Improving pre-algebraic thinking in preschoolers through patterning

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Abstract

The learning and generalization of patterns is an important aspect of mathematical thinking, such that the ability to identify and use patterns early in development predicts future success in algebra and math. Thus, understanding the critical factors that facilitate this relational knowledge is important for the development of instructional materials and for curriculum development. The aim of the present study was to examine the factors that facilitate the learning and transfer of pattern knowledge. In two experiments, 4- to 6-year-old children participated in a pre-post test design, in which they received training on novel patterns. Critically, we manipulated (1) the language with which children were exposed to novel patterns during training and (2) the perceptual format in which children were exposed to novel patterns. We find that 4-6 year old children were able to learn about novel patterns following this intervention, but faired best when trained on abstract ("A-B-A") rather than concrete ("red-blue-red") labels. Furthermore, the extent to which the training stimuli were grounded in visual representations affected both learning and generalization of this newly acquired pattern knowledge. This work has implications for instructional design and curriculum development in the classroom.

Keywords: Relational thinking, patterns, proportional reasoning, education, learning.

Introduction

The learning and generalization of repeating and numerical patterns is a central skill of early mathematical thinking (NCTM, 2000; Sarama & Clements, 2009) and is predictive of later math achievement (e.g., Charles, 2005; Kidd et al., 2014; Rittle-Johnson, Fyfe, Hofer, & Farran, 2016). This is particularly relevant given the low levels of mathematics achievement reported among students in the United States (NAEP, 2009; NCES, 2010; Siegler et al., 2012). Thus, improving students' math knowledge and reasoning ability early in a child's education is important. Furthermore, understanding what factors predict the best learning, generalization, transfer, and retention should be a fundamental component of instruction and curriculum development. The aim of the present study was to identify and test theoretically-grounded pattern training in an effort to facilitate relational learning in preschool-age children.

Patterning involves the ability to identify a predictable sequence or relational structure of a set of items, and is often introduced to children in the form of a repeating linear set (e.g., a string of two alternating colors: red-blue-red-blue; Rittle-Johnson et al., 2016). This early ability to classify patterns forms the foundation of later algebraic thinking, which in turn lays the foundation for abstract mathematical thinking (e.g., NCTM, 2000; Steen, 1988). Despite its importance, children struggle with these foundational concepts (Gentner & Medina, 1988; Son, Smith, & Goldstone, 2011). Only a handful of research to date has been conducted to systematically examine methods to optimize children's learning and transfer at this foundational level of learning (Fyfe, McNeil, & Rittle-Johnson, 2015; Kuwabara & Smith, 2012; Rattermann & Gentner, 1998; Son, Smith, Goldstone, & Leslie, 2012).

Although some of the factors facilitating pattern learning have been identified in prior research, very little research on patterning (rather than broader relational learning) has been conducted. The current study examines three theoretically-motivated ways of improving it: (1) Relational Language, (2) Instance Variability, and (3) Level of Idealization.

Relational Language. One factor that has emerged as a potential way to improve children's understanding of patterns is relational language. In one study, Fyfe et al. (2015) used physical manipulatives to train children on new patterns by showing them an exemplar and having children recreate that pattern themselves. Critically, they exposed children to either concrete (e.g., red-blue-red) or abstract labels (e.g., A-B-A) during training. Data from this study indicate that children in the abstract condition fared best (Fyfe et al., 2015). Support for this idea comes from work showing that young children in particular have difficulty grasping relevant relational information amidst varying perceptual cues, and that (more abstract) words may help them notice and comprehend the most relevant information during learning (Rattermann & Gentner, 1998; Son et al., 2008, 2012). The present study seeks to extend these findings, while also examining whether abstract labels may lead to better generalization and transfer *after* learning. That is, does the effect of abstract labels observed in Fyfe et al. (2015) extend beyond immediate training? Experiment 1 investigates these questions.

Instance Variability. Another line of research suggests that instance variability affects generalization (Hahn, Bailey, & Elvin, 2005; Heit & Hahn, 2001). Specifically, this research indicates that greater variability within a category may broaden category boundaries. However, it is unclear whether children's ability to learn about multiple patterns – and thus abstract this information more broadly – would benefit from higher vs. lower variability. According to Kuwabara and Smith (2012), relational judgments present children with attentional competition between the object(s) presented and the relation that they are tasked with learning. Therefore, perhaps lowering the competition through the use of identical (low variable) training items might promote children's learning of relations, rather than having them focus on the objects across instances. Experiment 2A examines the impact of instance variability on children's ability to learn and generalize patterns by asking whether decreasing the variability of pattern presentation during training result in better learning due to the lower demands on attention.

