiana University, Bloomington, IN 47405

## Abstract

The development of symbolic algebra transformed civilization. Since algebra is a recent cultural invention, however, algebraic reasoning must build on a foundation of more basic capacities. Past work suggests that spatial representations of number may be part of that foundation, but recent studies have failed to find relations between spatial-numerical associations and higher mathematical skills. One possible explanation of this failure is that spatial representations of number are not activated during complex mathematics. We tested this possibility by collecting dense behavioral recordings while participants manipulated equations. When interacting with an equation's greatest [/*least*] number, participants' movements were deflected upward [/*downward*] and rightward [/*leftward*]. This occurred even when the task was purely algebraic and could thus be solved without attending to magnitude (although the deflection was reduced). This is the first evidence that spatial representations of number are activated during algebra. Algebraic reasoning may require coordinating a variety of spatial processes.

**Keywords:** algebra; number and space; notations; mousetracking.

## Introduction

The invention of symbolic algebra transformed human civilization. Algebraic notation allows for accomplishments as mundane as buying paint for a new fence and as fantastic as discovering antimatter. But symbolic algebra is a recent cultural invention. Thus, it cannot rely on devoted neural machinery that evolved specifically for that purpose — an innate 'algebra module.' Instead, our capacity for symbolic algebra must be cobbled together from other cognitive capacities. But which?

One proposal is that higher mathematics, including symbolic algebra, builds on a foundation of space (e.g., Lakoff & Núñez, 2000; Sella et al, 2016). On this proposal, our evolutionarily ancient spatial abilities have been co-opted by culture to reason about abstract mathematical entities and relations. Indeed, early spatial abilities are known to predict life-long mathematical performance, from grades in elementary school to the choice of a mathematics-heavy college major.

One crucial aspect of this spatial foundation may be the ability to use space to make sense of number (Hubbard et al, 2005). Spatial representations of number could ground the highly abstract notion of numerical magnitude in the more basic, experiential notion of location. More complex forms of mathematics could then build on this foundation, from algebra to calculus and beyond (Núñez & Marghetis, 2015). The current study tests this account by examining whether spatial representations are activated during one canonical case of complex mathematical activity: solving equations.

## Mixed evidence for spatial-numerical associations in higher mathematics

There is considerable evidence that spatial representations of number are ubiquitous and automatic, at least during simple numerical tasks (Hubbard et al, 2005; Winter, Marghetis, & Matlock, 2015). These spatial representations involve both the horizontal and vertical axes. Among literate adults in Western cultures, for instance, processing lesser numbers facilitates subsequent responses on the left, while processing greater numbers facilitates responses on the right (Dehaene et al, 1993). Similarly, when German adults generate random numbers while undergoing upward and downward motion, they produce numbers that are significantly greater when moving upward and lesser when moving downward (Hartmann et al, 2011). In adults, these spatial representations have been shaped considerably by culture. The association between numerical magnitudes and horizontal locations, for instance, is reversed among Palestinians who read both words and numbers from right-to-left (Shaki et al, 2009). When we encounter a number, therefore, we automatically activate spatial representations of its magnitude.

But do these implicit spatial representations play any role in mathematics beyond the domain of simple numbers? The evidence is rather mixed. One point in favor of such a role is that the correlation between early spatial abilities and later school success in mathematics is mediated by the ability to map numbers to a linear path (Gunderson et al, 2012). Causal evidence comes from the finding that training students to map numbers to linear path improves calculation (Siegler & Ramani, 2009). There is also evidence that spatial representations of number play a role in mathematical communication. Mathematical experts produce gestures that express numbers as locations, even

when their speech does not contain any mention of space (Marghetis & Núñez, 2013).

