

# Tutorial: Mixed Models in R – An Applied Introduction

**Henrik Singmann (singmann@gmail.com)**  
Department of Psychology, University of Zürich  
Binzmühlestrasse 14/22, 8050 Zurich, Switzerland

**Keywords:** statistical analyses; repeated-measures; mixed model; multilevel model; hierarchical model

## Significance of the topic

In order to increase statistical power and precision, many data sets in cognitive science contain more than one data point from each unit of observation (e.g., participant), often across different experimental conditions. Such *repeated-measures* pose a problem to most standard statistical procedures such as ordinary least-squares regression, (between-subjects) ANOVA, or generalized linear models (e.g., logistic regression) as these procedures assume that the data points are *independent and identically distributed*. In case of repeated measures, the independence assumption is expected to be violated. For example, observations coming from the same participant are usually correlated – they are more likely to be similar to each other than two observations coming from two different participants.

The goal of this tutorial is to introduce a class of statistical models that is able to account for most of the cases of non-independence that are typically encountered in cognitive science – *linear mixed-effects models* (Baayen, Davidson, & Bates, 2008), or mixed models for short. Mixed models are a generalization of ordinary regression that explicitly capture the dependency among data points via random-effects parameters. Compared to traditional analyses approaches that ignore these dependencies, mixed models provide more accurate (and generalizable) estimates of the effects, improved statistical power, and non-inflated Type I errors (e.g., Barr, Levy, Scheepers, & Tily, 2013; Aarts, Verhage, Veenfliet, Dolan, & van der Sluis, 2014).

In recent years, mixed models have become increasingly popular. One of the main reason for this is that a number of software packages have appeared that allow to estimate large classes of mixed models in a relatively convenient manner. The tutorial will focus on `lme4` (Bates, Mchler, Bolker, & Walker, 2015), the gold standard for estimating mixed models in R (R Core Team, 2017). In addition, it will introduce the functionality of `afex` (Singmann, Bolker, Westfall, & Aust, 2017), which simplifies many aspects of using `lme4`, such as the calculation of *p*-values for mixed models. `afex` was specifically developed with a focus on factorial designs that are common in cognitive science.

Despite a number of high impact publications that introduce mixed models to a wide variety of audiences (e.g., Baayen et al., 2008; Judd, Westfall, & Kenny, 2012) the application of mixed models in practice is far from trivial. Applying mixed models requires a number of steps and decisions that are not necessarily part of the methodological arsenal of

every researcher. The goal of the workshop is to change this and to introduce mixed models in such a way that they can be effectively used and the results communicated.

## Structure of the workshop

The workshop is split into two parts main parts and one interlude. The focus of the first part is not on mixed models, but on the basic knowledge in statistical modeling with R that necessary for competently using mixed models. The second part focuses exclusively on mixed models. It introduces the key concepts and simultaneously shows how to fit mixed models of increasing complexity. Each part will take approximately 2 hours and 45 minutes (not including breaks). The time between the two parts will be used to provide a short introduction to the `tidyverse` (Wickham & Grolemund, 2017), a modern set of tools for data science in R that are especially useful in this context. A full overview over the topics in each part is given next.

Versions of this workshop have already been presented, once at UC Berkeley (as part of the “Data on the Mind” workshop)<sup>1</sup> and once in Zurich (for graduate students and post-docs). Therefore, preliminary versions of the workshop material are already publicly available.<sup>2</sup> These previous iterations have also shown that a full day is an appropriate duration that will allow participants to reach a level after which they can apply mixed models.

**Statistical Modeling** The first part is intended to provide a general introduction to statistical modeling in R in terms of linear or generalized linear models. Specifically, the first part will provide a practical overview over the steps necessary to perform a complete statistical analysis using ordinary regression or similar standard statistical models. The following steps will be introduced:

1. Identifying the appropriate probability distribution of ones’ data. For example, normal model versus binomial model (e.g., for accuracy data).
2. The difference between dealing with continuous and categorical covariates (especially in interactions), different factor coding schemes, and how to set up model formulas.
3. Estimating models.
4. Hypothesis tests of model terms (i.e., main effects and interactions).
5. Follow-up/post-hoc tests and custom contrasts. This section will focus on the `emmeans` package (Lenth, 2018).

The first part will end with a discussion on the limitations of most standard statistical models when repeated measures

<sup>1</sup><http://www.dataonthemind.org/2017-workshop>

<sup>2</sup>[https://github.com/singmann/mixed\\_model\\_workshop](https://github.com/singmann/mixed_model_workshop)

are involved. This will set the stage to introduce the three basic approaches for handling repeated measures (e.g., Gelman & Hill, 2007): complete pooling, no pooling, and partial pooling. The first two pooling approaches are the traditional methods for dealing with repeated-measures. Complete pooling ignores variability coming from sampling multiple units of observation (e.g., data is simply aggregated across participants and thereby individual variability ignored), whereas no pooling ignores all similarity across units of observation (e.g., a separate model is estimated for to the data of each participant). Partial pooling is the hierarchical approach implemented in mixed models which takes both variability as well as similarity across units of observation into account.