Level of Idealization. Although research suggests that perceptually rich and variable educational materials surround a child's learning environment (e.g., Van de Walle, 2007), presumably because these high-contrast items are attention-grabbing, this may be detrimental to children's learning. For example, children younger than 6 years of age had difficulty noticing relational matches of sets, instead focusing on the identity of the objects themselves (e.g., big square-small square would better match the pattern big circle-small circle, but children will instead pick an identity match, such as two squares of the same size; Gentner & Medina, 1988; also see Kotovsky & Gentner, 1996). Results from this line of research suggest that young children may have difficulty noticing relations due to their focus on the non-relevant perceptual properties of the objects in the task (Kanwisher & Driver, 1992; Kuwabara & Smith, 2012; Rattermann & Gentner, 1998; Son et al., 2012). Experiment 2B examines this question through the use of real-world objects as training stimuli.

Overview of the Current Study

The aim of the present study was to examine what factors facilitate pattern learning, generalization, and transfer in preschool-age children. Critically, we examine three theoretically-grounded methods of training. In Experiment 1, we investigate the impact of relational language on pattern learning and retention. We predict that children exposed to abstract labels (per Fyfe et al., 2015; Son et al., 2012) will learn better than those children exposed to concrete labels. Furthermore, we predict that this affect will hold across generalization and transfer. In Experiment 2, we manipulate the perceptual variability with which children are exposed to novel patterns during training in two distinct ways. For one group of children (2A), the variability within and across trials will be decreased, and we predict that this will lead to similar or higher learning as observed in Experiment 1. For a second group of children (2B), the items used during training will be idealized (e.g., cats and dogs rather than geometric shapes). We predict that this increase in idealization may hinder the learning of novel pattern concepts, despite evidence that these types of stimuli are often used in education settings.

Experiment 1

Method

Participants

Thirty-six 4- to 6-year-old children participated in this study (M_{Age}=5.0 years, *SD*=0.65 years). Children were randomly assigned to one of two conditions: Abstract Labels (*N*=17) or Concrete Labels (*N*=19). Children were tested in their own preschool, during regular school hours, or in a single visit to our laboratory on the main campus of a major Midwestern university. All children were tested in a quiet room with a single experimenter.

Materials

The task consisted of seven phases and participants completed each phase back-to-back in a single session. The first six phases were presented as a match-to-sample task, in which children saw a single pattern at the top of the screen and two answer "choices" at the bottom of the screen (Figure 1). One match was always a perceptual match (same shape/color but incorrect pattern) and the other match was always the correct pattern. The first block of trials was warm-up and consisted of two-item patterns, either A-A (triangle, triangle) or A-B (triangle, square). If children selected the incorrect answer during warm-up, they were told that the other match was correct. Feedback was not provided throughout the rest of the pattern task.

The second and fourth phases were identical and served as the pre- and post-test. The "sample" consisted of a three-item pattern and the matches were either perceptual (but incorrect pattern) or the correct pattern match. In between pre- and post-test, children saw a series of training trials (per Fyfe et al., 2015). For each "example" training trial, children saw two patterns side by side and the experimenter explained how each pattern – which differed

