Task dynamics reveal how fraction values are constructed

Richard Prather (prather1@umd.edu)
Department of Human Development and Quantitative Methodology, 3304 Benjamin Building
College Park, MD 27042 USA

Abstract

We evaluate how learners construct internal representations of fraction values. Symbolic numbers written using fraction notation are difficult for both children and adults to use. Errors made by learners suggest that even experienced adults can lack fluency with fractions. One such error is the Natural Number bias phenomenon: when the relative size of fractions values to be compared is incongruent with the relative size of the fraction components learners show a reaction time delay or decreased accuracy. For example, noting that 1/7 is smaller than 1/5 may take longer that noting that 3/10 is smaller than 5/10. We adjust the temporal dynamics of the fraction comparison task to characterize how learners construct fraction values from the constituent parts. We also create a mathematical model of the fraction value construction.

Keywords: dynamic systems; neural network; numerical cognition; decision making

Introduction

Learning about fractions, and the symbolic number system in general is about learning to construct meaning about magnitudes from symbols. Children must learn how a small set of symbols 0 – 9 can be used in various permutations to symbolize an infinite set of numbers. Learners commonly have initial difficulty in using fraction notation. The use of fraction notation is rife with errors for both children and adults. This difficulty may be due to a lack of fluency with the notation and a problem with how learners construct values from the symbolic notation. The common errors made with fractions may reveal the cognitive processes children employ in constructing fraction values

In this study, we evaluate how learners construct fraction values from the composite symbols. While there is not currently a comprehensive account of how learners construct fraction values, there are some relevant findings. Eye-tracking data suggests that, when comparing fractions, adults initially focus on the denominators of the two fraction values. (Huber, Moeller, & Nuerk, 2014; Obersteiner & Tumpek, 2016). There are descriptions of common errors made with fractions and hypothesis as to why these errors are made. One such error is the Natural number bias (e.g., Alibali & Sidney, 2015; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013; Vamvakoussi et al., 2012).

The *natural number bias* (NNB) refers to a behavioral phenomenon in which learners are slower and more error-prone when comparing symbolic numerical values when the value of the natural numbers components conflicts with the actual values to be compared. For example, a learner may incorrectly judge that the fraction *1/7* is a larger value than *1/5*. This error is thought to be due to interference from the fact that the natural number *7* is larger than *5*. Conversely, fraction pairs may be congruent, where relative values for the fraction and natural number do match, e.g., *2/3* is less than *6/7*. The natural number bias is typically demonstrated by higher error rates and slower reaction times for incongruent comparisons relative to congruent ones. The natural number bias has been observed in many populations, including elementary school-aged children (e.g., Meert, Gregoire, & Noel, 2010), high school students (e.g., DeWolf & Vosniadou, 2015), adults (e.g., Vamvakoussi, Van Dooren, & Verschaffel, 2012b), and even in expert mathematicians (e.g., Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013).

What aspect of the cognitive process of comparing symbolic fractions might explain the natural number bias phenomenon? The natural number bias shows both individual variation (Alibali & Sidney, 2015) and variation across situations (e.g., DeWolf & Vosniadou, 2015; Huber, Moeller, & Nuerk, 2014). Prediction of individual variation across learners, variation across contexts, specific stimuli, and development is necessary for a comprehensive account of the natural number bias.

Task Dynamics Approach

In the current study, we focus on how the construction of fraction values by learners can be evaluated by varying the task dynamics of the fraction comparison task. Given the evidence that fraction processing differs by the affordances of the problem, we manipulated the task dynamics of the fraction comparison task to more explicitly characterize the cognitive processes involved. Each fraction comparison depends on the relative values conveyed by two numerators and two denominators. We examine how variations in the presentation timing for each component are associated with changes in participants' behavior on the fraction comparison task. We vary time as a way to investigate what if anything is done with the incomplete information about the fraction values. We combine empirical experimentation with mathematical modeling to create an account of how participants combine components to create fraction values.

