# Sample-based Variant of Expected Utility Explains Effects of Time Pressure and Individual Differences in Processing Speed on Risk Preferences

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**Abstract**

While previous models of economic decision-making offer descriptive accounts of behavior, they often overlook the computational complexity of estimating expected utility. Here, we seek to understand how both environmental and individual constraints on cognition shape our daily decision. Informed by the predictions of a recently-proposed resource-rational process model of risky choice, *sample-based expected utility* (SbEU; Nobandegani, da Silva Castanheira, Otto, & Shultz, 2018), we reveal that both time pressure and individual differences in processing speed have a convergent effect on risk preferences during a risky decision-making task. Under severe time constraints, participants' risk preferences manifested a strong framing effect compared to little time pressure in which choice adhered to the classic fourfold pattern of risk preferences. Similarly, individual differences in processing speed, measured using an established task, predicted similar effects upon risk attitudes as extrinsic time pressure. These findings reveal a converging contribution of environmental and individual limitations on risky choice, and provide empirical support for SbEU as a resource-rational process model of risky decision making. Notably, SbEU serves as a single-process model of two well-established biases, and the transition between the two, in risky choice.

**Keywords:** Behavioral economics; Risky decision-making; Time pressure; Processing speed; resource-rational process models

## 1 Introduction

Our capacity to adapt our decision-making strategies—financial or otherwise—to environmental demands such as time pressure is an invaluable asset for successful behavior. From an online sale which expires in a few minutes, to the rapid trading of stocks in volatile financial markets, our decisions are inevitably constrained by time pressure. Furthermore, internal limitations in processing speed—that is, the speed with which an individual can perform any cognitive operation—should interact with these environmentally imposed limitations (Gigerenzer & Selten, 2002; Salthouse, 1985) as making a choice is widely thought to require a computation of the relative values of the options under consideration (Kahneman and Tversky, 1979). In light of these constraints, one might wonder if our apparent failures to abide by rational decision-making frameworks (e.g., expected utility theory) could reflect a strategic use of limited cognitive resources. To this end, a number of recent theories have proposed that human cognition, with all its apparent biases, can in fact be understood as optimal response—subject to computational and cognitive limitations (*rational minimalist program*, Nobandegani, 2018; Griffiths, Lieder, & Goodman, 2015; Icard, 2014).

Thus, it is of interest to better understand both how we have adapted our decision-making processes to meet these demands and to what extent our ostensibly irrational choices are shaped by these limitations. While previous work has investigated the effects of environmental constraints like time pressure on irrational choice (Guo, Trueblood, & Diederich, 2017), here we seek to corroborate the contributions of both environmental and individual limitations on risky decision-making.

Perhaps one of the most studied departures from rational theories of decision-making is the violation of *description invariance*, which posits that preferences should remain consistent across choices, regardless of the context in which available options are presented. For example, according to expected utility theory (von Neumann & Morgenstern, 2007), whether a decision is made to avoid a loss or seek gains, it should not change one's choice. However, this assumptions is challenged by a wealth of data supporting the framing effect: people tend to be risk seeking for losses and risk averse for gains (Tversky & Kahneman, 1981). Sensitivity to choice framing has been documented in a variety of real-world circumstances including consumer (e.g., Levin & Gaeth, 1988; Loke & Lau, 1992), and medical decisions (e.g., McNeill, Pauker, Sox, & Tversky, 1982; Moxey, O'Connell, McGettigan, & Henry, 2003). This classic pattern of choice—risk-seeking in the domain of losses and risk-aversion in the domain of gains—is perhaps most famously explained by the S-shaped utility function posited by prospect theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992)—a well-known descriptive model of choice behavior.

Prospect theory also explains another choice phenomenon: a decision-maker's risk preference depends not only on the framing of the problem (gains vs. losses), but also the probability of the outcome (small vs. large) associated with the risky option. For example, people buy lottery tickets for which winning in unlikely (low probability gain) but prefer to pay to insure their houses against unlikely disasters (low probability loss). On the other hand, when faced with highly probable outcomes, people prefer to select a sure gain over a probabilistic one—"something is better than nothing"—but prefer to risk it all when faced with two unfavorable options—"Ive got nothing to lose" (Di Mauro & Maffioletti, 2004; Fehr-Duda et al., 2010; Kahneman & Tversky, 1979; Markowitz, 1952; Scholten & Read, 2014; Tversky & Kahneman, 1992). According to prospect theory, the fourfold