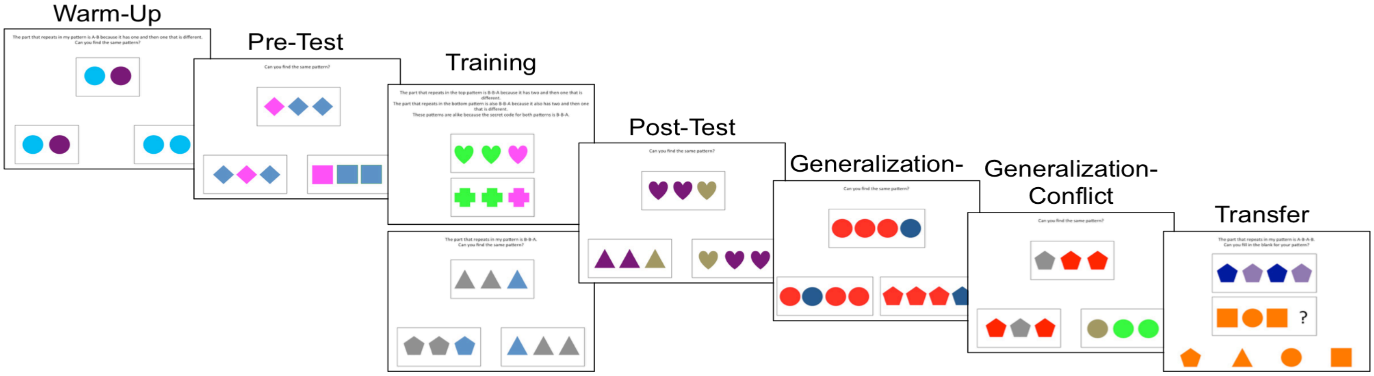

Figure 1. Example of the stimuli used across all phases in the Patterns Task.

perceptually – was the same pattern (Figure 1). After seeing the "example," children received two match-to-sample trials of the same pattern they had just learned (e.g., A-A-B). Following post-test, children received two blocks of generalization trials. The first block of generalization trials ("Gen-4") were similar to those seen in pre- and post-test, but consisted of four-item patterns. The second block of generalization trials ("Gen-Conflict") were three-item patterns and served as "conflict" trials – that is, rather than having to ignore a single perceptual variable (e.g., shape), children had to ignore two perceptual features (e.g., color and shape) in order to find the correct match (which matched in neither color nor shape; Figure 1). In the last phase of the patterns task, children completed a test of transfer. In this portion of the task, children saw a sample pattern (e.g., A-B-A-B) and were told to "fill in the blank" for their pattern (e.g., A-B-?-B) from a set of four potential answer choices (pattern completion; Figure 1).

Additionally, children participated in one of two Conditions: Abstract Labels or Concrete Labels. The patterns task was identical for all visual stimuli presented, with only the prompts during warm-up, training, and transfer differing across Conditions. In the Abstract Labels condition, patterns were presented as letters (e.g., A-B-B). In the Concrete Labels condition, patterns were presented with their corresponding shape or color label (e.g., red-blue-blue; per Fyfe et al., 2015).

Procedure

The procedure consisted of seven phases: warm-up, pre-test, training, post-test, generalization-4, generalization-conflict, and transfer. Warm-up consisted of four trials, two representing A-A and two representing A-B. Pre-test and post-test were identical in nature and consisted of 12 trials each. Four patterns were used: A-A-B, A-B-A, A-A-B, and B-B-A. Training consisted of eight patterns, with each trial containing an example, followed by two match-to-sample trials, for a total of 24 trials (16 scored). The patterns were the same as used in pre- and post-test. Generalization-4 consisted of 16 trials and presented children with four novel patterns: A-A-A-B, A-A-B-B, A-B-A-B, and A-B-B-B. Generalization-conflict consisted of 16 trials and the patterns were the same as those presented in pre- and post-test. Finally, the transfer pattern completion task consisted of eight trials and presented children with the same patterns as were presented in Generalization-4.

For all phases in the patterns task, half of all trials were color matches and half of all trials were shape matches (per Fyfe et al., 2015). The entire task was presented continuously on a Macbook laptop and children's answers were recorded.

Results

The analyses for Experiment 1 explored whether (a) children were more likely to make correct pattern matches (>chance-level) rather than perceptual non-pattern matches (see Figure 2) during the training, generalization, and transfer portions of the patterns task; (b) whether children demonstrated pre- to post-test gains; and (c) whether Condition affected these outcome variables.

We first examined whether children performed above chance-level (phases 2-6: 50%, phase 7: 25%) on our patterns task (Figure 2). We find that children scored at-chance on the pre-test (*M*=45.2%, *p*>.2), but above chance-level on training trials (*M*=75.7%), post-test trials (*M*=74.5%,), generalization-4 trials (*M*=69.3%), and transfer trials (*M*=51.7%, all *p's*<.001, *Cohen's d*>1.4). Children failed to perform above-chance on generalization-conflict trials (*M*=41.3%, *p*=.191).