We investigate how variation in the presentation of stimuli affects participants' behavior on the fraction comparison task as it relates to the Natural Number bias. Evidence of the natural number bias is demonstrated by evaluating reaction time in comparing fractions. Fraction pairs that are incongruent (e.g., 2/3 vs.2/4) take longer to compare than fraction pairs that are congruent (e.g., 3/7 vs. 5/7). We examine participant's performance regarding relative reaction time for congruent and incongruent fraction pairs. We include seven different fraction presentation styles across four experiments. Each fraction pair presentation contains four components: denominator 1, numerator 1, denominator 2, and numerator 2. The four components can be presented simultaneously or with a delay for one or more components (see Figure 1). We used seven unique sequences: No Delay, Single Numerator Delay, Single Denominator Delay, Double Numerator Delay, Double Denominator Delay, Mixed Delay, Single Fraction Delay.

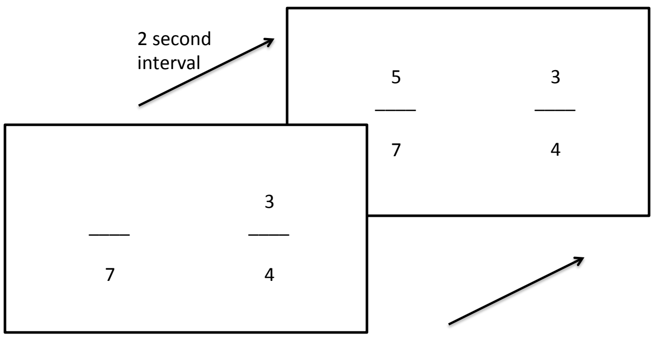

Figure 1. Schematic of dynamic fraction comparison task.

Behavioral Experiments

Experiment 1

In experiment one, we explored how variations in the fraction comparison task stimulus presentation may affect participants' comparison accuracy and speed. Participants completed the fraction comparison task across three conditions, No Delay, Single Numerator Delay, and Double Numerator Delay.

Method

Participants: Adults (n = 36) were recruited online via Amazon Mechanical Turk (age median 33, minimum 21, maximum 65). The experiment was designed to take approximately 20 minutes. Participants were compensated \$3.00. The analysis includes all participants who scored at least 70% overall (n = 34) to filter out participants who did not attend to the task. There was no time limit for each comparison

Procedure: Participants completed a fraction comparison task. Instructions indicated that two fractions would appear on the screen, participants were to indicate which fraction was a larger value via a button press. Participants were instructed that there may be a slight delay in the display of either fraction and they should respond as quickly and accurately as possible once both fractions were fully displayed. Fractions remained on the screen until the participant responded.

Fraction Comparison Task. Participants' only task was the fraction comparison task. Stimuli from this task were based on the stimulus set in prior work (Meert, Grégoire, & Noël, 2009). There were 64 unique comparisons, each viewed three times for a total of 192 comparisons for each participant. Fraction comparisons stimuli were blocked by presentation style: *No Delay, Single Numerator Delay, and Double Numerator Delay*. The order of the blocks was randomized, as was the order of the stimuli within each block. For *Single Numerator Delay* comparisons, the fraction on the left of the screen was initially presented with a blank numerator. The numerator then was displayed after a delay of 2 seconds. The delay length was chosen to be just long enough for participants to view the presented values. For the *Double Numerator Delay* comparisons, both fractions were initially presented with blank numerators.

Results and Discussion

We analyzed participant reaction time across all comparisons. For each participant we excluded reaction times longer than 6 seconds, representing less than 2% of trials. Outlier reaction times, sometimes as long as 30 seconds suggested the participant may not have been attending to the task. For each condition we complete a multiple linear regression predicting reaction time using trial type (congruent vs. incongruent) and fraction difference ratio, with the participant as a random effect. For the analyses, all reaction times were arcsine transformed.

For No Delay condition reaction time was predicted by congruency ($t = 5.89$, $p < 0.001$) but not fraction difference ($t = 1.25$, $p = 0.21$). Reaction times were consistent with the natural number bias in that incongruent trials had significantly longer reaction times (see Figure 2).

