

Semantic Processing in Fraction Comparison: An ERP Study

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Abstract

Fractions processing is a topic of major interest both in numerical cognition and mathematics education. The literature on the processing of common fractions has focused on whether fractions are compared by their magnitude or through their components. Only a few neuroimaging studies have looked at this question. The N400 component, traditionally seen in linguistic semantic congruency event-related-potential (ERPs) experimental designs, has been adapted to study arithmetic processing. Observing the N400, allows the study of how different arithmetic components affect overall processing. In this study, an N400 paradigm is used to investigate semantic congruency during a fraction magnitude comparison task (Match/Mismatch) in 24 adults. Behavioral results reveal interference by shared components across the compared fractions. EEG analysis results show an N400-like difference wave between Match and Mismatch conditions. Shared components modulate the latency of this N400 effect. These results show the N400 as a viable method for studying fractions.

Keywords: EEG; ERP; N400; Fractions; Fraction Processing; Mathematical Cognition

Introduction

Mathematical competence is a key cognitive skill for individual societal advancement, high wage earnings, and life achievement (Murnane, Willett, & Levy, 1995). On a larger scale, mathematical knowledge is crucial in the development of competitive 21st century workforces and in driving and sustaining overall societal advances in the information economy, medicine, and science (National Mathematics Advisory Panel, 2008).

Research in the development of mathematical ability has highlighted the understanding of fractions as a key component in the development of mathematics achievement (Siegler, Fazio, Bailey, & Zhou, 2013). Fraction knowledge is predictive of overall math achievement (Siegler et al., 2012) and might underlie the ability to develop understanding of concepts needed for higher level mathematics such as algebra (Booth & Newton, 2012). However, fractions remain one of the least understood topics by elementary school students (National Mathematics Advisory Panel, 2008). Furthermore, misconceptions about fractions are carried to high school and even college (Vamvakoussi, 2006).

Fractions are a bipartite representation of rational numbers in the quotient form a/b . Other representations of rational numbers include decimal numbers, ratios of the form $p:q$, and graphical illustrations. This distinction, although trivial, is central for understanding the context of fraction research. Some of the difficulties children encounter when learning fractions are because bipartite numbers are unintuitive. One key difficulty is the inability to perceive fractions' real magnitude and to fixate instead on numerator or denominator components as separate whole numbers (Lewis, Matthews, & Hubbard, 2015; Siegler et al., 2013). The application of rules and procedures learned from whole number arithmetic to fractions, has been called the "whole number bias" (Zhou & Ni, 2005).

Researchers have turned to study numerically fluent adults to understand whether they compare fractions by accessing the magnitude of the whole fraction (holistically) or by attending to the components separately (componentially). A numerical distance effect (NDE) paradigm, the increase of reaction time as the numerical distance between two numbers being compared decreases (Moyer & Landauer, 1967), has been a primary tool used in these fraction studies. The response time of a fraction comparison task can reflect either a numerical distance between the magnitude of a fraction and a target or a numerical distance effect between components of the two fractions being compared, thus discerning whether a componential or holistic comparison has been made.

One of the first studies to inquire into strategy use in adult fraction processing provided evidence of componential processing, instead of holistic magnitude, by showing a NDE between the denominators of unit fractions (those with a 1 numerator) being compared to the target $1/5$ fraction (Bonato, Fabbri, Umiltà, & Zorzi, 2007). More recent studies have highlighted how the nature of the stimuli, including unit fractions, can influence a componential comparisons (Toomarian & Hubbard, 2017; Zhang, Fang, Gabriel, & Szucs, 2014).

To better understand the role that various stimuli have on a fraction comparison task, two studies structured stimulus to either share a numerator ($2/5$ vs. $2/7$), a denominator ($2/3$ vs. $5/3$), or have no common components ($5/7$ vs. $2/9$). This study concluded that there was componential processing when the denominators were shared; but holistic processing

when the numerators were shared, suggesting a “hybrid” strategy (Meert, Grégoire, & Noël, 2010a, 2010b). Another study also limited componential strategies by restricting shared components across the fractions being compared and found a magnitude NDE, suggesting the magnitude of the whole fraction was being processed (Sprute & Temple, 2011). More than using full componential or holistic processing strategies, adult participants have the option of using generalized strategies for processing fractions depending on task demands. In fact, a study of trial by trial strategy used in fraction magnitude comparison showed that strategy used is dependent on stimuli, task, and level of expertise (Fazio, DeWolf, & Siegler, 2016).

