Who makes use of prior knowledge in a curriculum on proportional reasoning?

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Abstract

Understanding proportions is a time-intensive process that does not come cheap during late childhood and early adolescence. It is fostered by learning experiences in which students have opportunities to explore, discuss and experiment with situations involving proportions. Children must undergo many informal learning opportunities before they can gain from direct instruction on proportional reasoning. In this study, we aimed to determine whether physics curricula focusing on the concept of density prepares students for learning from a curriculum on proportional reasoning. A 2x2 design with the factors "physics curricula" (with, without) and "concept used to introduce proportional reasoning" (speed, density) was applied to 253 children from 12 classrooms at the beginning of grade 5. We expected the "density, with physics curriculum" group to outperform the other three groups. However, only the students who scored in the highest quartile on an intelligence measure gained from the prior knowledge they had acquired through the physics curricula. The results show that curricula on proportional reasoning are worthwhile for all students in early adolescence. However, more capable students can boost their proportional reasoning if they have the chance to acquire prior knowledge through a physics curriculum.

Keywords: proportional reasoning, prior knowledge, STEM

Theoretical Background

Proportional reasoning involves comparing ratios within or between quantities, and it is based on the formula a/b = c/d. The crucial step is understanding the multiplicative relationship between the quantities, which means knowing that increasing "a" by a certain factor requires either multiplying "c" or "b" with the same factor or dividing "d" by this factor. Most elementary school children erroneously compute differences rather than ratios. Multiplicative proportional reasoning strategies are considered a cornerstone in the cognitive development of adolescents because they are prerequisites for learning more advanced mathematics and for understanding scientific concepts in various formal domains. Moreover, proportional reasoning supports decision making in everyday life, such as when cooking or when evaluating sales.

As stated before, understanding proportions is a time-intensive process during late childhood and early adolescence. It emerges through repeated and varied experiences and enables mathematical terms and the associated ideas to become connected. The understanding of proportions is fostered by learning experiences in which students have opportunities to explore, discuss and experiment with situations involving proportions. Children must undergo many informal learning opportunities before they can gain from direct instruction on proportional reasoning.

The period from late childhood to adolescence is one of great change, not only in executive control and emotional regulation but also in cognitive competencies. Mastering science and mathematics competencies requires these cognitive tools and skills, which are expected to emerge during elementary school. For example, topics such as fractions, decimals, or ratios, which are at the focus of secondary school mathematics education, presuppose proportional reasoning abilities. These abilities emerge from extending the number concept beyond simple counting (Siegler & Lortie-Forgues, 2014). Elementary school children's competencies in solving mathematical problems addressing relations and proportions of numbers are highly predictive for secondary school performance, even more so than general cognitive abilities (Stern, 2009; Siegler et al., 2012).

Broadly applicable formal reasoning skills and learning strategies (e.g., proportional, logical and scientific reasoning and metacognitive knowledge) result from an interaction between brain maturation and education. Additionally, proportional reasoning, which is considered a domain-general competence, emerges from an interaction of cognitive development (stimulated by brain maturation) and exposure to learning opportunities (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998). These broadly applicable competencies can be fostered through direct instruction, but most children also acquire them incidentally by abstracting knowledge acquired during elementary school. However, children vary significantly in the ease with which they acquire these skills; these differences are attributed to person characteristics such as intelligence and learning opportunities. Earlier theories of cognitive development focused on universal maturation processes and assumed that all children reach a formal reasoning stage around puberty (Case, 1993). However, the considerable individual differences found within age groups with regard to formal reasoning tasks demonstrate the importance of domain-specific knowledge. Several studies have detected remarkable individual differences in proportional reasoning: While some eight-year-olds already master multiplicative strategies, some 15-year-olds still struggle, and adults with little or no standard schooling may never master these skills (Lawson, 1985). Thus, proportional understanding would not suddenly appear if there were no formally or informally acquired knowledge available upon which to build. For example, playing board games in preschool facilitates number-line understanding in elementary school (Siegler, & Ramani, 2008), and the number-line competencies of elementary school children predict later proportional reasoning and understanding of fractions (Siegler, Thompson, & Schneider, 2011). These and other longitudinal intervention studies with preschool and elementary school children have identified which learning opportunities support children in developing proportional reasoning competencies.