For Single Numerator Delay condition comparison reaction time was not predicted by congruency ($t = 0.95$, $p = 0.33$) or ratio difference ($t = 0.32$, $p = 0.74$). Reaction times were inconsistent with the natural number bias in that incongruent trials did not have significantly longer reaction times (see Figure 3).

For Double Numerator Delay condition comparison reaction time was not predicted by congruency ($t = 0.16$, $p = 0.86$) or fraction difference ($t = 1.88$, $p = 0.059$). Reaction times were inconsistent with the natural number bias in that incongruent trials did not have significantly longer reaction times (see Figure 4).

We find the natural number bias effect in the No Delay condition, in which fraction pairs were presented the same manner as previous work. However, for the Single Numerator delay and Double Numerator delay conditions, there is no evidence of a natural number bias effect. This result is despite the fact that the same fraction pairs are used.

Experiment 2

In the current experiment, each participant completed the fraction comparison task across three conditions, No Delay, *Single Denominator Delay*, and *Double Denominator Delay*. For *Single Denominator Delay* comparisons, the fraction on the left of the screen was initially presented with a blank denominator. The denominator then was displayed after a delay of 2 seconds. For the *Double Denominator Delay* comparisons, both fractions were initially presented with blank denominators.

Method

Participants: Adults ($n = 35$). Recruitment and payment was the same as in Experiment 1. The analysis includes all participants who scored at least 70% overall, $n = 33$.

Procedure: Procedure was identical to Experiment 1. Fraction comparisons stimuli were blocked by presentation style: *No Delay, Single Denominator Delay, and Double Denominator Delay*.

Results and Discussion

We analyzed participant reaction time across all comparisons for each participant we excluded reaction times longer than 6 seconds, representing less than 3% of trials. For the analyses, all reaction times were arcsine transformed. For each condition we complete a regression predicting reaction time using trial type (congruent vs. incongruent), fraction ratio, numerator ratio, and denominator ratio, with the participant as a random factor.

For No Delay condition reaction time was predicted congruency ($t = 2.19$, $p = 0.028$) and fraction difference ($t = 2.60$, $p < 0.01$). Reaction times were consistent with the Natural number bias in that incongruent trials had significantly longer reaction times (see Figure 5).

For Single Denominator Delay condition reaction time was predicted congruency ($t = 3.41$, $p < 0.001$) but not fraction difference ($t = 1.58$, $p = 0.11$). Reaction times were consistent with the Natural number bias in that incongruent trials had significantly longer reaction times (see Figure 6).

For Double Denominator Delay condition reaction time was predicted congruency ($t = 4.97$, $p < 0.001$) but not fraction difference ($t = 1.01$, $p = 0.31$). Reaction times were consistent with the Natural number bias in that incongruent trials had significantly longer reaction times (see Figure 7).

We find the natural number bias effect in the No Delay condition, in which fraction pairs were presented the same manner as previous work. For the Single Denominator and Double Denominator conditions, we also find that incongruent trials had significantly longer reaction times, consistent with the Natural Number bias.

Experiment 3

In the current experiment, participants completed the fraction comparison task across three conditions, No Delay, Mixed Delay, and Single Fraction Delay. For Mixed delay condition, one fraction had a delayed numerator while the other fraction had a delayed denominator. For Single Fraction delay condition, both the numerator and denominator of one fraction value were delayed.

Method

Participants: Adults ($n = 31$). Recruitment and payment was the same as in Experiment 1. The analysis includes all participants who scored at least 70% overall, $n = 27$.

Procedure. The procedure was identical to Experiment 1. Fraction comparisons stimuli were blocked by presentation style: *No Delay, Mixed Delay, and Single Fraction Delay*.

Results and Discussion

We analyzed participant reaction time across all comparisons. For each participant we excluded reaction times longer than 6 seconds, representing less than 3% of trials. For each condition we complete a multiple linear regression predicting reaction time using trial type (congruent vs. incongruent) and fraction difference ratio, with the participant as a random effect. For the analyses, all reaction times were arcsine transformed.