On the other hand, there have been a number of failures to find any relation between spatial-numerical associations and higher mathematical ability (e.g., Cipora & Nuerk, 2013; Cipora, Patro, & Nuerk, 2015). Indeed, there is little evidence that spatial representations of number are activated at all during mathematical activities that are more complex than simple numerical judgments — judging a number’s relative magnitude, or determining whether it is even or odd. In contrast to these simple tasks, real mathematical activity seldom involves single numerals in isolation. Algebra, in particular, is often the first time that students begin to think about numbers, not just in terms of their magnitudes, but also in terms of their structural interrelations. As these more complex notions come to the fore, spatial representations of numerical magnitude may fade into the background.

Moreover, many of these more complex notions may *also* have a spatial underpinning. During mental calculation, for instance, subtraction and addition are associated with leftward and rightward motion, respectively (Knops et al, 2009; Marghetis, Núñez, and Bergen, 2014); and during algebraic reasoning, space is used to represent the hierarchical syntax of algebraic expressions (Landy & Goldstone, 2007). If space is playing these other roles, then the association between space and *number* might reasonably be expected to fade. The neural circuitry responsible for representing spatial location cannot be all things at once. On this account, as space is co-opted for new roles — arithmetic, algebraic syntax — its association with numerical magnitude might diminish. Number-space associations may be limited to simple numerical judgments, disappearing as mathematical complexity increases.

### The present study

The current literature, therefore, appears to support conflicting accounts. On one hand, associations between number and space are activated automatically during a variety of simple tasks, and spatial processing more generally has been implicated in higher-level mathematical thinking. On the other hand, individual differences in spatial representations of number do not appear to correlate with mathematical expertise. This presents a puzzle. What is happening to these spatial representations of number as people transition from simple judgments of isolated numerals to more complex mathematical activities?

To resolve this puzzle, we analyzed the spatial dynamics of individuals’ manipulations of algebraic equations, using a methodology that we have dubbed Dense Recording of Algebraic Manipulations (DREAM). In this approach, participants manipulate equations using click-and-drag dynamic algebra software; we record the moment-to-moment details of these manipulations, including the precise mouse trajectories used to rearranging equations.

In the current study, algebraic equations were displayed on a computer screen (e.g.,  $x + 3 = 7$ ), and participants could rearrange these equations by clicking and dragging symbols as if they were physical objects. We also manipulated whether participants performed a task that was focused on magnitude (“Click and drag the greatest/least number”) or algebraic structure (“Solve for  $x$ ”). Throughout, we recorded the fine-grained details of these interactions, including the precise spatial locations at which individual numbers were clicked<sup>1</sup>. By varying the numbers in the equations, we could see whether numerical magnitude had a systematic effect on the location of symbolic manipulations.

We foresaw several possible outcomes. If spatial representations of number play no role in higher mathematics — or play a role only in development — then spatial representations might not be activated at all while equations are manipulated, particularly if the goal is to solve the equation. If, on the other hand, spatial representations of number continue to play a functional role in algebra — perhaps by grounding the meaning of otherwise arcane symbolic manipulations — then we might find traces of spatial-numerical associations in the fine details of how equations are manipulated. In particular, greater numbers might be clicked higher or more rightward, while lesser numbers might be clicked lower or more leftward.

## Methods

### Design

Participants manipulated equations with a computer mouse, moving terms as if they were virtual objects. This was implemented with the Graspable Math software ([www.graspablemath.com](http://www.graspablemath.com)). Think of files and folders on a computer desktop, which can be reorganized and rearranged by clicking and dragging. Graspable Math offers the same functionality but for equations. In response to these manipulations, the software automatically adjusts the equation to maintain its validity. For instance, given the equation ‘ $x + 2 = 4$ ,’ as the 2 is dragged to the far side of the 4, the + symbol changes automatically into the – symbol as it crosses the equal sign, so that the final state of the equation would be ‘ $x = 4 - 2$ ’ (Fig. 1a). This allows users to focus on how and why they want to rearrange equations. In addition, clicking on the equals sign flips an equation (e.g.,  $x = 2 \rightarrow 2 = x$ ), and clicking on an operation performs that operation (e.g.,  $x = 4 - 2 \rightarrow x = 2$ ; Fig. 1b).