Even though adults engage a hybrid strategy when comparing fractions, it is still an open question whether this strategy is the goal that educational instruction should aim to foster in young learners. One study found 10 and 12 year-olds, like adults, are able to compare fractions through their magnitude (holistically) but they were susceptible (as seen in longer reaction times) to interference from shared components between the two fractions being compared (Meert et al., 2010a). One additional study also concluded that the componential aspects of fractions might interfere with accessing the overall magnitude of fractions (DeWolf & Vosniadou, 2015). This study suggested that there is an “on-line” magnitude calculation that emerges out of a ratio estimation of the two fraction components rather than retrieving a magnitude in long term memory. Neuroimaging methods are fit to provide insights into the temporal aspects of this “on-line” processing.

Only a few neuroimaging studies have looked at fraction processing. Two functional MRI studies provided similar evidence of magnitude numerical distance effects in a fraction comparison task (holistic processing) (Ischebeck, Schocke, & Delazer, 2009; Jacob & Nieder, 2009). These studies observed that intraparietal sulcus (IPS) activation is modulated by the numerical distance between the magnitudes of two fractions being compared, and not by the components. Alternately, one EEG study concluded that amplitude and latency of the P3 component during a fraction comparison of common fractions to a 1/5 target (a simple condition) demonstrated evidence of componential processing that a comparison of fractions and decimals to the same target did not (a complex condition) (Zhang et al., 2012). In this study, the complex condition also showed longer latency and more negative amplitude of the N2 component over frontal electrodes than the simple condition, suggesting a more taxing cognitive demand. However, it is not clear whether the patterns observed on the complex condition comes from a different type of strategy or from the cognitive load of switching between fractions and decimals. Given the high temporal resolution of EEG, the use of event-related-potentials (ERPs) has much to offer as a method to study fraction processing.

Some numerical cognition studies have made use of the N400 component, traditionally seen in linguistic semantically incongruent comparisons, to study numerical processing in arithmetic tasks (Niedeggen, Rösler, & Jost 1999; Niedeggen, & Rösler, 1999). These studies have seen a similar ERP component in numerically incongruous trials (e.g. $7 \times 4 = 28$ vs. $7 \times 4 = 26$) as in linguistic ones (e.g. “The candle has burned” vs. “The ball has dreamed”), when the evoked potential of the incongruous trials is subtracted from that of the congruent ones. This evoked potential difference has been named the “arithmetic” N400.

The arithmetic N400 allows the examination of how variations of incongruous stimuli modulate the amplitude and latency of the ERP, for example how unrelated errors like $3 \times 6 = 19$ differ from related ones $3 \times 6 = 24$ (Niedeggen et al., 1999). Similar interference effects are seen in behavioral tasks such as two digit number comparisons with slower reaction times when the comparison of unit digits between the two numbers are incongruous to the comparison of tenths digit (e.g. 47 vs. 62 , $4 < 6$ while $7 > 2$) (Nuerk, Weger, & Willmes, 2001). The authors of this study concluded that the unit-decade-compatibility between the numbers being compared interfered with the number judgment for both small and large numerical distance. Given this trend in multi digit number comparisons, an N400 paradigm for fraction comparison could distill the role that shared components play in overall processing.

Despite the potential of the N400 component in studying fraction processing, the N400 paradigm has not yet been applied to fraction comparison tasks. Doing so can help understand how erroneous solutions arise and how they are influenced by the stimulus. In the present study, an N400 paradigm was used during a fraction comparison task where participants judged the magnitude equivalence of two fractions on a match/mismatch task paradigm while EEGs were recorded. The main aim of the study is to show the use of the N400 as a viable paradigm for studying fractions and to explore how different fraction stimuli can affect the reaction time, accuracy, and the event-related-potentials of magnitude comparisons.