For No Delay condition reaction time was predicted congruency ($t = 1.72$, $p = 0.08$) and fraction difference ($t = 2.88$, $p < 0.01$). Reaction times were consistent with the Natural number bias in that incongruent trials were not significantly longer reaction times (see Figure 8).

For Mixed Delay condition reaction time was predicted congruency ($t = 3.52$, $p < 0.001$) but not fraction difference ($t = 1.52$, $p = 0.12$). Reaction times were consistent with the Natural number bias in that incongruent trials had significantly longer reaction times (see Figure 9).

For Single Fraction Delay condition reaction time was predicted congruency ($t = 2.62$, $p = 0.02$) and fraction difference ($t = 3.44$, $p < 0.001$). Reaction times were consistent with the Natural number bias in that incongruent trials had significantly longer reaction times (see Figure 10).

We find that for Mixed Delay and Single Fraction delay conditions reaction times were consistent with the Natural Number bias. For the No Delay condition, though incongruent trials were slower than congruent trials this difference did not reach $p = 0.05$ significance.

Experiment 4

In this experiment, we sought to replicate the lack of natural number bias in numerator delay trial types observed in Experiment 1. We also sought to provide within subject evidence that delayed numerator trials take less response time than delayed denominator trials. Participants completed the fraction comparison task across three conditions; No Delay, Double Numerator Delay and Double Denominator Delay.

Method

Participants: Adults ($n = 37$). Recruitment and payment was the same as in Experiment 1. The analysis includes all participants who scored at least 70% overall, $n = 30$.

Procedure: Procedure was identical to Experiment 1. Fraction comparisons stimuli were blocked by presentation style: *No Delay, Double Numerator Delay, and Double Denominator Delay*.

Results and Discussion

We analyzed participant reaction time across all comparisons. For each participant we excluded reaction times longer than 6 seconds, representing less than 2% of trials. For each condition we complete a multiple linear regression predicting reaction time using trial type (congruent vs. incongruent) and fraction difference ratio, with the participant as a random effect. For the analyses, all reaction times were arcsine transformed.

For No Delay condition reaction time was predicted congruency ($t = 4.80$, $p < 0.001$) and fraction difference ($t = 2.39$, $p = 0.01$). Reaction times were consistent with the Natural number bias in that incongruent trials had significantly longer reaction times (see Figure 11).

For Double denominator Delay condition reaction time was predicted congruency ($t = 6.48$, $p < 0.001$) but not fraction difference ($t = 1.53$, $p = 0.12$). Reaction times were consistent with the Natural number bias in that incongruent trials had significantly longer reaction times (see Figure 12).

For Double Numerator Delay condition reaction time was predicted congruency ($t = 2.64$, $p < 0.01$) but not fraction difference ($t = 1.34$, $p = 0.17$). In this case reaction times were inconsistent with the Natural number bias in that incongruent trials had significantly faster reaction times (see Figure 13). Post-hoc contrast of participants' mean reaction times across conditions was not significant in comparing congruent ($M = 1.49$) and incongruent ($M = 1.41$) trials, $t(29) = 1.29$, $p = 0.21$.

We also directly compared reaction times between numerator delay and denominator delay conditions. Reaction times for numerator delay trials were significantly faster ($M = 1.65$) than denominator delay trials ($M = 1.95$), $t(29) = 2.62$, $p = 0.013$.

This experiment replicates out finding from Experiment 1 that when numerators are delayed there is not an observed Natural Number bias. We again replicate the Natural number bias for No Delay condition, consistent with prior work. Additionally, we find that in general numerator delayed trials were faster than denominator delayed trials. These results further support the account that numerator and denominator information is treated differently in constructing fraction values.