The full system is quite powerful and can be explored online ([www.graspablemath.com](http://www.graspablemath.com)). The current study used a simplified version that included only the dragging and clicking interactions described above. We recorded where

<sup>1</sup> Technically, participants clicked *numerals* that *denoted* numbers, and it was the numerals’ denotations that had magnitude. For simplicity of presentation, however, we shall conflate numerals with their denotations and refer to them as *numbers*.

and when interactions occurred, including x,y coordinates of the mouse cursor. Here we focus on where, exactly, participants clicked on numbers, to investigate whether this spatial behavior was affected by the numbers' magnitude.

$$\begin{array}{l}
 x + 2 = 4 \\
 x = 4 - 2 \\
 x = 2
 \end{array}$$

*Figure 1. Manipulating equations using Graspable Math. (a) As an equation is rearranged, it's updated automatically to remain valid. Here, '+2' is dragged from left to right; the sign is switched as it crosses the equals sign. (b) Operations are triggered by clicking the operator.*

On each trial, an algebraic equation appeared on the screen (e.g., ' $x + 3 = 5$ '). Participants performed one of two tasks, assigned between-subjects.

In the Algebra task, participants had to solve for the variable by clicking and dragging to simplify the equation (Fig. 1). For instance, given  $x + 3 = 5$ , one might start by dragging the 3 to the other side of the equation. Note that this does not require attention to numerical magnitude, only to the algebraic relations between the terms.

In the Magnitude task, participants were presented with the exact same equations, but their task was to find the least number — or the greatest number, depending on the block — and indicate their selection by dragging it to other side of the equation. This click-and-drag response was chosen so that the two tasks involved comparable interactions with identical stimuli.

### Participants

Volunteers participated in exchange for partial course credit ( $N = 69$ ,  $M_{\text{age}} = 19$  years, 51 women, 18 men). A target sample size of 68 was determined in advance on the basis of similar studies of number and space (e.g.,  $n = 44$  in Fischer et al, 2010).

### Materials

For both tasks, items consisted of equations in the form  $x \pm b = c$  ( $N = 112$ ). Values of  $b$  and  $c$  ranged from 1 to 9,

excluding 5. The value of  $b$  was always different from  $c$ , so one number was always greater than the other, producing 56 combinations of values for  $b$  and  $c$ . Each combination was used to create two equations: one with addition (e.g.,  $x + 2 = 3$ ) and one with subtraction (e.g.,  $x - 2 = 3$ ).

### Procedure

Participants gave informed consent, completed a brief tutorial on how to manipulate equations with the mouse, and read task instructions. This was followed by practice trials chosen randomly from the full list of items ( $n = 4$ ). They then completed the experimental trials ( $n = 224$ ). Each item appeared twice, ordered randomly across four blocks. For the Magnitude task, initial target magnitude (*greater, lesser*) was assigned randomly and switched halfway through.

Each trial began with the appearance of a fixation symbol at the top-center of the screen. Clicking on this fixation symbol triggered the appearance of an equation toward the bottom of the monitor. The equation appeared either on the left or right of the screen and with the variable either on the left or right of the equal sign (i.e., ' $x + 2 = 3$ ' or ' $3 = x + 2$ '), assigned randomly. Participants were then free to manipulate the equation using the computer mouse. Trials in the Magnitude task ended automatically when a number was dragged across the equal sign and released. In the Algebra task, trials ended automatically when participants had solved for  $x$ . Participants finished by answering a series of standard demographic questions along with four questions about mathematical experience: Did they study calculus in high school? In college? What was their grade? And what was their SAT score? No other measures were collected.

### Analysis

We focused on *where* numbers were clicked, specifically the first number manipulated during each trial. Our primary measure was the *deflection* of these locations, relative to where the participant would click typically (i.e., standardized by participant). A value of zero thus indicated no deflection; negative values, deflections downward or leftward; and positive values, deflections upward or rightward. Analyses used linear mixed-effects models, with centered predictors and the maximal converging effects structure justified by the design (Barr et al, 2013).