Children also demonstrated significant pre- to post-test gains on our patterns task (difference score = posttest – pretest). That is, children's difference score differed significantly from zero (*M*=29.3%, *p*<.001, *Cohen's d*=2.1), and this was confirmed by comparing pre- to post-test accuracy as well ($M_{pretest}$=45.2% vs. $M_{posttest}$=74.5%; *p*<.001). In fact, although children failed to score above chance-level at pre-test (*p*>.2), children performed significantly above 50% at post-test (*p*<.001), indicating that their matching at post-test favored a pattern, rather than perceptual, match. Interestingly, children's gains did not differ by condition: That is, children in the Abstract Labels (M=33.8%) demonstrated statistically similar gains, albeit numerically higher, as in the Concrete Labels condition (*M*=25.2%, *p*>.3).

Critically, we also examined the role of Condition in our outcome variables. A repeated measures ANOVA examined the within-subjects variable of Phase (6) and the between-subjects variable of Condition (2).

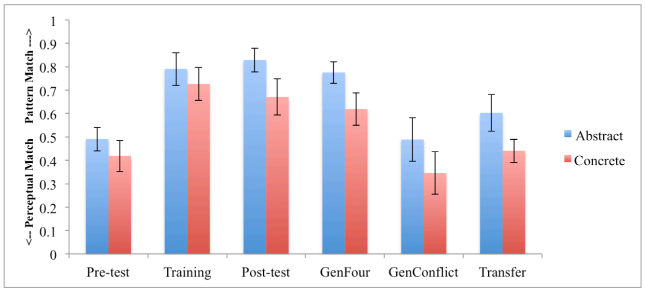

Figure 2. Accuracy on the Patterns Task in Experiment 1 as a function of Phase and Condition.

Unsurprisingly, there was a main effect of Phase ($p<.001$, $n^2_p>.3$). Condition did not significantly interact with Phase ($p>3$); however, a marginal main effect of Condition ($p=.095$, $n^2_p=.08$) indicated a general advantage for those children in the Abstract > Concrete condition throughout this task.

Discussion

Results from Experiment 1 replicated and extended previous findings (e.g., Fyfe et al., 2015), indicating that preschool-age children can learn novel pattern relationships following a short training. In fact, children not only performed well at training and post-test, but generalized and transferred this new relational knowledge across subsequent phases. Importantly, we observed greater gains and further generalization and transfer following training with Abstract Labels, as compared to Concrete Labels. However, relational language is only one factor that may influence children's understanding of patterns. Other theoretically-grounded factors are explored next.

Experiment 2

Experiment 2 sought to further explore what factors may impact children's ability to learn, retain, and generalize novel pattern across two sub-experiments.

2A: Instance Variability. Research suggests that instance variability affects generalization (Hahn et al., 2005; Heit & Hahn, 2001). This research indicates that greater variability within a category may broaden category boundaries. Relational judgments create competition for children's attention, such that young children are unsure of whether to pay attention to the object itself or the relation that they are tasked with recognizing. Thus, it is unclear whether children's ability to learn about multiple patterns – and thus abstract this information more broadly – would benefit from higher vs. lower variability. Experiment 2A examines the impact of instance variability on children's ability to learn and generalize patterns. Here, we train children using less variable exemplars (within and across training trials; compared to Experiment 1) in an effort to promote higher success on this task.

2B: Level of Idealization. This avenue warrants further study considering the widespread use of concrete instantiations and visual displays in real-world classrooms (Van de Walle, 2007). These high-contrast items presumably grab children's attention an effort to keep them interested in the materials being taught (Fisher, Godwin, & Seltman, 2014; Peterson & McNeil, 2012). The impact of perceptual variables on children's ability to learn about novel patterns is particularly relevant given recent research suggesting the role of children's underdeveloped ability to filter out irrelevant information in classification and categorization tasks (Deng & Sloutsky, 2015, 2016; Plebanek & Sloutsky, 2017). Because young children allocate their attention to both relevant and irrelevant aspects of novel categories (i.e., a novel pattern), this in turn affects how that information is encoded. If the goal of math education is to direct children's attention to the relation being taught, perhaps less interesting and less concrete learning examples may serve better (Son et al., 2012). Therefore, Experiment 2B trained children on novel patterns through the use of concrete, visually engaging representations (e.g., real world objects rather than geometric shapes).

