The Explanatory Value of Mathematical Information in Everyday Explanations

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Abstract

With two experiments, we begin an inquiry into the perceived explanatory value of mathematical entities in everyday explanations. This work is motivated by a philosophical debate about the role mathematical entities play in explanation. Simply put, are the mathematical entities themselves explanatory, or is mathematical talk elliptical or shorthand for talk about the physical entities we are concerned with? Across the two experiments, we found clear evidence that situational factors affected how the mathematical entities were considered. However, when those situational factors are accounted for, participants tended to see more explanatory value for mathematical entities that point to other objects involved in the explanation as opposed to mathematical entities that assume the explanatory role themselves.

Keywords: explanation; mathematical explanation; indispensability argument; nominalism; platonism

As scientists, we often appeal to mathematical entities within the explanatory frameworks we adopt. These entities can take a variety of forms, from simple numerals (e.g., '7' and 'thirty') and functions (e.g., '*f(x)*') to complex computational models. Most cognitive scientists, but by no means all, recognize the usefulness of this mathematical information, and there has been extensive commentary on its role and how it should be interpreted. In recent years, philosophers have taken up this question with an increased focus on historical and contemporary case studies in the natural sciences (e.g. Lange, 2016; Pincock, 2011). However, outside of these formal, scientific frameworks, there is arguably a less well developed sense of what role mathematical entities play in explanations.

In this paper, we consider how a live philosophical debate about the explanatory role of mathematical entities relates to everyday explanations. Do mathematical entities contribute to the explanatory work themselves or are they "merely" drawing out the structure necessary for the explanation, identifying the relevant conceptual entities that are actually doing the explanatory work?

Within philosophy of mathematics, platonists affirm the existence of mind-independent and abstract mathematical objects, while nominalists deny that there are any such entities (see Cowling, 2017, for a general discussion of the platonist-nominalist debate). An influential line of argument in defense of platonism is the "Indispensability Argument", which posits that an ontological commitment to mathematical entities of the sort held by platonists is warranted because mathematical entities like numbers and functions play an indispensable explanatory role (Colyvan, 1998). Put differently, platonists make a claim about what exists—namely, that along with concrete entities like electrons and tables, there are also imperceptible, non-spatiotemporal mathematical entities. In contrast, nominalists deny that mathematical entities exist while acknowledging that we must nevertheless explain their usefulness in explanations. We examine whether this distinction that has motivated philosophical debate plays a role in everyday explanation.

In most scientific frameworks, mathematical entities are used to provide formal descriptions of processes and components theorized within conceptual frameworks. For instance, in the categorization literature numerous mathematical models have been proposed to account for how individuals organize items into coherent classes. These models vary from rather simple computations of feature overlap among the items to complex systems of probabilistic computation. They employ mathematical entities in a variety of ways, but there is no assumption that the explanatory value of the models rests on a commitment to the existence of those mathematical entities. Instead, the mathematical entities reference the things, e.g. the features, that are doing the explanatory work. We describe this approach as a *nominalist friendly* (NF) position. On nominalism and the various accounts that have been developed to account for mathematical explanation, see Burgess and Rosen (1997).

One can also accept an ontological commitment to the mathematical entities and allow them to assume explanatory relevance. In this case, the mathematical entities themselves ground the explanation as opposed to simply representing the physical-causal entities and their relations. For instance, consider the explanation for why certain species of cicadas emerge from their nymph state in either 13 or 17 year cycles. The explanation for these life cycles can be understood in terms of avoiding predation (the cicadas would evolve to have a life cycle that minimizes overlap with the life cycle of predators), but that explanation ultimately rests on the fact that 13 and 17 are prime numbers. The mathematical reality of prime numbers is that they cannot be factored. The explanation for the life cycle of these species of cicada thus relies on a commitment to the mathematical entities as having particular qualities and would therefore be no less real or existent than familiar objects like chairs and racecars (Baker, 2005). We describe this stance as a *platonist friendly* (PF) position.