Method

Participants

Twenty-four right-handed, native English-speaking participants (15 female, $M = 20.7$ years, $SD = 5.31$) with no history of neurological disorders, brain injuries or developmental disabilities were recruited from the University of Alabama to participate in this experiment. All participants gave written consent and were paid \$25 for participating.

Task

Participants judged whether an initial fraction (probe) presented on the screen had the same numerical value

(magnitude) as a subsequent fraction (target). Participants indicated whether the target fraction was a “match” or a “mismatch” by pressing either the right or left buttons (counterbalanced across participants) on a gamepad controller. All stimuli were presented on white color over a black background in Times New Roman font size 96.

The first fraction was randomly selected from an array of the first five multiples of the unit fractions 1/2, 1/3, and 1/4. There were 15 possible probe fractions (see Table 1).

Table 1: Fraction stimuli per condition. Mismatch combination with shared components are in bold.

First Fraction	Target	Condition
2/4, 3/6 , 4/8 , 5/10, & 6/12	1/2	Match
	1/3 or 1/4	Mismatch
2/6, 3/9, 4/12 , 5/15, & 6/18	1/3	Match
	1/2 or 1/4	Mismatch
2/8 , 3/12, 4/16 , 5/20, & 6/24	1/4	Match
	1/2 or 1/4	Mismatch

The target fraction was limited to unit fractions 1/2, 1/3, and 1/4 for both the match and mismatch trials. The range of target fractions was restricted to these three unit fractions in order to limit the amount of double digit multiples that could be presented (all unit fractions smaller than 1/4 have only double digit multiples). Some probe-target combos shared a component across their numerators and denominators. Four groups were created across condition (match vs. mismatch) and component (shared vs. nonshared): MatchShared (MAS), MismatchShared (MMS), MatchNonshared (MANS), and MismatchNonshared (MMNS).

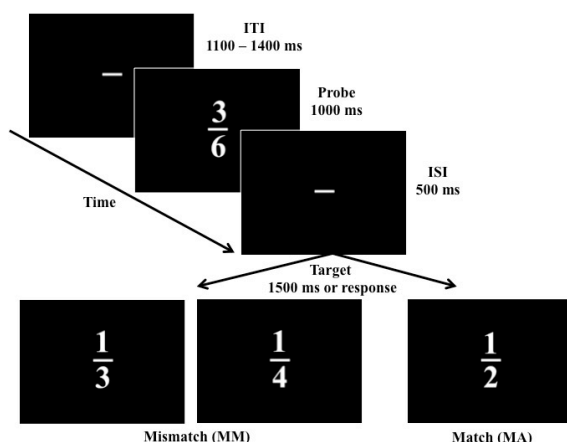


Figure 1. Experimental progression.

Trials begun with a fixation line centered on the screen. This fixation line overlaid on the fraction line between the two numbers of the fraction presented. Following there was an inter trial stimulus (ITI) of 1100 millisecond plus a random jitter between 1 and 300 milliseconds. Immediately

after, the probe is presented for 1 second followed by an inter stimulus interval (ISI) of 500 milliseconds. Whether the trial was match or mismatch was randomly selected after the probe was presented. Finally, the target fraction was presented for 1500 milliseconds or until a response was detected (see Figure 1). Responses with reaction time greater than 1500 milliseconds were marked incorrectly. Participants completed 60 randomized trials per block. There was a total of 10 blocks, with a total of 600 trials.