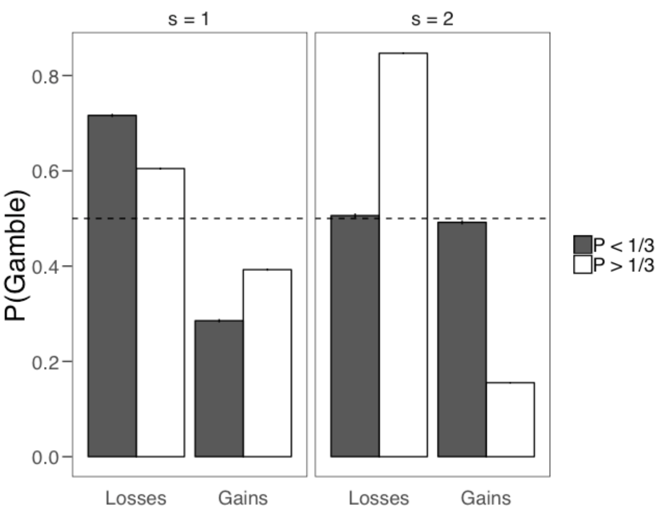


Figure 1: Sample-based expected utility (SbEU; Nobandegani et al., 2018) model predictions for the differential effect of the number of samples on choice. With limited samples (Left) model predicts a framing effect whereas, with more samples, the model predicts more of a fourfold pattern.

pattern of choice arises from the interplay between the S-shaped utility function and the subjective over-weighting of small probabilities (below 1/3) and underweighting of large probabilities (above 1/3) (Tversky & Kahneman, 1992).

While prospect theory offers a descriptive account for the framing effects, it fails to explain either how the decision-making process evolves over time, or how time constraints might bear upon the decision-making process. In order to answer questions about the role of time in these risky choices one must turn to dynamic models of choice. Sequential sampling models are a class of models which assume that choice preferences are estimated by the simulation of an action's potential consequences and where samples are simulated outcomes (Shadlen & Shohamy, 2016). In such models, each simulation takes a non-negligible amount of time and cannot be run in parallel, making time a valuable resource for the decision-maker (Lieder, Griffiths, & Hsu, 2018; Nobandegani, da Silva Castanheira, Otto, & Shultz, 2018). Thus, both total available time and the speed at which these simulations (i.e., samples) are run are directly proportional to the total number of potential outcomes considered (i.e., samples).

If sampling is costly in terms of elementary mental processes, then the number of effective 'samples' an individual is able to draw in a fixed amount of time should also vary in accordance with individual differences in the speed at which an individual processes information—a well-documented capacity limitation termed "processing speed"—which varies considerably across individuals (Kail & Salthouse, 1994). Accordingly, we leverage time pressure manipulations and these individual differences in processing speed to investigate the effect of limiting the number of samples on risky decision-making. Using these two manipulations, will test the effect of varying the number of samples on risky decision-making. Our hypotheses on the directionality of the effect of the time pressure are chiefly informed by a recently-proposed resource-rational process-level model of risky decision-making, *sample-based expected utility* (SbEU; Nobandegani et al., 2018). Extending an earlier model by Lieder et al. (2018), SbEU posits that an agent rationally adapts their strategies depending on the amount of time available for deciding.

Recently, Lieder et al. (2018) proposed a rational process model of risky choice. This model estimates the difference in expected utility of two prospect by using importance sampling, whereby outcomes are sampled in proportion to both its objective probability and its utility (e.g., important outcomes are overrepresented). Lieder et al.'s model, however, was developed under restrictive technical assumptions, making it only optimal when a large number of samples can be drawn. Fortunately, recent developments have determined an optimal sampling distribution which holds for both small and large number of samples (Nobandegani et al., 2018). This is of particular importance as mounting empirical evidence suggests that decision-makers draw very few samples (e.g., Vul, Goodman, Griffiths, & Tenenbaum, 2014); thus, providing an opportunity to explore the effect of limiting cognitive resources (i.e., available samples) on risk preferences.