We use this philosophical debate to background an initial inquiry into how lay people use and evaluate mathematical entities in everyday explanations. We want to be clear that we do not think that people ponder the ontological commitments they are making as they produce or evaluate these kinds of explanations. However, there may be an effect tied to whether the explanations induce genuine ontological commitments to mathematical entities. Indeed, platonists who endorse the indispensability argument standardly assert that, without PF-friendly claims, certain proposed explanations will seem non-explanatory and that, generally, PF explanations are superior to NF explanations (Colyvan, 2018). As we consider below, whether the mathematical entities are represented with regard to their number theoretic value or are merely non-referring placeholders for information about the items they reference will, according to platonists, impact explanatory processes.

Psychologists have examined why people engage in explanation, what implications explaining has for other cognitive activities, and what cognitive structures underlie explanation. There is evidence that people value explanations that are simple and provide coverage in terms of how widely the explanation can be applied (Lombrozo, 2012). There is also evidence that explanatory processes rely on structured internal representations (Chin-Parker & Bradner, 2017; Johnson, Johnston, Koven, & Keil, 2018). These two aspects of explanation suggest that the ontological commitment could indeed play a role in how people regard explanations. For instance, if an explanatory relationship is represented in terms of the number theoretical values (e.g. $5 < 6$), it might be considered simple and widely applicable. If the mathematical entities facilitate the kind of structured representations implicated in explanatory processes, there could be a preference for PF explanations.

However, insights from the psychological study of mathematical reasoning complicate this simplistic rendering of the situation. This literature is vast, so we focus here on two issues. First, there is variability in the ability of people to use and understand mathematical information (Rittle-Johnson, 2017). This variability in mathematical reasoning would likely impact whether an individual is able to easily use the mathematical information to instantiate the requisite representations that the explanatory processes operate over. Second, how the information is presented also impacts mathematical reasoning (Koedinger, Alibali, & Nathan, 2008). In a simple problem, people tend to be more successful when the relevant information is grounded, when it has a clear relationship to concrete referents. When the information is presented in a more abstract manner, e.g. algebraic notation, people are less able to solve the problem. At the same time, the more abstract mathematical entities can facilitate more complex mathematical reasoning. Given these patterns, we expect the type of explanation may interact with the content of the explanation.

We use the logical form of the sentence to determine the ontological commitment of a mathematical statement. For example, ‘Thirteen is prime’ is PF because it entails that there is something that is prime, which is logically equivalent to the claim that thirteen—a mathematical entity—exists. A NF stance would, consequently, be one in which mathematical terms only appear in non-subject positions—e.g., ‘There are thirteen dogs’. Here, 'thirteen' merely modifies the subject, dogs, and the sentence directly entails that there are dogs, but does not, without auxiliary logical assumptions, entail that there is a number thirteen. We note, however, that this assumption is a familiar point of controversy among philosophers and linguists and it is far from clear that lay persons are sensitive to the complex relationship between syntactic position and ontological commitment even if such a view is defensible upon sustained philosophical analysis (see Hofweber, 2016, for a recent discussion). In taking on this account of ontological commitment for the present study, we are, in part, investigating whether certain factors that philosophers of mathematics to take to be of paramount importance are represented in everyday explanatory practices.

To begin our inquiry (Experiments 1a and 1b), we asked participants to generate, and subsequently evaluate, explanations for a series of scenarios. The scenarios varied in terms of their content so that we could assess the generalizability of the participants' ontological commitments across situations. By asking the participants to both generate and evaluate explanations, we were also able to assess whether those commitments vary across different explanatory processes. Thus, the first experiments allowed us to examine whether there is a consistent preference for one type of explanation over the other, or whether the explanation and, in turn, commitment to mathematical entities varies between individuals, situations, and how the information is used. Because of the exploratory nature of this inquiry, we focus on describing the patterns of participant responses relevant to these topics as opposed to testing a priori hypotheses.