EEG Acquisition and Analysis

The experiment took place in a sound attenuated experiment room. Neurobs Presentation (www.neurobs.com) was used for presenting the stimulus. A Logitech F310 game controller was used as the input device. Participants used their right and left index fingers to provide responses. EEG Data was collected using a BrainVision 32 Channel ActiChamp system (www.brainvision.com), with Easy Cap recording caps using Ag/AgCl electrodes. The 32 electrodes were attached according to the international 10-20 system at the locations FP1/2, F7/8, F3/4, FZ, FT9/10, FC1/2, FC5/6, T7/8, C3/4, CZ, TP9/10, CP1/2, CP5/6, P7/8, P3/4, PZ, O1/2, OZ and referenced to CZ. BrainVision Recorder was used to record data (electrode impedance < 10 kΩ, 0.5-70 Hz, 500 samples/sec). A custom MATLAB script using ERPLAB (erpinfo.org/erplab/) and EEGLAB (scn.ucsd.edu/eeglab) functions were used to analyze data. A mass univariate analysis was conducted using the Mass Univariate ERP toolbox in Matlab with a family wise alpha level of .05 (Groppe, Urbach, & Kutas, 2011). Data was downsampled to 128 Hz using a boxcar filter. Inferential statistics were conducted with JASP (<https://jasp-stats.org>).

During the analysis the continuous EEG data was re-referenced to average. 200ms pre-stimulus period was used for baseline. A simple voltage threshold algorithm was used for artifact detection. Epochs exceeding the $\pm 50 \mu V$ threshold were excluded (8% of trials). Only the epochs that preceded a correct response were included in the subject-level averaged ERPs (94% of trials). The averaged ERPs were low pass filtered with a 30 Hz (zero-phase, 12 dB/octave) filter. The reported EEG amplitudes are in μV (microvolts).

Results

Behavioral Results

A repeated measures ANOVA of condition (match, mismatch) x component (shared, nonshared) on accuracy showed main effects of condition $F(1, 23) = 11.25, p < .005$, and component $F(1, 23) = 25.65, p < .001$, and a significant interaction between condition and component, $F(1, 23) = 65.52, p < .001$. Posthoc comparisons indicated a mean difference for both condition ($M = -0.029, SE = 0.012$) $p_{\text{bonf}} = .018$ and component ($M = -0.034, SE = 0.013$) $p_{\text{bonf}} = .004$.

Paired samples t-test showed significant difference for all groups MAS > MMS > MMNS > MANS (see Figure 2a).

A repeated measures ANOVA of condition (match, mismatch) x components (shared, nonshared) on RT revealed a main effect of condition $F(1, 23) = 143211.5, p < .05$. The main effect of component did not reach significance $p = .057$ but there was a significant interaction between condition and component $F(1, 23) = 117726.5, p = .003$. Posthoc comparison indicated a mean difference for both condition ($M = 0.21, SD = 0.023$) $p_{\text{bonf}} < .001$ and component ($M = 0.049, SD = 0.013$) $p_{\text{bonf}} = .001$. Paired samples t-tests revealed a significant difference between mean RT of MMS and MMNS, MAS and MMS, as well as for MNS and MMNS (see Figure 2b).