Accordingly, we used SbEU to generate predictions of people's behavior for a mixture of gambles (i.e., both gains and losses and large and small outcome probabilities) under both conditions of time pressure—in which they can draw very few samples ($s = 1$)—and less constrained conditions—in which they can draw more samples ($s = 2$). Both prospects and time conditions modeled are conceptually identical to those experienced by participants during the task. As depicted in Fig. 1, drawing more samples to estimate the expected utility results in moving from a 'pure' framing effect (Fig. 1a) to the classic fourfold pattern of risk preferences (Fig. 1b). This prediction is in line with the empirical work which suggests that time pressure reduces the amount of information one can process (Miller, 1960; Zur & Breznitz, 1981), as the fourfold pattern requires integrating both outcome and outcome probability information (Kahneman & Tversky, 1979, 1979). Thus, informed by the SbEU's predictions, we sought out to test whether the effects of time pressure on economic choice would conform to the hypothesized pattern. Furthermore, as these predictions are not specific to external time pressure, but any internal constraint on the amount of information that can be processed per unit time, we simultaneously test if differences in cognitive capacity (i.e., processing speed) can also predict a similar pattern of results.

# Method

## Participants

Data were collected online using Amazon's Mechanical Turk; 100 (41 Female) US-based adult volunteers (mean age =

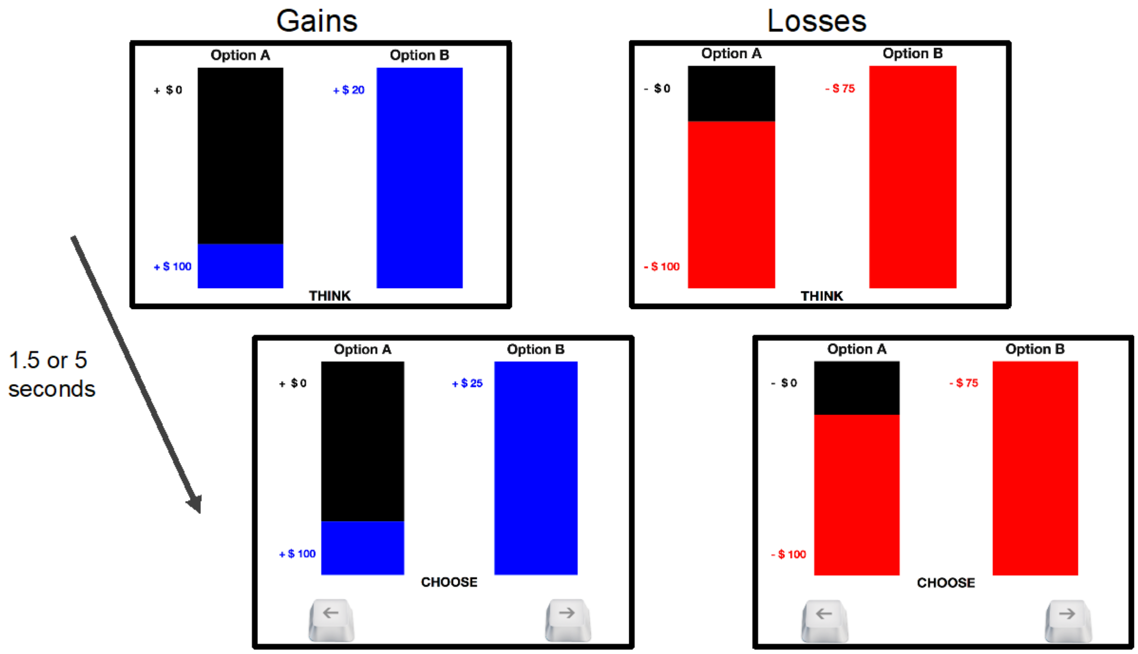


Figure 2: Screenshots of the Gambling task. Participants were given instructed to think about the gamble presented to them before being prompted to respond. The time allotted to think about the problem varied between time pressure conditions: under severe time pressure participants were given 1.5 second to think whereas under light time pressure participants had 5 seconds. Gambles were represented as bar charts where the probability of an outcome was depicted as proportional to the size of the colored portion. Color was used to emphasize the frame of the problem: red represented losses and blue represented gains.

34.77, SD = 9.88), recruited via Amazon's Mechanical Turk (Crump, McDonnell, & Gureckis, 2013), participated in the experiment for a base remuneration of $3.00 USD and a cash bonus—computed in proportion to the outcomes of all trials, with a mean overall payment of $5.85 USD. This study was approved by the McGill University Research Ethics Board.