Experiment 2 presents a more controlled examination of the issue. We used modified versions of the cicada life-cycle scenario (Baker, 2005) and asked participants to rate the explanatory value and complexity of various explanations the cicada life cycle. As prior, the explanations varied in terms of whether the mathematical entities made a NF or PF commitment. Also, we varied whether the mathematical terms were developed using a more or less specific example. This manipulation was intended to affect the ease with which participants could represent the information provided in the explanation.

When the explanation was developed using a more specific example, we expected that the participants should find the explanation to be less complex and they should give higher explanatory ratings for PF explanations. This prediction rests on the idea that the grounded mathematical terms could be more easily incorporated into an internal representation and the PF explanation would provide better coverage because it reflects the existence of those entities. However, when the explanation was developed with a less specific example, we expected that the participants would rate it as more complex and they should give higher explanatory ratings for the NF explanations. If the participant has more difficulty representing the situation due to the development of the mathematical terms, an explanation that does not rely on the existence of those mathematical terms should be seen as more explanatory.

In sum, we predict a main effect of the information in the explanation (specific vs. non-specific) on the complexity ratings, and an interaction between the development and the ontological commitment (PF vs. NF) for the explanatory ratings. These predictions rest on the assumption that the explanatory value will reflect both the generalizability and simplicity of the explanation. In order to get a better understanding of the individual differences in play, we also asked participants to report their comfort with mathematics and belief about the existence of mathematical entities.

Experiments 1a and 1b

Methods

Participants Undergraduate students participated as partial fulfillment of a requirement for an introductory psychology course. Thirty participants completed Exp. 1a. Forty participants completed Exp. 1b. Two participants in Exp. 1b failed to complete the explanation generation task, but they did provide ratings of the explanations.

Materials and Procedure The two studies used the same materials, but the method of data collection differed. In Exp. 1a, participants completed the study in small groups. Materials were projected onto a screen, and participants wrote out their responses in prepared packets. In Exp. 1b, participants completed the study on-line by completing a questionnaire created using the Qualtrics platform. See the *Appendix* for the full set of materials used in Exp. 1a and Exp. 1b.

Four scenarios were developed for this experiment. Each scenario presented a set of initial conditions that included mathematical entities (e.g. "The editor of the Daily News has 127 remaining newspapers to deliver and only three paperboys to deliver them.") and then a specific why-question related to those conditions (e.g. "Why can't the editor distribute the papers equally to each of the paperboys?"). The scenarios were designed such that it was possible to answer the question by positing the existence of the mathematical entities (a PF explanation), but a suitable explanation could be made without such a commitment (a NF explanation). In Exp. 1a, the order of the scenarios was balanced, and in Exp. 1b, the order of the scenarios was randomized. In both cases, participants were presented with the scenario and why-question and asked to generate a response.

After responding to all of the scenarios, the participants were told that other students had also generated explanations and those explanations needed to be evaluated. The participants were presented with the same four scenarios – the order of the situations was again balanced (Exp. 1a) or randomized (Exp. 1b). Each scenario was accompanied by two short explanations. One of the explanations reflected PF commitment (e.g. "Because 127 is not divisible by three") and the other reflected NF commitment (e.g. "Because if he gives each paperboy 42 papers, there will be one paper remaining"). In Exp. 1a, participants were asked to select which of the two explanations they considered to be the better explanation. In Exp. 1b, the participants were asked to rate how explanatory each explanation was. Along with each explanation was a slider that could be adjusted from 0 ("not explanatory") to 10 ("ideally explanatory").

Results

Explanation Generation The explanations generated by the participants were coded as to whether they rested on a PF claim, a NF claim, or whether the claim was ambiguous. The explanations were independently coded by two of the study authors, and disagreements were resolved through discussion including the third author. The inter-rater agreement was 87% for the responses from Exp. 1a and 84% for responses from Exp. 1b. Disagreements were easily resolved.