Experiment 1-4 Discussion

The results from experiments 1 – 4 demonstrate that the natural number bias in fraction comparison depends on the dynamics of the stimulus presentation. When both fractions are presented with no delay, we find the same natural number bias previously reported. Fractions in which the natural number component relative magnitude is incongruent with the fractions relative magnitude take participants longer to compare. Across the experiments, we also present six other styles of stimulus presentation, Single Numerator Delay, Single Denominator Delay, Double Numerator Delay, Double Denominator Delay, Mixed Delay, Single Fraction Delay. We find that for some presentation styles the natural number bias is not present. For both single and double numerator delays we found no difference in participants' reaction time in comparing congruent and incongruent fraction. We find this for independent samples in experiment 1 and experiment 4. For every other presentation style, singe denominator delay, double denominator delay, mixed delay, and single fraction delay, the natural number bias effect is present.

Mathematical Modeling

To further characterize the governing dynamics of the fraction comparison we constructed a mathematical model of the task. The purpose of the models is to examine how hypothesized internal representations and their dynamics give rise to the patterns of behavior reported in experiments 1 – 4. The model is constructed to simulate the fraction comparison task using a multilayered dynamic systems neural network. The model makes fraction comparisons corresponding to each of the seven trial types: No Delay, Single Numerator Delay, Single Denominator Delay, Double Numerator Delay, Double Denominator Delay, Mixed Delay, Single Fraction Delay.

The model study includes three conditions to evaluate our hypothesis regarding the construction of fraction values:

Model Condition A: construction of fraction values requires all components to be present.

Model Condition B: construction of fraction values begins as soon as the denominator is present.

Model Condition C: construction of fraction values requires all components to be present, but the model will use shortcuts when common components are presented first (e.g. x/10 vs. x/10).

We assume that in processing a fraction, the natural number values are relatively quickly represented while the fraction value is constructed slowly depending on the presentation timing of fraction components. This process involves inhibiting the representation of the natural numbers. Thus in the first milliseconds of viewing 7/18, the representation of 7 and 18 is relatively strong, while the representation of 7/18 is relatively weak. The important question is what processes are involved in constructing the fraction value. We hypothesize that the lack of natural number bias for numerator delay conditions is because when both denominators are present, the fraction value construction begins. Where when either denominator is delayed then no progress is made in constructing the fraction value.

Consider how the model condition results may differ on numerator delay trials. In single and double numerator delay the participants view both denominators for a specified time, though no information is given that indicates which fraction will be larger. One possibility is that the fraction value is constructed through the denominator more so than the numerator. In that case 10/X is less useful information than X/10. When given X/10 the possible values of X are limited. In this case the denominator, as kind of a sense of how big each part of the whole is, needs to be set. The relative size of the parts (the denominator) is then more useful information than the number of parts (the numerator).

Results and Discussion

Model Condition A

We evaluate the number of time-steps taken in completing each comparison. The model completed the same comparison trials as participants. Model decisions were significantly faster for congruent trials compared to incongruent trials for every type of trial; No Delay trials (M = 257 vs. M = 339 time-steps; t(62) = 4.23, p < 0.001), Single Numerator Delay trials (M = 266 vs. M = 352 time-steps), t(62) = 4.57, p = <0.001), Double Numerator Delay trials (M = 270 vs. M = 380 time-steps), t(62) = 5.22, p < 0.001), Single Denominator Delay trials (M = 270 vs. M = 346 time-steps, t(62) = 4.07, p < 0.001), Double Denominator Delay trials (M = 300 vs. M = 351 time-steps), t(62) = 2.89, p < 0.001), Mixed Delay trials (M = 296 vs. M = 372 time-steps, t(62) = 4.54, p < 0.001), Single Fraction Delay trials (M = 263 vs. M = 347 time-steps, t(62) = 4.47, p < 0.001). We also compared double numerator delay (M = 334) to double denominator delay (M = 326) and found no significant difference between the two, t(62) = 1.86, p = 0.066.

Model Condition B.