## Results

One participant was removed for poor accuracy (72%). Accuracy was high among remaining participants ( $M = 96\%$ , 95% CI [86%, 100%]). One additional participant was removed for corrupted data. Before analysis, we removed trials where the participant did not arrive at the correct response (4% of trials), followed by those that were three standard deviations faster or slower than each participant's mean (1.4% of trials).

### Overall spatial deflection due to numerical magnitude

We first investigated whether numerical magnitude caused systematic *spatial deflections* in click locations. For each trial, we calculated a measure of overall spatial deflection by summing the deflection along the vertical and horizontal axis (i.e., a signed Manhattan distance). On this measure, positive values indicate deflections that are, overall, congruent with our predictions for greater numerical magnitudes (i.e., rightward and upward), and negative values indicate deflections congruent with predictions for lesser magnitudes (i.e., leftward and downward). This spatial deflection was analyzed with a model that included fixed effects of Relative Magnitude (i.e., whether the selected number was *greater* or *less* than the equation's other number), Task (*Algebra* vs. *Magnitude*), and their interaction; and random intercepts and slopes for both participants and items.

There was no effect of Task ( $p > .9$ ). As predicted, interactions with numerals were deflected spatially by their magnitude,  $b = .25 \pm .03$  SEM,  $t = 8.8$ ,  $p < .0001$ . These deflections were congruent with canonical spatial representations of numerical magnitude. When participants manipulated the lesser number in an equation, they clicked a location that was deflected in the congruent left-downward direction ( $M = -0.09$ ); when they manipulated the greater number, they clicked more right-upward ( $M = 0.11$ ).

The size of this spatial deflection, moreover, was moderated by the task,  $b = -0.14 \pm 0.06$  SEM,  $t = -2.4$ ,  $p = .02$ . The size of the magnitude-based spatial deflection in the Magnitude task ( $b = 0.30 \pm 0.03$  SEM) was significantly larger than in the Algebra task ( $b = .18 \pm .04$  SEM), even though the magnitude-based deflection was significant in both tasks (both  $ps < .0001$ ). Thus, magnitude induced an overall spatial deflection of numeral manipulations, and the size of this deflection was task-dependent.

### Axis-specific spatial deflections

We next investigated whether this task-sensitive spatial deflection was specific to either the vertical or horizontal axis. Along the vertical axis, there was no evidence that responses differed by Task,  $b = 0.001 \pm .02$  SEM,  $p > .9$ . By contrast, a number's relative magnitude had a systematic impact on where it was clicked,  $b = .18 \pm .02$  SEM,  $t = 7.7$ ,  $p < .0001$ . When the selected number was greater than the other number in the equation, it was clicked 0.18 standard deviations higher than when it was less than the other number. This spatial-numerical deflection was moderated by the task, as revealed by a significant interaction,  $b = -0.14 \pm .05$  SEM,  $t = -2.3$ ,  $p = .02$ . Additional analyses confirmed that a spatial-numerical deflection occurred for both tasks, and differed only in size. In the Magnitude task, greater numbers were clicked higher than lesser numbers,  $b = 0.23$

$\pm 0.04$  SEM,  $p < .0001$ . In the Algebra task, greater numbers were still clicked significantly higher, but the deflection was dampened,  $b = 0.12 \pm .03$  SEM,  $p = .0001$ . Thus, there was spatial-numerical deflection in both tasks, but the amount of deflection was greater with explicit attention to magnitude.

On the horizontal axis, the effect of Magnitude was smaller but still significant ( $b = 0.06 \pm 0.02$  SEM,  $t = 2.9$ ,  $p < .01$ ). While there was no evidence that this magnitude-based deflection was moderated significantly by the Task ( $b = -0.02 \pm 0.04$  SEM,  $t = -0.5$ ,  $p > .6$ ), additional analysis revealed that the magnitude-based horizontal deflection was only reliable in the Magnitude task,  $b = 0.07 \pm .03$  SEM,  $p = .01$ . In the Algebra task, by contrast, there was no evidence of a magnitude-based deflection along the horizontal axis,  $b = 0.05 \pm 0.04$  SEM,  $p = .17$ .