The distribution of the explanatory claims was similar across the two studies. In Exp. 1a, 62% of the explanations were NF, 31% PF, and 8% ambiguous. In Exp. 1b, 65% were NF, 31% PF, and 4% ambiguous. The results indicated that people tend to rely more on NF claims, but that they also will invoke PF claims when deemed appropriate. None of the participants in either study generated PF explanations for all four scenarios, 8% generated three PF explanations, 22% generated two PF explanations, 49% generated a single PF explanation, and 16% generated no PF explanations. The scenario being explained had an effect on the type of explanation generated. In the "paperboy" scenario, participants readily generated PF explanations (85% of the explanations), while in the other three scenarios, they tended to rely on NF explanations (over 75% of the explanations for each scenario).

Explanation Selection, Exp. 1a The variability between participants and among scenarios is also evident in Exp. 1a when the participants were asked to select one explanation, NF or PF, as more explanatory. No participant consistently selected the PF explanation for every scenario while only six participants consistently selected the NF.

Table 1: Explanation Selection in Exp. 1a

Scenario	PF Selection	NF Selection
Championship	13/30	17/30
Fishing	5/30	25/30
Paperboy	22/30	8/30
Wheat	9/30	21/30

In order to assess explanatory preference, the selection data for each situation were compared to an assumed equal distribution of explanation types using a one-sample binomial test. In both the "fishing" and "wheat" scenarios, the participants showed a consistent preference for the NF explanations (both *ps* $< .05$). In the "paperboy" scenario, the participants showed a clear preference for the PF explanation ($p < .05$). Only the "championship" scenario had a distribution that indicated that the participants had no preference for the type of explanation.

We did not find evidence that participants made a consistent ontological commitment across the generation and selection tasks. When the participant generated a PF (or NF) explanation for a particular scenario, they subsequently selected the same type of explanation for that scenario only 52% of the time.

Explanation Rating, Exp. 1b The participant ratings for the PF and NF explanations for each scenario were analyzed using a 2 (type of explanation) X 4 (scenario) repeated measures ANOVA. There was no overall effect of the type of explanation, $F(1, 37) = 0.37$, $p = .55$, $\eta_p^2 = .01$, a significant effect of the scenario, $F(3, 111) = 6.70$, $p < .001$, $\eta_p^2 = .15$, and a significant interaction between the type of explanation and the scenario, $F(3, 111) = 8.54$, $p < .001$, $\eta_p^2 = .18$. As can be seen in *Figure 1*, the explanations for the "championship" scenario were significantly lower than the ratings for the other three scenarios (all *ps* $< .01$).

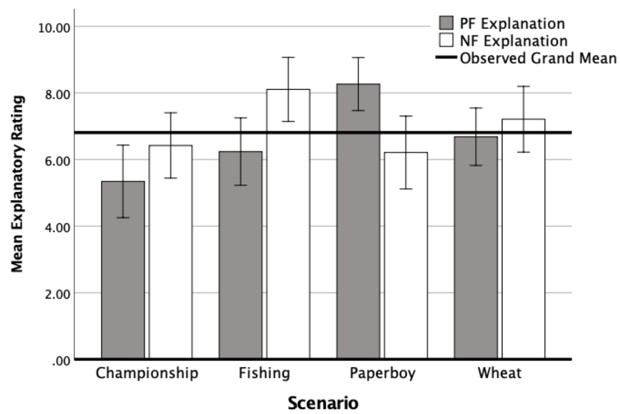

Figure 1: Mean Rating for PF and NF Explanations by Scenario from Exp. 1b

Note. Error bars represent 95% confidence intervals.

The other scenarios did not differ significantly from one another. Paired sample *t*-tests revealed that the ratings for the PF and NF explanations in the "championship" and "wheat" scenarios were not different; $t(39) = -1.25$, $p = .22$, and $t(39) = -0.64$, $p = .53$, respectively. However, the type of explanation did affect the ratings in the "paperboy" scenario, $t(39) = 3.14$, $p < .01$, and "fishing" scenario, $t(39) = -3.03$, $p < .01$, although in opposite directions.

In Exp. 1b, the participants showed a more consistent pattern of commitment to a particular type of explanation than in Exp. 1a. Overall, the participants gave a higher rating to the explanation that matched the type of explanation they had generated 66% of the time. However, this was primarily driven by participants that generated NF explanations and subsequently rated the NF explanations as better. The participants that initially generated PF explanations rated the PF explanations as better only 50% of the time.