We evaluate the number of time-steps taken in completing each comparison. Model completed the same comparison trials as participants. Model decisions were significantly faster for congruent trials compared to incongruent trials for No Delay trials (M = 257 vs. M = 338 time-steps, t(62) = 4.18, p < 0.001), Single Denominator Delay trials (M = 270 vs. M = 346 time-steps, t(62) = 4.08, p < 0.001), Double Denominator Delay (M = 299 vs. M = 348 time-steps, t(62) = 2.81, p < 0.001), Mixed Delay trials (M = 296 vs. M = 376 time-steps, t(62) = 4.46, p < 0.001), Single Fraction Delay trials (M = 263 vs. M = 341 time-steps, t(62) = 4.46, p < 0.001).

Model decisions were not significantly faster for congruent trials compared to incongruent trials for Single Numerator Delay trials (M = 233 vs. M = 221 time-steps, t(62) = 0.48, p = 0.63), or for Double Numerator Delay (M = 253 vs. M = 247 time-steps, t(62) = 0.21, p = 0.83). We also compared double numerator delay (M = 250) to double denominator delay (M = 324) and found numerator delay trials were significantly faster, t(63) = 4.42, p < 0.001.

Model Condition C.

For model condition C we investigate how the model would behave if it used a natural number comparison shortcut. Construction of fraction values requires all components to be present, but the model will use shortcuts if common components are presented first. (e.g. x/10 vs x/10). Model condition C is meant to address the possibility that the behavioral data is due to strategy use by participants and not delayed fraction value construction. We find that when this strategy is used, there is still a natural number bias for No Delay, Single and Double Numerator delay conditions. This result is counter to what is found in Model condition B.

We evaluated the number of time-steps taken in completing each comparison. Model completed the same comparison trials as participants. Model decisions were significantly faster for congruent trials compared to incongruent trials for No Delay trials the model decisions were significantly faster for congruent trials (M = 338 vs. M = 386 time-steps, t(62) = 2.23, p < 0.01), Single Numerator Delay trials (M = 307 vs. M = 362 time-steps, t(62) = 3.14, p < 0.01), Double Numerator Delay trials (M = 338 vs. M = 392 time-steps, t(62) = 3.74, p < 0.01), Mixed Delay trials (M = 373 vs. M = 423 time-steps, t(62) = 2.91, p < 0.01), Single Fraction Delay trials (M = 329 vs. M = 362 time-steps, t(62) = 1.63, p = 0.10).

Model decisions were not significantly faster for congruent trials compared to incongruent trials for Single Denominator Delay trials the model decisions were not significantly different than congruent trials (M = 351 time-steps) compared to incongruent trials (M = 382 time-steps), t(62) = 1.55, p = 0.12. For Double Denominator Delay trials the model decisions were not significantly faster for congruent trials (M = 352 time-steps) compared to incongruent trials (M = 360 time-steps), t(62) = 0.59, p = 0.55). We also compared double numerator delay trials (M = 365) to double denominator delay trials (M = 356) and found no significant difference between the two, t(63) = 1.45 , p = 0.15.

Model Study Conclusions

We compare three separate model approaches to fraction value construction and comparison. Model condition B, which constructs fraction values from denominator components, matched participant data. Mathematical modeling results suggest that the lack of natural number bias when numerators are delayed may be because of the fraction value construction process. The mathematical model version in which fraction value construction only requires a denominator to initiate replicates the behavioral data from experiments 1 – 4. The mathematical version that simply incorporates fraction components, as they are available does not match the behavioral data. We interpret this as additional evidence that participants begin the process of fraction value construction even when only aware of the denominator value.

General Discussion

Using a dynamic stimulus presentation, we reveal a novel behavioral phenomenon in which the natural number bias is not present when numerator values are briefly delayed. We further demonstrated using a series of mathematical models that the behavioral data are consistent with the hypothesis that learners construct fraction values beginning with the denominator value. Learners may start to construct the fraction value even when the numerator is unknown. Prior accounts of the natural number bias cannot explain this effect. The present a hypothesis, described via a mathematical model, is that learners' construction of fraction values requires knowledge of the denominator first. Modeling results demonstrate how such a fraction construction paradigm leads to the pattern of behavioral data observed in experiments 1 – 4. A model using a fraction construction paradigm that simply combines fraction components once they are available does not produce similar results. Together we take this as evidence that the dynamic fraction display provides empirical data that allows for more comprehensive characterization of the underlying processes in fraction comparison.