Method

Participants

Forty-eight 4- to 6-year-old additional children participated in this study (M_{Age}=4.8 years, SD=0.71 years). Children were assigned to one of two conditions: Experiment 2A: Instance Variability (N=29) or Experiment 2B: Real Objects (N=18). Data from Experiment 1's Abstract Labels condition was included in data analyses. Children were tested in their own preschool, during regular school hours, or in a single visit to our laboratory on the main campus of a major Midwestern university. All children were tested in a quiet room with a single experimenter.

Materials

The materials were identical to those used in Experiment 1 with the following differences: For Experiment 2A (Instance Variability), training trials used the same language as the Abstract Labels condition in Experiment 1, but visual stimuli were consistent in color and shape across training; that is, the comparison example and two solves within a single training cluster used the same geometric shape and color consistently (see Figure 3). For Experiment 2B (Real Objects), training trials used the same language as the Abstract Labels condition in Experiment 1, but visual stimuli consisted of real-world objects rather than geometric shapes (e.g., dogs, planes, couch, etc.; see Figure 3). Items varied from trial to trial.

Procedure

The procedure was identical to that of Experiment 1. Both Experiment 2A (Instance Variability) and Experiment 2B (Real Objects) conditions were run using the prompts and wording as the Abstract Labels condition of Experiment 1.

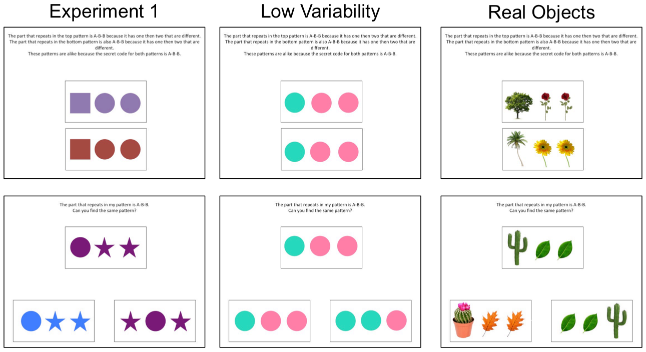

Figure 3. Examples of stimuli used in Experiment 2 training trials.

Results

The analyses for Experiment 2 explored whether either Instance Variability or Real Objects affected children's ability to learn, retain, and generalize pattern concepts, as well as how this compared to children's performance in the Abstract Labels condition of Experiment 1.

Experiment 2: Collapsed. A repeated measures ANOVA examining the within-subjects variable of Phase (6) and the between-subjects variable of Experiment (3: 1, 2A, 2B) revealed, unsurprisingly, a main effect of Phase (p<.001, n^2_p>.2) and an interaction between Phase and Experiment (p<.001, n^2_p>.1; Figure 4). Therefore, data from Experiments 2A and 2B were analyzed separately.

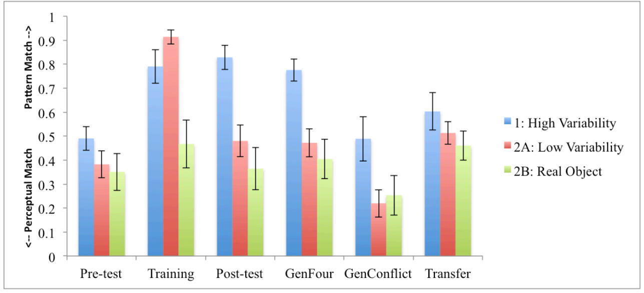

Figure 4. Accuracy on the patterns task across training conditions in Experiments 1 (Abstract Labels) and 2 (Low Variability, Real Objects).

Experiment 2A: Instance Variability. Children in Experiments 1 and 2A did not differ in accuracy at pre-test (p>.1). Low variability promoted children's ability to learn novel patterns during training ($M_{Experiment1}$=79.0% vs. $M_{LowVariability}$=91.4%, p=.069). However, unlike the children in Experiment 1, the children trained with low variability could not maintain this pattern knowledge in post-test or generalization (*p's*<.02, *Cohen's d's*>0.78; see Figure 4). Finally, children in Experiment 1 demonstrated greater gains (M=37.2%) than those in trained with low variability (M=9.8%; t(44)=3.25, p=.002, *Cohen's d*=1.0). Of note, children's gains in the Low Variability condition did not statistically exceed zero (difference score: t(28)=2.0, p=.054, *Cohen's d*=1.0), indicating that they did not make any pre- to post-test gains, as observed in Experiment 1.