Experiment 2

Methods

Participants Participants ($n = 173$) were obtained using the Mechanical Turk platform. They had to have above a 98% positive approval rating and successfully completed at least 100 tasks within the system. Eight participants were removed for not following directions. The questions included in this study were embedded within an unrelated memory study. Participants were paid for their participation.

Design Explanations varied in terms of commitment of the mathematical entities (either NF or PF) and how the mathematical terms were developed in the explanation (whether they contained a specific or non-specific example). Combining these factors created four conditions, and participants were randomly assigned to one condition.

Materials and Procedure Participants completed the study online using the Qualtrics platform. Each participant read a short passage about cicadas that provided some basic information about their appearance and diet. Importantly, they were informed about the cicada life-cycle being either thirteen or seventeen years. The description ended with the statement, "A question that has interested scientists is why cicadas have this particular life-cycle." Four different explanations were created to address that question.

All of the explanations consisted of six sentences, the first and last sentences were identical across all of the explanations. The second sentence reintroduced the idea that the life cycle of the cicadas was either thirteen or seventeen years. In PF explanations that point was connected to the notion that these numbers are prime:

> *Interestingly, 13 and 17 are prime numbers – this means that no smaller value (such as 2 or 3) can be divided into these numbers.*

In the NF explanations, the number of years was connected to the notion that those numbers could not be evenly segmented:

> *Interestingly, cicadas' life-cycles are 13 or 17 years long – this means that these periods cannot be segmented evenly into durations of two years, durations of three years, and so on.*

In all explanations, the next (third) sentence noted that the length of the life cycle minimized overlap with potential predators. In the PF explanations, this point was explicitly tied to the fact that the length of the life-cycle was a prime number. In the NF explanations, the point was tied to the length of the life-cycle generally. The fourth and fifth sentences developed the idea raised in the third sentence with either a specific or non-specific example. For instance, in the specific PF explanation, the example described how a predator with a three year life cycle would overlap with cicadas with a thirteen year life cycle only once every 39 years, but it would overlap every life cycle with a cicada that had a twelve year life cycle. In the non-specific PF explanation, the development of the explanation relied on algebraic notation:

> *If a predator of the cicada had a life-cycle of x years (where x is equal to 2, 3, or 4), it would threaten cicadas with a 13 year life-cycle only once every 13*x years because that number would be the first number that can be divided by both x and 13.*

In the NF explanations, the specific and non-specific examples differed similarly except the examples referred to how the life-cycle could be segmented as opposed to the characteristics of prime numbers.

Immediately following the explanation, two rating scales were presented. The first scale asked participants, "How well does the above account explain the cicada life-cycle?". The participants could move a slider along a scale from 1 ("Not at all explanatory") to 9 ("Very explanatory"). The second scale asked, "How complex would you consider the explanation provided above?". The scale went from 1 ("Extremely simple") to 9 ("Extremely complex").

Following the critical questions, the participant was asked, "Would you consider yourself to be a scientist?" and "Are numbers real?" (Yes/No options for both measures). There was also a measure where the participant was asked to report their comfort with math from 1 ("Not comfortable") to 9 ("Very comfortable").

Results

The participants in the study predominately reported that they did not consider themselves scientists, (7.5% responded "yes" and 92.5% responded "no") and that they considered numbers to be real (95.4% responded "yes" and 4.6% responded "no"). Overall, they reported that they were "reasonably comfortable" with math (m = 5.62, s = 2.23), but there was some variability in those responses. Importantly, the reported comfort with math did not meaningfully vary by condition, $F(3, 169) = 1.71$, $p = .16$, $\eta^2 = .03$.