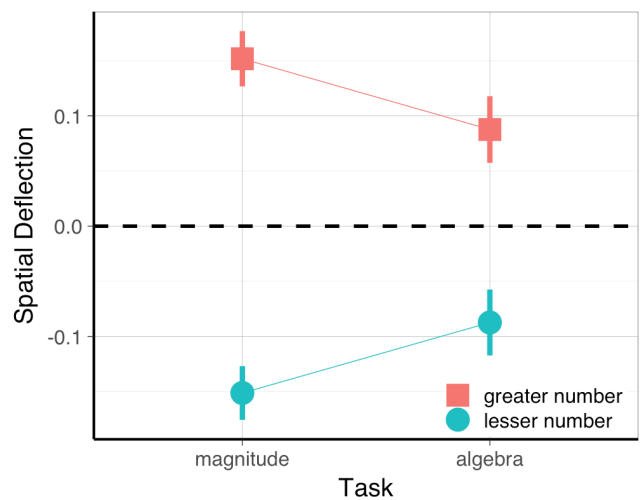


Figure 2: Magnitude-based spatial deflection while manipulating equations. The vertical axis indicates mean spatial deflection, normalized for each subject (i.e., z-scored). Interactions with greater numbers (red squares) were deflected upward and rightward; interactions with lesser numbers (blue circles), downward and leftward. This occurred in both tasks, but it was significantly more pronounced in the Magnitude task. (Error lines = SEM.)

## Discussion

We investigated whether symbol-mediated algebraic reasoning activates spatial representations of number, using dense recordings of algebraic manipulations (DREAM). Manipulations of algebraic equations were deflected *upward* and *rightward* when interacting with greater numbers, and *downward* and *leftward* when interacting with lesser numbers. The strength of this magnitude-based deflection, however, was moderated by the task. Spatial deflection was greatest when the task required explicit attention to numerical magnitude, and was dampened when the task

required algebraic reasoning. This was true even though the two tasks involved interacting with identical equations using comparable movements. In sum, when manipulating equations, people automatically activate a spatial representation of numerical magnitude, and the strength of this activation depends on the task's mathematical demands.

Spatial deflection along the horizontal axis was less pronounced than along the vertical axis. One explanation of this finding is that algebraic notation uses horizontal spacing for another purpose: to indicate syntactic hierarchy. In algebraic notation, higher-precedence operations are often written with little space between operands (e.g.,  $3 \cdot x \cdot y$ ) or no space at all (e.g.,  $3xy$ ), while lower-precedence operations often introduce additional space between operands (e.g.,  $3 + x + y$ ). Thus, during equation manipulation, the horizontal axis may be co-opted to represent algebraic structure, dampening horizontal representations of numerical magnitude (Landy & Goldstone, 2007; Landy, Allen, & Zednik, 2014). By contrast, on purely numerical or arithmetic tasks, numerical magnitude *does* deflect hand movements along the horizontal axis: to the left for lesser magnitudes, and to the right for greater magnitudes (Marghetis et al, 2014; Faulkenberry, 2016).

### A new DREAM for studying algebraic reasoning

The study reported here is the first to use a methodology that we have dubbed dense recording of algebraic manipulations (DREAM) to gain insight into the cognitive processes at work during algebraic reasoning. Similar computer mousetracking approaches have been used to study the dynamics of simple numerical judgments (e.g., Faulkenberry, 2016; Song & Nakayama, 2008) and mental arithmetic (e.g., Marghetis et al, 2014). DREAM extends this mousetracking methodology to a domain where manual interaction with external symbols is not just an artificial feature of the experimental design, but an integral part of the mathematical activity itself. One contribution of this study is to introduce this data-rich paradigm, which we hope can open new avenues of inquiry into mathematical cognition.