Experiment 2B: Real Objects. Children in Experiments 1 and 2B did not differ in accuracy at pre-test (p>.1). Children run in the Real Objects condition failed to choose the pattern match across all phases. Children performed at chance-level during pre-test (M=35.1%, p=.07), training (M=46.7%, p>.7), post-test (M=36.4%, p>.1), Gen-4 (M=40.5%, p>.2), and transfer (M=46.0%, p>.5; see Figure 4). Children were significantly below chance – meaning they statistically chose the perceptual match more often than the pattern match – for Gen-Conflict trials: M=25.3%; p=.008, *Cohen's d*=.02). Finally, children did not show any gains from pre- to post-test (difference score vs. 0: t(19)=0.23, p>.8), indicating that they failed to learn the novel pattern concepts during training. Unsurprisingly, children in the Real Objects condition varied significantly from those in Experiment 1 (t(34)=3.34, p=.002, *Cohen's d*=1.13).

Discussion

Results from Experiment 2 extend the findings from Experiment 1 and implicate two factors that detract from children's ability to learn about novel patterns. First, although Instance Variability training led to equal performance at training and small gains from pre- to post-test, this was not maintained during generalization. Second, training in the Real Objects condition did not lead to children's learning of novel patterns, with a tendency to select the perceptual match over the pattern match.

General Discussion

Understanding the critical factors that facilitate early relational knowledge is important for the development of instructional materials and for curriculum development. The aim of the present study was to examine the factors that facilitate the learning and transfer of pattern knowledge.

In two experiments, 4- to 6-year-old children participated in a pre-post test design, in which they received pattern training. Critically, we manipulated (1) the relational language with which children were exposed to novel patterns during training and (2) whether the perceptual format in which children were initially exposed to novel patterns affected their learning of those materials. We find that 4-6 year old children were able to learn about novel patterns following this intervention, but faired best when trained on abstract ("A-B-A") rather than concrete ("red-blue-red") labels (extending and replicating findings from Fyfe et al., 2015; also see Rattermann & Gentner, 1998). It seems that the use of more abstract language directed children's attention to the higher-order relational structure being taught.

Furthermore, when variability across trials was minimized during training (2A: Instance Variability), children successfully learned novel pattern concepts and showed modest gains from pre- to post-test; however, this minimal variability negatively impacted their ability to generalize these concepts following training. These results suggest that, while it may be tempting to reduce attentional demands by limiting the variability that children see when learning novel patterns, this is not beneficial in the long-run. These findings also add to a greater body of work indicating that instance variability affects generalization (Hahn et al., 2005; Heit & Hahn, 2001), such that a broader variability across trials, as in Experiment 1, resulted in broader category boundaries.

The idealization of the training stimuli also detrimentally impacted children's ability to learn about novel patterns (2B: Real Objects). Children failed to learn the pattern match during training, suggesting that they diffused attention to both relevant and irrelevant aspects of the items (i.e., objects > relations, per Kuwabara & Smith, 2012; Rattermann & Gentner, 1998). Children showed more advanced relational reasoning when objects were abstract and simple and showed less relational reasoning when objects were rich and detailed. These simpler stimuli may have weakened the competition from the objects themselves, promoting relational learning. Much like the abstract language in Experiment 1, it seems that perceptual instances that are appropriately vague allow children to attend to relations > objects across learning exemplars.

The present study contains several limitations. First, the sample was not large enough so as to examine age effects, but this may be an interesting future direction, as one would expect to see developmental changes across the preschool years. Second, Experiment 2 did not examine the use of Concrete Labels, as in Experiment 1, so a 2x3 cross-experiment analysis could not be conducted. Finally, the specificity of this pattern task does not speak to (a) the extent to which this training holds in the long-term, or whether extended training could facilitate different patterns of learning (e.g., Carvalho & Goldstone, 2017) or (b) the link between patterning and broader accounts of analogical reasoning (e.g., Richland, Morrison, & Holyoak, 2006).

In sum, the present study investigated what factors impact pattern learning, generalization, and transfer in preschool-age children. This study adds to limited prior research examining patterning as an early indicator of relational thinking. This work has implications for instructional design and curriculum development in the classroom.

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References

Carvalho, P.F., & Goldstone, R.L. (2017). The sequence of study changes what information is attended to, encoded, and remembered during category learning. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, doi: http://dx.doi.org/10.1037/xlm0000406

Charles, R. I. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. Journal of Mathematics Education Leadership, 8, 9–24.