The explanatory ratings were analyzed using a 2 (specificity) X 2 (commitment) ANOVA (see Figure 2). There was no effect of specificity on the explanatory rating, $F(1, 169) = 0.12$, $p = .73$, $\eta_p^2 = .001$. The mean for the specific explanations ($m = 6.41$, $s = 2.03$) was similar to the mean for the non-specific explanations ($m = 6.32$, $s = 2.05$). There was a significant effect of the mathematical commitment on the ratings, $F(1, 169) = 3.85$, $p = .05$, $\eta_p^2 = .02$. The mean for the NF explanations ($m = 6.67$, $s = 1.86$) was significantly higher than the mean for the PF explanations ($m = 6.06$, $s = 2.17$). There was no interaction between the specificity and mathematical commitment of the explanations, $F(1, 169) = 1.19$, $p = .28$, $\eta_p^2 = .01$.

The complexity ratings were similarly analyzed (see Figure 3). There was no effect of specificity on the complexity rating, $F(1, 169) = 1.88$, $p = .17$, $\eta_p^2 = .01$. The

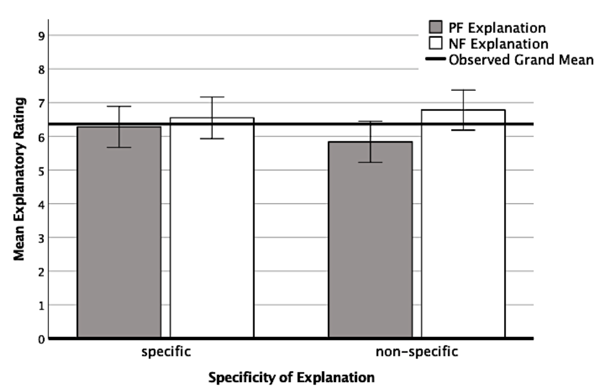

Figure 2: Mean Explanatory Ratings from Exp. 2
Note. Error bars represent 95% confidence intervals.

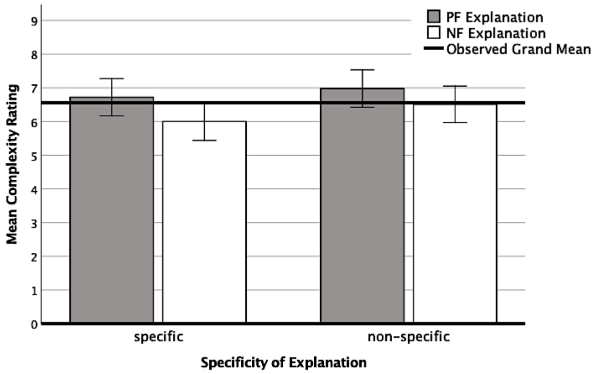

Figure 3: Mean Complexity Ratings from Exp. 2
Note. Error bars represent 95% confidence intervals.

mean for the specific explanations (m = 6.36, s = 1.90) was similar to the mean for the non-specific explanations (m = 6.74, s = 1.82). There was a significant effect of the mathematical commitment on the ratings, $F(1, 169) = 4.49$, $p = .04$, $\eta_p^2 = .03$. The mean for the NF explanations (m = 6.26, s = 1.88) was significantly lower than the mean for the PF explanations (m = 6.85, s = 1.81). There was no interaction between the specificity and the mathematical commitment, $F(1, 169) = 0.21$, $p = .65$, $\eta_p^2 = .001$.

The participant ratings of how explanatory and complex the explanations were had a weak, negative relationship, r = -0.12, p = .10. There was no relationship between the participants' reported comfort with math and their explanatory (r = -0.02, p = .80) or complexity (r = -0.01, p = .88) ratings. There were too few people that reported themselves to be scientists (or to not believe numbers to be real) to assess how those factors might have impacted their ratings of the explanations.

Discussion

Experiments 1a and 1b showed that there is variability in the mathematical commitments people are willing to make when generating or evaluating explanations for relatively simple situations. It was clear that the variability was not simply an individual difference issue – i.e. there was no evidence that some people always use and value PF entities and other people do not. This result suggests, contrary to some philosophical discourse, that there are not distinctively nominalist or platonist reasoners.