Algebraic reasoning is powerful because it transforms difficult conceptual tasks into a series of simple, robust physical manipulations of stable external symbols (Hutchins, 1995). Indeed, it is a canonical example of a cognitive accomplishment that depends on distributing the cognitive load across time and space. This requires coordinating skull-internal processes (perception, planning) with external processes like writing and gesturing. At its core, therefore, the practice of algebra demands the skillful use of hands: writing and erasing equations; using a finger to point to some aspect of an equation. DREAM allows us to analyze this distributed 'manual labor' that is a natural part of algebraic activity.

### Soft-assembling space for mathematics

This is the first evidence that spatial representations of number are activated during algebraic reasoning. Previous research, however, has documented other spatial processes that play a role in algebraic reasoning. The conventions of our algebraic notation use horizontal spacing to indicate syntactic hierarchy: higher-precedence operations are compressed (e.g.,  $xy$ ), while lower-precedence operations introduce more space between symbols (e.g.,  $x + y$ ). Participants are sensitive to these conventions (Landy & Goldstone, 2007; Landy et al, 2014). Once participants master the basic syntax of algebra, moreover, they retrain their visual system so that they literally *see* equations as consisting of visual objects that respect the syntactic hierarchy of algebra (e.g.,  $x \cdot a + y \cdot b$  is perceived as two objects: 'x • a' and 'y • b'; Marghetis, Landy, & Goldstone, 2016). The current study adds to this list of spatial processes that are deployed to solve algebraic equations.

This menagerie of spatial processes raises the question of how they are all brought into coordination. We favor an account where these different brain-based spatial resources are soft-assembled: they are brought into coordination in a way that is both transient and situated, responding to the demands of the task and the material environment (Clark, 2008). On this account, the development of mathematical expertise is not merely a process of piling new insights on top of old. Instead, the mathematical expert learns to combine, flexibly, a range of spatial processes, sometimes deploying one representation, other times another.

This account raises just as many questions as it answers. First, what is the time course of these processes? Are they all activated at once, or are they brought online sequentially in a cascade of activations? Marghetis and colleagues (2014), for instance, documented how, when individuals perform exact symbolic arithmetic (e.g.,  $2 + 7$ ), they first activate a spatial representation of the magnitude of the first number, then of the arithmetic operation, and finally of the solution. A similar cascade may occur in algebra.

Second, given our limited cognitive resources, how do all these mathematical facets—magnitude, arithmetic, algebraic syntax—become coupled to space without conflicting with each other? The spatial impact of relative magnitude was dampened significantly when the goal was to solve the equation rather than to judge relative magnitude. This suggests that spatial representations of number may fade over time, particularly when it comes to mathematical activities, like algebra, that foreground structural relationships over numerical magnitudes.

This fading may occur on multiple timescales, from the developmental to the momentary. On a developmental timescale, mathematical expertise might involve redeploying spatial resources to represent arithmetic or algebraic relations, pushing aside representations of magnitude. On shorter timescales, the activation of spatial

representations may be task-dependent, as it was in this study, or change from moment-to-moment—for instance, as an individual goes from identifying the symbol they intend to manipulate, to actually moving that symbol. Thus, as attention shifts away from magnitude or as other concepts acquire spatial associations, a symbol may become “semantically bleached” of its spatial-numerical content.

Indeed, the current results leave open the question of whether these spatial-numerical associations play a functional role in algebraic reasoning. Taken to the extreme, our results are consistent with an account wherein, for higher mathematics, spatial representations of number are largely epiphenomenal, playing a diminished role as spatial circuits are re-deployed to represent other aspects of the mathematical content (e.g., hierarchical algebraic structure).