Deng, W., & Sloutsky, V.M. (2015). The development of categorization: Effects of classification and inference training on category representations. *Developmental Psychology, 51*, 392-405.

Deng, W., & Sloutsky, V.M. (2016). Selective attention, diffused attention, and the development of categorization. *Cognitive Psychology, 91*, 24-62.

Fisher, A. V., Godwin, K. E., & Seltman, H. (2014). Visual environment, attention allocation, and learning in young children: When too much of a good thing may be bad. *Psychological Science*, *25*, 1362–1370.

Fyfe, E.R., McNeil, N.M., & Rittle-Johnson, B. (2015). Easy as ABCABC: Abstract language facilitates performance on a concrete patterning task. *Child Development, 86*(3), 927-935.

Gentner, D., & Medina, J. (1998). Similarity and the development of rules. *Cognition, 65*(2), 263-297.

Hahn, U., Bailey, T.M., & Elvin, L.B.C. (2005). Effects of category diversity on learning, memory, and generalization. *Memory & Cognition, 33*(2), 289-302.

Heit, E., &Hahn, U. (2001). Diversity-based reasoning in children. *Cognitive Psychology*,**43**, 243–273.

Kanwisher, N., & Driver, J. (1992). Objects, attributes, and visual attention: Which, what, and where. *Current Directions in Psychological Science*, 1(1).

Kidd, J. K., Pasnak, R., Gadzichowski, K. M., Gallington, D. A., McKnight, P., Boyer, C. E., & Carlson, A. (2014). Instructing first-grade children on patterning improves reading and mathematics. Early Education & Development, 25, 134–151.

Kotovsky, L., & Gentner, D. (1996). Comparison and categorization in the development of relational similarity. Child Development, 67, 2797–2822.

Kuwabara, M., & Smith, L.B. (2012). Cross-cultural differences in cognitive development: Attention to relations and objects. *Journal of Experimental Child Psychology, 113*(1), 20-35.

National Center for Education Statistics (2010). *The Nation's Report Card: Mathematics 2009: National Assessment of Educational Progress at Grades 4 and 8.* (NCES 2010-451). Institute of Education Sciences, U.S. Department of Education, Washington, D.C.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA.

Peterson, L.A., & McNeil, N.M. (2012). Effects of Perceptually Rich Manipulatives on Preschoolers' Counting Performance: Established Knowledge Counts. *Child Development, 84*(3), 1020-1033.

Plebanek, D.J., & Sloutsky, V.M. (2017). Costs of selective attention: When children notice what adults miss. *Psychological Science, 28*, 723-732.

Rattermann, M.J., & Gentner, D. (1998). *The effect of language on similarity: The use of relational labels improves young children's performance in a mapping task.* In K. Holyoak, D. Gentner, & B. Kokinov (Eds.), Advances in analogy research: Integration of theory & data from the cognitive, computational, and neural sciences (pp. 274-288). Sophia: New Bulgarian University.

Rittle-Johnson, B., Fyfe, E.R., Hofer, K.G., & Farran, D.C. (2016). Early math trajectories: Low-income children's mathematics knowledge from age 4 to 11. *Child Development, 88*(5), 1727-1742.

Rueda, M.R., Posner, M.I., & Rothbart, M.K. (2005). The development of executive attention: Contributions to the emergence of self-regulation. *Developmental Neuropsychology, 28*, 573-594.

Sarama, 1., & Clements, D. H. (2009). *Early childhood mathematics education research: learning trajectories for young children.* New York: Routledge.

Siegler, R.S., Duncan, G.J., Davis-Kean, P.E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M.I., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science, 23*.

Son, J.Y., Smith, L.B., & Goldstone, R.L. (2011). Connecting instances to promote children's relational reasoning. *Journal of Experimental Child Psychology, 108*, 260-277.

Son, J.Y., Smith, L.B., Goldstone, R.L., & Leslie, M. (2012). The importance o being interpreted: Grounded words and children's relational reasoning. *Frontiers in Psychology, 3*, 45.

Steen, L. A. (1988). The science of patterns. Science, 240, 611–616.

Van de Walle, J.A. (2007). *Elementary and Middle School Mathematics: Teaching Developmentally*. New York: Pearson.