The variation in Exp. 1a and 1b appeared to be largely driven by differences among the scenarios. Across both samples and all measures, participants readily committed to PF explanations for the "paperboy" scenario. It involves the simplest mathematical relations as the explanatory value rests on a single mathematical operation. The "fishing" scenario tended to be the one where explanations ontologically committed to mathematical entities were least valued, and that scenario involves multiple operations across several potential numerical values. The other two scenarios tended to show less consistent patterns of response. This suggests to us that the complexity of the structure of putative explanations might drive much of the variation seen in the participants' preferences for the different types of mathematical explanations. Further study, and more careful control, of the various factors that differentiate these kinds of everyday explanations should provide more clarity as to why people shift in the ontological commitments.

In Experiment 2, we did not find the predicted effect of the specificity on the rated complexity of the explanations. We also did not find the predicted interaction between the specificity and the type of mathematical commitment on the explanatory value. However, among the non-specific explanations, the explanatory ratings differed between the PF and NF conditions (p = .03). Even though our manipulation of specificity did not have the expected effect on the complexity ratings, the participants responded to the PF and NF explanations differently when the information was non-specific.

The main results of Experiment 2 were that participants considered the PF explanations to be less explanatory and more complex than the NF explanations. It is possible that the mathematical relations underlying prime numbers are more difficult for people to grasp than we had anticipated. If that is the case, the results across the two experiments align; with more difficult mathematical relations, people perceive the explanations are being less explanatory. This assessment fits with recent work by Johnson, Johnston, Koven, and Keil (2017) and aligns with findings that there is a negative relationship between complexity and explanatoriness in non-mathematical explanations (Lombrozo, 2012). However, that relationship may not always hold (Johnson, Valenti, & Keil, 2017). Alternatively, it is possible that the participants were receptive to the more verbal depictions of the mathematical relations found in the NF explanations (see Koedinger & Nathan, 2004). This would suggest that it could relate more generally to how easily participants are able to represent the relations that underlie the explanation.

The present results suggest that lay people often find NF explanations satisfactory and, in certain instances, preferable to PF explanations. So, if platonists seek to defend the existence of mathematical entities because of their explanatory value or because of the manifest superiority of platonist over nominalist explanations, the present study provides preliminary evidence that such claims cannot be substantiated by our everyday explanatory practices, which are often quite friendly to would-be nominalists. We fully recognize we are not able to resolve the philosophical debate that backgrounds this study, but it does provide an interesting glimpse into how people use mathematical information in explanations.

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Appendix: Exp. 1a and 1b Materials

Championship Scenario

Central High School hosts the league basketball championship game every three years and they host the league volleyball championship every four years. Eighteen years ago, they won both championships on their home court. Why can't they duplicate that feat this year?

(PF) Because the 18 is a multiple of three, but not four.

(NF) Because it will be another six years until they host both championship games again.

Fishing Scenario

Lana has $30 and wants to buy a fishing rod, fishing reel, and fishing line. There are two rods priced at $21 and $22. There are three reels priced at $7, $8, and $9. Fishing line is $2. Lana wants to spend exactly $30. Why should Lana buy the $21 rod?

(PF) Because the sum of 22, 7, and 2 is greater than 30.

(NF) Because any way of combining the $22 rod purchase with the purchase of a fishing reel and fishing line requires spending more than $30.

Paperboy Scenario

The editor of the Daily News has 127 remaining newspapers to deliver and only three paperboys to deliver them. Why can't the editor distribute the papers equally to each of the paperboys?

(PF) Because 127 is not divisible by three.

(NF) Because, if he gives each paperboy 42 papers, there will be one paper remaining.

Wheat Scenario

Fred needs 86 lbs of wheat for winter and he can't afford to waste any money on unused wheat. Wheat comes in bags of 8 lbs. He has 54 lbs of wheat already. Why can Fred avoid buying any unnecessary bags of wheat?

(PF) Because Fred must buy 32 lbs of wheat, and thirty-two divided by eight is four.

(NF) Because Fred must buy 32 lbs of wheat, and if Fred buys four 8 lb bags, he will have 32 lbs.