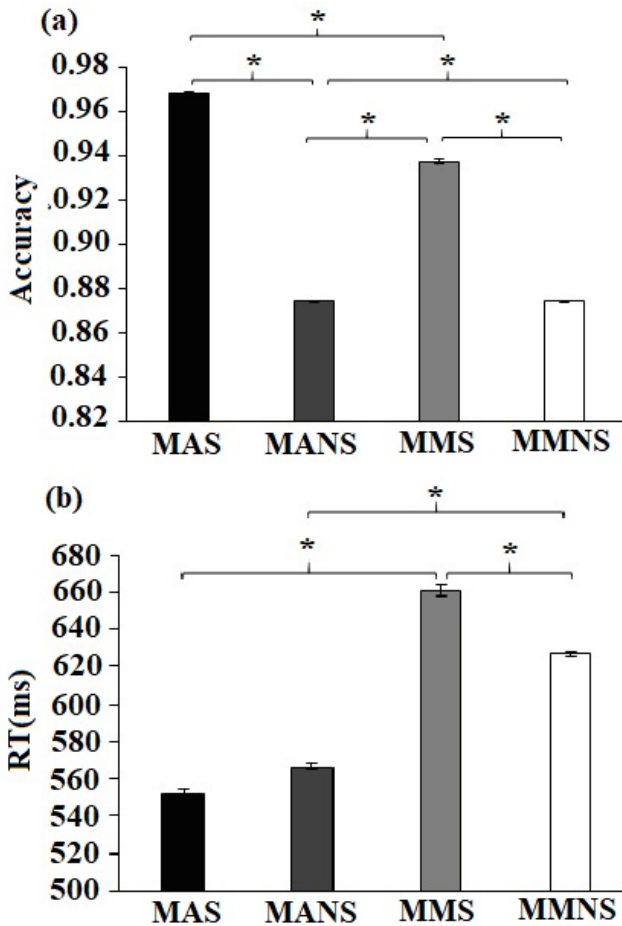


Figure 2: Group mean (a) accuracy and mean (b) reaction time comparisons for all four groups. Error bars are standard errors of the mean. * $p < 0.05$.

EEG Results

A mass univariate analysis was conducted using spatiotemporal clustering. Permutation-based cluster differences for all 32 electrodes were calculated on the 0 to 600ms time range. Average Match vs. Mismatch differences were calculated. Follow-up comparisons of MAS vs. MMS and MANS vs. MMNS) were performed. 10,000 iterations were used. The threshold for clustering inclusion was 0.05.

The Match vs. Mismatch average comparison showed mainly a centro-parietal cluster difference from 280ms to 420ms (see Figure 3).

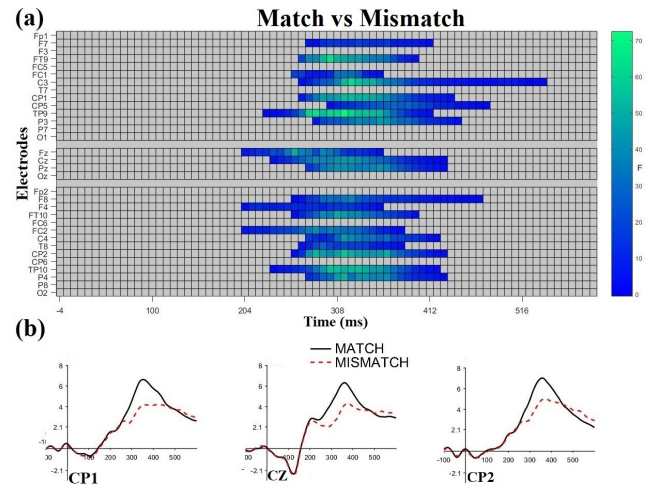


Figure 3: Raster plot of significant cluster differences between the match and mismatch trial groups (a). Grand average of ERP waveforms for electrodes where the N400 is traditionally seen (b).

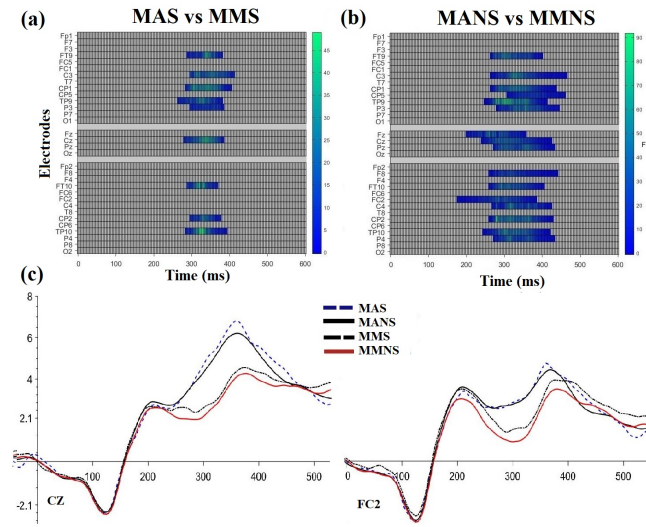


Figure 4: Raster plot showing significant cluster differences between (a) MatchShared (MAS) and MismatchShared (MMS) and between (b) MatchNonshared (MANS) and MismatchNonshared (MMNS) trial groups. (c) Grand average ERP waveforms for electrodes CZ and FC2 showing each of the four conditions (MAS, MANS, MMS, MMNS).