Research has rarely examined how knowledge about proportional reasoning is represented in a broader network. Represented as a domain-general principle, it should be transferable to isomorphic problems in various contexts. This kind of transferable knowledge is difficult to acquire and requires intensive instruction (Bransford & Schwartz, 1999). In particular, guided inquiry stands out as an effective means to train the transfer of domain-general principles across situations and time (Chen & Klahr, 1999, 2008). Similarly, inquiry-based science and math learning has been shown to be a successful means for developing domain-specific content knowledge throughout preschool (Leuchter, Saalbach, & Hardy, 2014), elementary school (Hardy, Jonen, Möller, & Stern, 2006), and secondary school.

A longitudinal focus in researching such a complex concept as proportional reasoning would be optimal. However, studies often concentrate on short-term interventions to identify the learner characteristics and instructional factors that affect learning outcomes. These findings do not necessarily capture a generic understanding of proportional reasoning that can be transferred to superficially different but structurally isomorphic problems. Thus, unless learners have acquired expertise, they rarely develop representations of abstract formal structures such as domain-general proportional reasoning (Chi & VanLehn, 2012).

Embedding a general principle such as proportional reasoning in various contexts can support learners in developing an abstract understanding of general principles that can flexibly be used in novel contexts and situations (Alfieri, Nokes-Malach, & Schunn, 2013; Gentner, 2010).

The Current Study

The current study builds on the Swiss MINT Study. (MINT is the acronym for Mathematics, Informatics, Natural Science, and Technology.) In this longitudinal study, elementary school teachers were trained in implementing physics curricula developed by a team of science education experts (https://verlage.westermanngruppe.de/spectra/reihe/KINTBOX). The inquiry-based curricula included four different basic physics topics: floating & sinking, air & atmospheric pressure, sound & spreading of sound, and stability of bridges. Classes started in third and fourth grade. The curricula were tailored to develop children’s domain-specific conceptual content knowledge on these four topics (Möller, & Jonen, 2005).

In every curriculum, children engaged frequently in experimentation to explore the different basic physics concepts. This inquiry-based approach was accompanied by a strong emphasis on instructional guidance and teacher-led classroom discussion. Teachers who agreed to participate in the study underwent four half-day trainings conducted in small groups. In total, the four curricula encompassed 60 classroom lessons. The floating & sinking curriculum, for instance, introduced the concepts of water displacement and object density over 15 lessons.

The children were engaged in extensive guided experimentation activities within and across the four curricula, and they encountered many examples of proportional reasoning. In the swimming and floating curriculum, for example, they immersed pieces of different materials with a similar size or similar materials with different sizes into water to examine how these two characteristics influence floating ability. Through this inquiry-based process, children learned about the concept of density. Therefore, through this curriculum, elementary school children gained not only content knowledge but also experience with regard to the domain-general concept of proportions. None of the four curricula involved general, direct remarks about proportional reasoning.

In this study, we want to find out whether children who studied physics curricula that implicitly included proportional concepts better comprehend proportional reasoning in a subsequent teaching unit than those who studied the traditional way. We expected that the manifold guided experimentation activities not only fostered children’s domain-specific content knowledge but also helped them understand abstract mathematical concepts such as (the domain-general principle of) proportional reasoning. Thus, we expected a significant main effect of “physics curricula”.

We wanted to distinguish between a more general and a more specific effect of prior physics curricula. Therefore, two curricula on proportional reasoning were developed:

one based on the concept of speed, which was not part of the physics curriculum, and one based on the concept of density, which was central in the unit on floating and sinking. The curricula on proportional reasoning were applied either to classes that were part of the previously described Swiss MINT study (and therefore had undergone the physics curricula) or classes that underwent regular science education (which usually does not include physics at all). Thus, we expected a significant interaction between "physics curricula" and "concept used to introduce proportional reasoning".

This led us to the following research questions and hypotheses:

1. Does early science learning affect later mathematical learning (for proportional reasoning)?
   We assumed that later math learning is affected positively (but that this effect depends on the problem context chosen for the intervention; see next point). In other words, we assumed that students with early science learning understand proportions better after receiving instruction on proportional reasoning.
2. In what way does early science learning prepare students for future learning? Does it work more generally or more specifically?
   For the familiar problem context of density, we expected a greater advantage; for the non-familiar problem context of speed, we expect only marginal group differences. We predicted that students who underwent the physics curricula were able to link the new information to their knowledge about physics.

In short, we predicted that children who underwent the physics curricula and were taught proportional reasoning with density scored highest on a transfer test on proportional reasoning.