If participants begin construction of the fraction value when only presented with the denominator, what are they doing during that time? There are several possibilities, which may be difficult to distinguish, as they may not make different behavioral predictions. To borrow an educational metaphor of fractions being slices of a pie, then knowledge of the denominator gives you the number of slices the pie must be cut into, but not the number of slices to be taken. Thus the denominator allows for an estimate of how many parts the whole will be divided into.

Does the present data represent a 'true' measure of fraction comparison? Evidence from other work suggests that behavior in fraction comparison is context dependent. The range of fraction comparisons presented to the participant may constrain behavior. Stimulus sets that contain fractions with common components, as used here, may lead to the use of shortcuts that are not employed in other cases. Prior work showed adaptive processing of fractions that depended on the context of the fraction comparison (Huber et al., 2014). For example, if a stimulus set contained only same-denominator comparisons participants would process the fraction values via components, attending more to the numerators. In this case, numerators were informative while denominators were not. In other work, participants were also 'experts,' defined as adults with a degree in mathematics (Obersteiner, Hoof, Verschaffel, & Dooren, 2016; Obersteiner et al., 2013). These participants did not show the typical natural number bias when comparing fraction values.

Though the current work provides insight into how learner's construct fraction values variation in context could affect behavior. Of course, that is the case for fraction comparison as it is with arithmetic tasks in general (Prather & Alibali, 2009) and any tasks.

What do the conclusions of this study this suggest about the natural number bias and fractions? The results suggest that participants' processing of fractions values relies on the presence of a denominator but not a numerator. When numerator components were delayed, the natural number bias was not present. We found the natural number bias with no delay, the delay of a single fraction, or with the delay of one or both denominators. We interpret this to suggest that the participants in these cases are unable to begin to process both fraction values. Consider the case of a single fraction delay. In this case, no information of either component for one fraction is present at the outset. The reaction times of this condition are indistinguishable from single denominator delay. We interpret this as evidence that the participants cannot use an isolated numerator. On the other hand, a single numerator delay condition trials were both faster overall in terms of reaction time and did not show the natural number bias.

References

Alibali, M. W., & Sidney, P. G. (2015). Variability in the natural number bias: Who, when, how, and why. *Learning and Instruction*, *37*, 56–61.

DeWolf, M., & Vosniadou, S. (2015). The representation of fraction magnitudes and the whole number bias reconsidered. *Learning and Instruction*, *37*, 39–49.

Huber, S., Moeller, K., & Nuerk, H. C. (2014). Adaptive processing of fractions - Evidence from eye-tracking. *Acta Psychologica*, *148*, 37–48.

Meert, G., Grégoire, J., & Noël, M.-P. (2009). Rational numbers: componential versus holistic representation of fractions in a magnitude comparison task. *Quarterly Journal of Experimental Psychology*, *62*(8), 1598–1616.

Obersteiner, A., Hoof, J. Van, Verschaffel, L., & Dooren, W. Van. (2016). Who can escape the natural number bias in rational number tasks? A study involving students and experts. *British Journal of Psychology*, *107*(3), 537–555. 1

Obersteiner, A., & Tumpek, C. (2016). Measuring fraction comparison strategies with eye-tracking. *ZDM - Mathematics Education*, *48*(3), 255–266.

Obersteiner, A., Van Dooren, W., Van Hoof, J., & Verschaffel, L. (2013). The natural number bias and magnitude representation in fraction comparison by expert mathematicians. *Learning and Instruction*, *28*, 64–72.

Prather, R. W., & Alibali, M. W. (2009). The development of arithmetic principle knowledge: How do we know what learners know? *Developmental Review*, *29*(4), 221–248.

Vamvakoussi, X., Van Dooren, W., & Verschaffel, L. (2012). Naturally biased? In search for reaction time evidence for a natural number bias in adults. *Journal of Mathematical Behavior*, *31*(3), 344–355. http://doi.org/10.1016/j.jmathb.2012.02.001