## Conclusion

Are spatial representations of number really as ubiquitous as some have assumed, or are they limited to simple numerical tasks? Using dense behavioral recordings of equation manipulations, we found that numerical magnitude did, indeed, cause deflections that suggest a bottom-to-top and left-to-right spatial representation of number. This occurred even when the task was entirely algebraic, though the deflections were more pronounced when the task *did* require attending to magnitude. Our capacity for algebraic reasoning depends on a host of skills and processes — many of which are spatial — that must be brought in and out of coordination during situated reasoning. This singular ability would be impossible without the capacity to cobble together such processes both flexibly and dynamically.

## Acknowledgments

We thank C. J. Casper, V. Updike, B. Graves, and S. Malpeddi for data collection, and B. Rogers for comments.

## References

- Cipora, K., & Nuerk, H (2013). Is the SNARC effect related to the level of mathematics? *QJEP*, 66, 1974-1991.
- Cipora, K., Patro, K., Nuerk, H. (2015). Are Spatial Numerical Associations a Cornerstone for Arithmetic Learning? *Mind, Brain, & Education*, 9, 190-206.
- Clark, A. (2008). *Supersizing the Mind*. Oxford U. Press.
- Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General*, 122, 371.
- Faulkenberry, T. J. (2016). Testing a direct mapping versus competition account of response dynamics in number comparison. *J. of Cognitive Psychology*, 28, 825-842.
- Gunderson, E., Ramirez, G., Beilock, S., Levine, S. (2012). The relation between spatial skill and early number knowledge. *Developmental Psychology*, 48, 1229-1241.
- Hartmann, M., Grabherr, L., Last, F. (2011). Moving along the mental number line. *JEP: HPP*, 38, 1416-1427.
- Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Rev. Neuroscience*, 6, 435-448.
- Hutchins, E. (1995). *Cognition in the Wild*. MIT Press.
- Knops, A., Thirion, B., Hubbard, E., Michel, V., Dehaene, S. (2009). Recruitment of an area involved in eye movement during mental arithmetic. *Science*, 324, 1583
- Lakoff, G., & Núñez, R. (2000). *Where Mathematics Comes From*. New York: Basic Books.
- Landy, D, Goldstone, R. (2007). How abstract is symbolic thought? *Journal of Experimental Psychology: Learning, Memory, & Cognition*, 33, 720–733.
- Landy, D, Allen, C, Zednik, C (2014). A perceptual account of symbolic reasoning. *Frontiers in Psychology*, 5, 275.
- Marghetis, T., & Núñez, R. (2013). The motion behind the symbols: a vital role for dynamism in the conceptualization of limits and continuity in expert mathematics. *Topics in Cognitive Science*, 5, 299-316.
- Marghetis, T, Núñez, R, Bergen, B (2014) Doing arithmetic by hand: Hand movements during exact arithmetic reveal systematic, dynamic spatial processing. *Quarterly Journal of Experimental Psychology*, 67, 1579-1596.
- Núñez, R & Marghetis, T (2015). Cognitive linguistics and the concept (s) of number. In R. Cohen-Kadosh & K. Dowker (eds.), *Oxford Handbook of Numerical Cognition*. Oxford: Oxford University Press.
- Sella, F., Sader, E., Lollot, S., & Cohen Kadosh, R. (2016). Basic and advanced numerical performances relate to mathematical expertise but are fully mediated by visuospatial skills. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 42, 1458–1472.
- Shaki, S., Fischer, M. H., & Petrusic, W. M. (2009). Reading habits for both words and numbers contribute to the SNARC effect. *PBR*, 16, 328-331.
- Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games—but not circular ones—improves low-income preschoolers’ numerical understanding. *Journal of Educational Psychology*, 101, 545.
- Song, J. J., & Nakayama, K. (2008). Numeric comparison in a visually-guided manual reaching task. *Cognition*, 106, 994-1003.
- Winter, B., Marghetis, T., & Matlock, T. (2015). Of magnitudes and metaphors. *Cortex*, 64, 209-224.
- Zorzi, M., Priftis, K., & Umiltà, C. (2002). Neglect disrupts the mental number line. *Nature*, 417, 138-139.