Method

Participants

Participants included 253 children from 12 classrooms at the beginning of grade 5 (age: M = 10.73 years, SD = 0.55). Participants per cell of the 2x2-design are as follows (see below): density/without physics curricula: n = 66, density/with physics curricula: n = 62, speed/without physics curricula: n = 66, speed/with physics curricula: n = 59).

The children in our sample came from different regions of Switzerland. All of them were part of the Swiss MINT Study and either had completed all four physics themes with the aforementioned curricula prior to the curriculum on proportional reasoning or were part of a waiting group that had not yet started with the physics curricula. Whole schools rather than individual teachers volunteered to be a part of the Swiss MINT Study (and it was not the students who chose a particular school, educational track or curriculum). Nevertheless, it can be assumed that teachers (and school teams) taking part in the current study were STEM oriented and that there were no differences between the "waiting group" stage and the "applying the curricula" stage. Attempts were made to minimize the differences between the student populations by parallelizing the catchment areas of schools at the "waiting group" stage and the "applying the curricula" stage (rural, agglomeration, city and average socioeconomic status of a particular area), as in Switzerland, students are assigned to schools according to their place of residence. Teachers were recruited through a mailing list, and the teachers and classes participated voluntarily during their class time. They received no monetary compensation.

We chose fifth-grade classes for the study as this is a time in which the development of the understanding of proportional reasoning increases, and only towards the end of fifth grade (that is, at the end of our intervention) is proportional reasoning an explicit part of the official study curriculum. Thus, we were able to test whether and to what extent proportional reasoning can develop without formal instruction and to what extent physics experimentation experience additionally boosts this development.

Procedure

A 2x2 design with the factors "physics curricula" (with, without) and "concept used to introduce proportional reasoning" (speed, density) was applied. The curriculum on proportional reasoning (both speed and density) consisted of 3 lessons (45 minutes each) that were based on the idea of concreteness fading (Goldstone & Son, 2005). In the speed group, children were faced with two cars that traveled the same distance in different times, while in the density group, children were shown cubes of the same size but different weights. Afterwards, the children were faced with different combinations of time/distance and weight/volume. The dependent variable was a test on proportional reasoning that was applied at the end of the curriculum (subsequently called the transfer test, as this test consisted of untrained word problems). To control for differential effects, a measure of general intelligence was applied.

Material

Intervention: The intervention was designed in a way that is scientifically proven to be most effective. With the intervention, we followed some basic principles: We tried to make students focus on the underlying mathematical structure of a problem and on multiple solution and representation strategies. We promoted the use of external representations in learning and calculating proportions. We explicitly compared and contrasted the different solutions and representations (see Ziegler and Stern, 2014). Furthermore, we tried to implement self-explanations (Jitendra et al., 2008 on schema-based instruction). To accomplish this, we combined direct instruction with phases of working alone (vs. in pairs). Furthermore, the lessons were based on the idea of concreteness fading (Goldstone & Son, 2005).

Problem presentation influences task difficulty and often determines whether a student can solve a problem (see Boyer et al., 2008). Our participants therefore solved and received feedback on two different problem types: comparison problems and missing values problems (see Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005). The two problem contexts were tightly parallelized, and the same values were used in both of them.

Pre-test, post-test and transfer test (see table 1): In a pre-test, prior knowledge about the two problem contexts was assessed, i.e., students' knowledge on density and speed. Additionally, the N2 subscale of a cognitive ability test (kognitiver Fähigkeitstest, KFT; Heller & Perleth, 2000) was administered at pre-testing. After the above-described intervention (proportional reasoning introduced in the context of density vs. speed), a post-test on the understanding of proportions (very similar to the pre-test—the same in structure as the tasks solved during the intervention) was administered. Additionally, a transfer test (proportion problems embedded into word problems) had to be solved (see table 1). All participants of the 2x2 design solved the same test versions. Both tests (post and transfer) took place one to two days after the intervention.