MAS vs. MMS comparison revealed a main cluster between 280ms and 400ms with the most positive peak at 340ms. The difference was observed in a set of electrodes; central CZ, left fronto-central FT9, FC1, C3, CP1, TP9, and P3, and only FT10, CP2, and TP10 on the right hemisphere (see Figure 4a). MANS vs. MMNS comparison revealed a

significant frontal cluster from 200ms to about 380ms in electrodes FZ and FC2. The main visible cluster runs from 250ms to 450ms with the most positive peak also at 320ms occurring over central electrodes FZ, CZ, PZ and bilaterally over frontal and central electrodes F8, FT9, FT10, FC2, C3, C4, CP1, CP2, CP5, and temporal and parietal electrodes TP9, P10, P3, and P4 (see Figure 4b).

Discussion

Participants judged whether 15 different fractions matched in magnitude to three different unit fractions: 1/2, 1/3, and 1/4. A subdivision of the match and mismatch groups was made based on whether there was a shared number between numerators and denominators (shared vs. nonshared).

The match trials showed lower reaction times compared to mismatch trials, indicating that the mismatch trials were more taxing. In addition, the presence of shared components seemed to interfere with comparisons across mismatch magnitude judgments as seen in significant higher RTs and lower accuracy for MMS trials but not in the MMNS trials.

This is possibly because sharing a component between two mismatch fractions is perceived as a conflict that needs to be resolved before a correct judgment about the match or mismatch can be made. This requires recruitment of executive resources for inhibition. Similar effects were observed when numerical judgments about double digit numbers were made, with interference based on the single-digit values of the numerals that make up the numbers processed (Nuerk et al., 2001). These results point to the presence of a Stroop-like conflict between two fractions' magnitudes (which don't match) and their numerical components (which match). The absence of this conflict in match shared trials might account for the observed lower reaction times and higher accuracy.

EEG analysis showed there was an arithmetic N400-like effect between congruent (Match) and incongruent (Mismatch) trials complementing the behavioral results. Furthermore the shared/nonshared status influenced the ERP waveforms; while the shared conditions did not show a difference between match and mismatch in the 200 to 300ms window, a match/mismatch difference was observed on the nonshared condition as early as 200ms. This shows that when there is a component shared between the two fractions, the magnitude match judgment is delayed, which explains the RT effects observed. This hints at the presence of an executive process necessary to inhibit the information about the shared components, a pattern also seen in a multiplication N400 study (Niedeggen & Rösler, 1999).

The N400 effect observed, the influence of shared components on Match-Mismatch judgments' RT and the accompanying ERP effect showing delayed processing when a component is shared between two fractions point to a hybrid model of adult fraction processing where holistic processing is subject to component's interference. This finding is in agreement with previous fraction comparison studies with shared components (Meert et al., 2010b) and with the slower incongruent fraction judgment seen in

children (Van Hoof, Lijnen, Verschaffel, & Van Dooren, 2013). Additionally, the fact that higher levels of inhibition have been shown to be predictive of fraction comparison proficiency (Gómez, Jiménez, Bobadilla, Reyes, & Dartnell, 2015) shows that inhibition might be a cognitive function necessary for successfully carrying out fraction magnitude processing.

Limitations and Future Directions

The main limitation of the study is the restricted set of fraction stimuli selected for both probes and target. Only three unit fractions were used as targets. This limited the possibility of observing a numerical distance effect (NDE). While this was a practical adaptation for using EEG, future studies should incorporate a wider range of probe and target fractions (the rest of unit fractions and non-unit fractions as well). Additionally, future studies can look at the processing of relative magnitude across fractions and not just magnitude equivalence. A second limitation was the uneven number of trials across shared and nonshared subdivisions. Trials with shared components were one fourth as frequent as nonshared.

Conclusion

The results indicate the existence of an N400 effect between match and mismatch conditions during a fraction validation task. The fraction N400 can be a useful tool for the future studies on fraction processing. In addition, the study showed that quick and successful comparisons of fractions in adults require the inhibition of peripheral associations between fraction components. Even though inhibition of unrelated information during numerical performance is not considered a skill of central importance, it might have a key role in the development of rational number understanding.

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