**Interlude** The interlude will introduce the `tidyverse` (Wickham & Grolemund, 2017), specifically packages `tidyr`, `dplyr`, `ggplot2`, and `broom`. Those will be used to perform a no pooling analyses of a repeated measures data set introduced at the end of part 1.

**Mixed Models** The goal of the second part if to introduce the concepts necessary to specify and estimate mixed models. The following topics will be discussed:

1. Partial pooling and the distinction between fixed-effects and random-effects.
2. How to specify the correct random-effects structure. Here we will first introduce the notion of the *maximal random-effects structure* (Barr et al., 2013). Then we will discuss alternative views suggesting model selection for this goal (e.g., Matuschek, Kliegl, Vasishth, Baayen, & Bates, 2017).
3. How to deal with convergence problems.
4. Different methods for testing terms in mixed models: Kenward-Roger approximation, Satterthwaite approximation, likelihood-ratio tests, and parametric bootstrap.
5. Mixed-models with alternative distributional assumptions (e.g., binomial models).

If time permits, part 2 will also provide an overview of different ways of visualizing mixed models and discuss the problem of calculating effect sizes or pseudo effect sizes for mixed models. Pointers to R packages that allow the estimation of mixed models within a Bayesian statistical framework are also provided.

**Participation Requirements** Attendees are expected to have a basic knowledge of R. For example, attendees should be able to load their data into R, calculate some summary statistics, estimate a linear regression on their data, and create some basic plots. For a basic R tutorial see for example: [www.cyclismo.org/tutorial/R/](http://www.cyclismo.org/tutorial/R/)

## Credentials of the Organizer

Henrik Singmann is a postdoc at the University of Zurich in the lab of Klaus Oberauer and interested in cognitive and statistical models for cognitive science and related disciplines. Most of his substantive work is on aspects of higher-level cognition such as reasoning or memory.

He is also the author of a number of R packages such as `MPTinR` (for estimating multinomial processing tree models), `rtdists` (for evidence accumulation models), or `bridgesampling` (for estimating marginal likelihoods). Most importantly, he is also the main developer behind `afex`, a package for the convenient analysis of factorial experiments that offers a user-friendly interface for the analysis of mixed models. The workshop is loosely based on his recent introductory chapter to mixed models (Singmann & Kellen, in press).

## References

- Aarts, E., Verhage, M., Veenvliet, J. V., Dolan, C. V., & van der Sluis, S. (2014). A solution to dependency: using multilevel analysis to accommodate nested data. *Nature Neuroscience*, *17*(4), 491–496. doi: 10.1038/nn.3648
- Baayen, H., Davidson, D., & Bates, D. (2008). Mixed-effects modeling with crossed random effects for subjects and items. *Journal of Memory and Language*, *59*(4), 390–412. doi: 10.1016/j.jml.2007.12.005
- Barr, D. J., Levy, R., Scheepers, C., & Tily, H. J. (2013). Random effects structure for confirmatory hypothesis testing: Keep it maximal. *Journal of Memory and Language*, *68*(3), 255–278. doi: 10.1016/j.jml.2012.11.001
- Bates, D., Mchler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, *67*(1). doi: 10.18637/jss.v067.i01
- Gelman, A., & Hill, J. (2007). *Data analysis using regression and multilevel/hierarchical models*. Cambridge; New York: Cambridge University Press.
- Judd, C. M., Westfall, J., & Kenny, D. A. (2012). Treating stimuli as a random factor in social psychology: A new and comprehensive solution to a pervasive but largely ignored problem. *Journal of Personality and Social Psychology*, *103*(1), 54–69. doi: 10.1037/a0028347
- Lenth, R. (2018). *emmeans: Estimated marginal means, aka least-squares means*. (R package version 1.1, <https://CRAN.R-project.org/package=emmeans>)
- Matuschek, H., Kliegl, R., Vasishth, S., Baayen, H., & Bates, D. (2017). Balancing type i error and power in linear mixed models. *Journal of Memory and Language*, *94*, 305–315. doi: 10.1016/j.jml.2017.01.001
- R Core Team. (2017). *R: A language and environment for statistical computing*. Vienna, Austria: R Foundation for Statistical Computing. (<http://www.R-project.org/>)
- Singmann, H., Bolker, B., Westfall, J., & Aust, F. (2017). *afex: Analysis of factorial experiments*. (R package version 0.18-0. <http://cran.r-project.org/package=afex>)
- Singmann, H., & Kellen, D. (in press). An introduction to linear mixed modeling in experimental psychology. In *New methods in neuroscience and cognitive psychology*. Psychology Press.
- Wickham, H., & Grolemund, G. (2017). *R for data science: Import, tidy, transform, visualize, and model data*. Sebastopol CA: O’Reilly.