Table 1: Schematic view of the experimental design

Pre-test	Intervention: (3x45 min) problem context of density or speed	Post-test and transfer test (one to two days after the intervention)
- Physics knowledge on density - Prior understanding of speed - Cognitive ability test (kognitiver Fähigkeitstest, KFT; Heller & Perleth, 2000), subscale N2		- Post-test: knowledge on proportional reasoning (in the same problem context as during the intervention) - Transfer test: knowledge on proportionality in new problem contexts

Results

Pre-Test

Keeping in mind our research question, "Who makes use of prior knowledge in a curriculum on proportional reasoning?" it was important to check whether prior knowledge was actually still available. Indeed, this was the case: The physics knowledge test consisting of the themes floating & sinking revealed significantly higher knowledge in the group with prior experience with the physics curricula (and no significant difference between the speed vs. density conditions). For the group with prior knowledge, the test can be considered a long-term follow-up from the physics curricula; the students completed the physics curricula up to two years prior to the actual proportional reasoning curriculum. For the group without prior experience with the physics curricula, the test and the themes were new. For the group with prior physics knowledge, the solution rate was 40%, whereas the solution rate of the group without prior experience was slightly over 20%. Therefore, for the problem context of density, we can build on the differences in knowledge between the groups with and without physics curriculum experience.

For further analyses, we formed subgroups; i.e., we grouped participants into quartiles according to the results on the N2 subscale of the cognitive ability test. The four groups of the 2x2-design did not significantly differ in their level of cognitive abilities, $F(3,242) = 0.33$, $p = 0.8$. Additionally, when looking at each quartile separately, we find no significant difference between the four groups of the 2x2 design. Therefore, the distribution of cognitive abilities and quartile groups is comparable between the four groups of the 2x2 design.

When they had no prior experience with the physics curricula, participants of all quartiles scored similarly low, and their scores closely overlapped. Cognitive abilities were not reflected in the results regarding physics understanding when physics had not yet been formally taught. However, when the physics curricula had been applied, intelligence differences unfolded, with the highest quartile scoring significantly higher (47%, right) than the other quartiles (see figure 1 and note that error bars indicate standard errors of the mean). Thus, the cognitive ability test did its share only in the group with prior physics instruction.

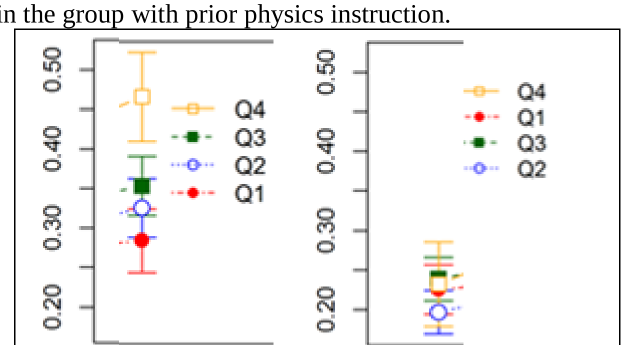

Figure 1: Results of the physics test on floating and sinking. Left: group with prior experience with the physics curricula (for this group, the test can be considered a long-term, i.e., up to one year, follow-up test). Right: group without experience with the physics curricula. Depicted are solution rates; error bars indicate standard errors of the mean. Q1-4 refers to grouping participants into quartiles according to their results on the cognitive ability test, with Q4 indicating the quartile with the highest cognitive ability.

The test on prior knowledge about speed revealed no difference between the groups (with/without physics curricula) as a whole and when split into quartiles. "Speed" was not part of the physics curricula. Prior knowledge about speed, however, was positively correlated with participants' results on the cognitive ability test ($r = .32$). Cognitively abler participants scored higher on the test of prior knowledge about speed. See figure 2 for the differences between the quartiles. The average solution rates were around 50%.

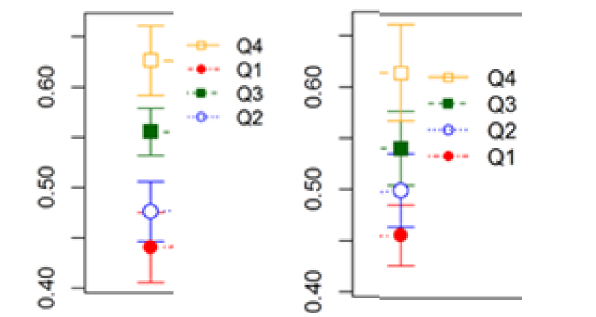

Figure 2: Test of prior knowledge about speed. Left: group with prior experience with the physics curriculum. Right: group without experience with the physics curriculum. Depicted are solution rates; error bars indicate standard errors of the mean. Q1-4 refers to grouping participants according to their results on the cognitive ability test, with Q4 indicating the quartile with the highest cognitive ability. No difference between the groups (with/without physics curriculum) is found. “Speed” was not part of the physics curriculum.

Post-Test

The post-test was very similar to the pre-test and had the same structure as the tasks solved during the intervention. Therefore, the post-test can be viewed as a manipulation check of the implemented proportional reasoning curriculum. This manipulation check was positive in that participants were able to redo tasks that were administered during the curriculum (80% of participants solved all tasks correctly with no mistakes, and there were no differences between the four groups of the 2x2 design).

Transfer-Test

Coming to the core results of this study, overall, no significant interaction was observed between groups (with and without prior physics curricula experience) and context of the intervention (values of the ANOVA comparing the four cells of the 2x2 design: $F(3,236) = 0.94$, $p = 0.42$)). Therefore, the main effect of “physics curricula” turned out not to be significant. Additionally, the interaction between “physics curricula” and “concept used to introduce proportional reasoning” turned out not to be significant.

Therefore, the findings corroborate neither the more general nor the more specific hypothesis of a positive influence of prior physics curricula on the learning of proportional reasoning. This encouraged us to more closely examine whether the subgroups possibly gained from the physics curricula with regard to learning proportional reasoning. Indeed, we did find effects: When grouping participants by quartiles according to their cognitive ability measure, we found the expected positive effect of prior physics learning for students in the highest quartile. Those in the highest quartile who underwent the physics curricula scored higher on the proportional reasoning transfer test than students in the lower three quartiles ($p < .05$). No such effects were found for students in the highest quartile who did not undergo the physics curricula. In the transfer test, the solution rates were relatively low (mean solution rate just slightly over 20%); see figure 3. Similar distributions were found when we only looked at the density groups without evaluating the speed groups: The highest quartile with prior knowledge scored significantly higher than the three lower quartiles, and for those without prior knowledge, the quartiles did not differ. Taken together, for the most intelligent students, the findings corroborate the more general hypothesis. Therefore, physics curricula can serve as preparation for future learning of proportional reasoning.

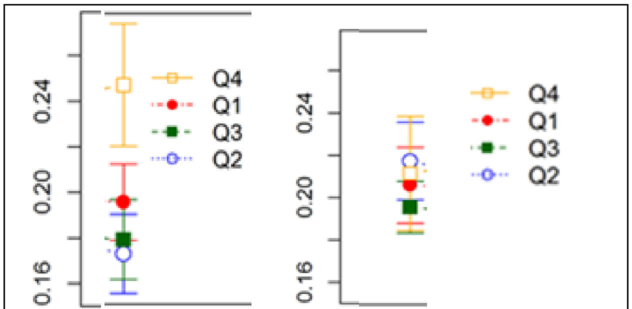

Figure 3: Results of the proportional reasoning transfer test by physics curriculum (with/without). Left: group with prior experience with the physics curriculum. Right: group without prior experience with the physics curriculum. The transfer test consisted of entirely novel proportional word problems. Depicted are solution rates; error bars indicate standard errors of the mean. Q1-4 refers to grouping participants according to their results on the cognitive ability test, with Q4 indicating the quartile with the highest cognitive ability.

Discussion

Our results confirm what has been demonstrated many times: Transfer does not come cheap. The study focused on learning opportunities that foster the emergence of consecutive competencies in related fields. Contrary to our expectation, we did not find a general advantage of physics learning (more precisely, density learning) for learning proportional reasoning taught by referring to density.

However, students scoring in the highest quartile of the intelligence measure were able to make use of the prior knowledge they had acquired during the physics curriculum. We therefore conclude that intelligence differences can unfold students’ individual potential in combination with sufficient prior knowledge. If a child has high cognitive ability and encounters many examples of proportional reasoning situations, he or she will be prepared for a subsequent formal learning situation on proportional reasoning. It is the combination of high intelligence and prior experience or specific prior knowledge that leads to the ability to exploit a learning situation better.

The results show that curricula on proportional reasoning are worthwhile for all students in early adolescence. However, more capable students can boost their proportional reasoning if they have the chance to acquire prior knowledge in a physics curriculum.

Future work could focus on the question of how physics curricula can better support students in understanding proportions. It is possible that introducing the abstract structure of the mathematical concept prior to inquiry-based instruction would have an even greater effect.

References

Alfieri, L., Nokes-Malach, T. J., & Schunn, C. D. (2013). Learning through case comparisons: A meta-analytic review. *Educational Psychologist*, 48(2), 87-113.

Ben-Chaim, D., Fey, J. T., Fitzgerald, W. M., Benedetto, C., & Miller, J. (1998). Proportional reasoning among 7th grade students with different curricular experiences. *Educational Studies in Mathematics*, 36(3), 247-273.

Boyer, T. W., Levine, S. C., & Huttenlocher, J. (2008). Development of proportional reasoning: where young children go wrong. *Developmental psychology*, 44(5), 1478.

Bransford, J. D., & Schwartz, D. L. (1999). Rethinking transfer: A simple proposal with multiple implications. Review of Research in Education, 24, 61-100.

Case, R. (1993). Theories of learning and theories of development. *Educational Psychologist*, 28(3), 219-233.

Chen, Z., & Klahr, D. (1999). All other things being equal: Acquisition and transfer of the control of variables strategy. *Child Development*, 70(5), 1098-1120.

Chen, Z., & Klahr, D. (2008). Remote transfer of scientific-reasoning and problem-solving strategies in children. In R. V. Kail (Ed.), *Advances in child development and behavior* (Vol. 36, pp. 419-470). Amsterdam: Elsevier.

Chi, M. T. H., & VanLehn, K. A. (2012). Seeing deep structure from the interactions of surface features. *Educational Psychologist*, 47(3), 177-188.

Gentner, D. (2010). Bootstrapping the mind: Analogical processes and symbol systems. *Cognitive Science*, 34, 752-775.

Goldstone, R. L., & Son, J. Y. (2005). The transfer of scientific principles using concrete and idealized simulations. *The Journal of the Learning Sciences*, 14(1), 69-110.

Hardy, I., Jonen, A., Möller, K., & Stern, E. (2006). Effects of instructional support within constructivist learning environments for elementary school students' understanding of "floating and sinking". *Journal of Educational Psychology*, 98(2), 307-326.

Heller, K. A., & Perleth, C. (2000). *KFT 4-12+ R kognitiver Fähigkeitstest für 4. bis 12. Klassen, Revision.* Beltz Test.

Jitendra, A. K., Star, J. R., Starosta, K., Leh, J. M., Sood, S., Caskie, G., ... & Mack, T. R. (2009). Improving seventh grade students' learning of ratio and proportion: The role of schema-based instruction. *Contemporary Educational Psychology*, 34(3), 250-264.

Lawson, A. E. (1985). A review of research on formal reasoning and science teaching. *Journal of Research in Science Teaching*, 22(7), 569-617.

Leuchter, M., Saalbach, H., & Hardy, I. (2014). Designing science learning in the first years of schooling. An intervention study with sequenced learning material on the topic of 'floating and sinking'. *International Journal of Science Education*, 36(10), 1751-1771.

Möller, K., & Jonen, A. (2005). *Die KiNT-Boxen–Kinder lernen Naturwissenschaft und Technik. Klassenkisten für den Sachunterricht, 1.* See also: https://verlage.westermanngruppe.de/spectra/artikel/A421/Die-KiNT-Boxen-Klassenkisten-I-Schwimmen-und-Sinken for the KiNT Box of swimming and floating.

Siegler, R. S., & Lortie-Forgues, H. (2014). An integrative theory of numerical development. *Child Development Perspectives*, 8(3), 144-150.

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fraction development. *Cognitive Psychology*, 62, 273-296.

Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental science*, 11(5), 655-661.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., ... & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological science*, 23(7), 691-697.

Smith, C. L. (2007). Bootstrapping processes in the development of students' commonsense matter theories: Using analogical mappings, thought experiments, and learning to measure to promote conceptual restructuring. *Cognition and Instruction*, 25(4), 337-398.

Stern, E. (2009). The development of mathematical competencies: Sources of individual differences and their developmental trajectories. In W. Schneider & M. Bullock (Eds.), *Human development from early childhood to early adulthood: Evidence from the Munich Longitudinal Study on the Genesis of Individual Competencies (LOGIC)* (pp. 221-236). Mahwah, NJ: Erlbaum.

Van Dooren, W., De Bock, D., Hessels, A., Janssens, D., & Verschaffel, L. (2005). Not everything is proportional: Effects of age and problem type on propensities for overgeneralization. *Cognition and Instruction*, 23(1), 57-86.

Ziegler, E., & Stern, E. (2014). Delayed benefits of learning elementary algebraic transformations through contrasted comparisons. *Learning and Instruction*, 33, 131-146.