## Processing Speed Measurement

Individual differences in processing speed were assessed using a computerized Digit-Symbol Coding task (Mathias et al., 2017; Salthouse, 1985) which we adapted for use online. Participants were asked to indicate whether or not the digit-symbol pair presented in the center of the screen matched according to the key of associations presented to them. In order to asses processing speed, participants were given 90 seconds to respond to as many trials as correctly and quickly as possible. To ensure participants were taking the task seriously and to minimize exclusions due to random responding, participants were asked to complete the task a second time if their accuracy was below 70%. We subsequently only analyzed the data from a participant's final attempt at the task.

## Risky Decision-Making Task

Participants were presented with 120 pairs of binary choices, 60 of which were presented during the light-time pressure (LTP) block and the remaining 60 were presented in the severe time pressure block (STP). Time pressure was manipulated by allowing the participants either 1.5 seconds (STP blocks) or 5 seconds (LTP blocks) to think about their choice. After this lock-out period, participants had a 1 second window to respond in both time pressure conditions; this response window was implemented to minimize the variability in response times and isolate the effects of processing speed on decision-making. Participants were prompted to think about their choice before the response window opened which was signaled by a switch in the cue—from "think" to "choose"—and the image of two arrow keys. The order of presentation of the two time pressure blocks was counterbalanced across participants.

Each pair of options involved a certain option and a risky option with probability $p$ of winning the indicated amount and probability $1-p$ of winning nothing; all gambles were of equal expected value except for 12 "catch" trials in which the expected value greatly favored an option (expected value $= \pm 90$). Half of the stimuli were framed as losses and half of the stimuli were framed as gains. In both frames, the outcome probability of the risky options varied between extremely likely (0.90, 0.95 or 0.99) or extremely unlikely (0.10, 0.05, 0.01).

Information about each pair of options were presented in a manner similar to that used by Tymula et al. (2012): at the start of each trial participants were presented with two stacked bar-graphs where framing was depicted by the color of the bars (red for losses, and blue for gain) and the outcome probability was depicted by the proportion of the bar which was colored (either red or blue), while the amounts ranged from \$1 to \$200 (see Fig. 2). The outcomes of gain trials were added to total earnings while the outcomes of loss trials were subtracted from total earnings—making the task incentives compatible. Participants were paid a bonus in proportion to their total earnings.

### Data Analysis

In order to ensure that participants' choices were not made randomly but were based on the information presented, participants with less than 75% accuracy on catch trials across both conditions (operationalized as the proportion of choices which maximize expected value) were excluded from the sample, resulting in the exclusion of 21 participants. Participants who also failed to score above 70% accuracy during the last run of the digit-symbol task were also excluded from the sample—one in total. Finally, six participants were excluded for failing to meet the specified deadline resulting in a total exclusion of 28 participants of the 100 collected.

We used a mixed-effects logistic regression to predict risky versus certain choice on the basis of 1) the framing of the problem (losses or gains) 2) the outcome probability (coded as $>0.5$ or $<0.5$), and 3) time pressure condition (light or severe), and all two- and three-way interactions between these predictors. This regression model then gives us two terms of interest: the two-way interaction between probability and framing—an estimate of the fourfold pattern of choice effect since it represents the extent to which mean differences between gain and loss frames depend on outcome probability (large or small), and the three-way interaction between probability, framing and time pressure, which indicates the extent to which the presence of fourfold pattern is modulated by time pressure. Similarly, two additional regression models were run to test the effects of individual differences in processing speed on choice within each time pressure condition. Specifically, to assess the influence of individual differences in processing speed on choice, a similar regression was run for each time pressure condition except with normalized processing speed score added as an independent variable instead of time pressure condition. For all regressions, all categorical independent variables were effect coded and entered as both fixed and random effects. These regressions were estimated using the lme4 package (Pinheiro & Bates, 2002) for the R programming language.

## Results

As predicted, under little time constraints participants exhibited a fourfold pattern of choice: they were both sensitive to the framing of the problem ($\beta = 1.44$, SE = 0.10, $p < .001$), and the interaction between the outcome probability and the framing of the problem ($\beta = -0.37$, SE = 0.18, $p = 0.04$).

However, under strong time pressure participants exhibited a marked framing effect, becoming less sensitive to outcome probability ($\beta = 0.28$, SE = 0.08, $p = 0.001$). Thus, the effect of time pressure on risky choice, surprisingly, changed participant's preferences from one ostensibly irrational pattern

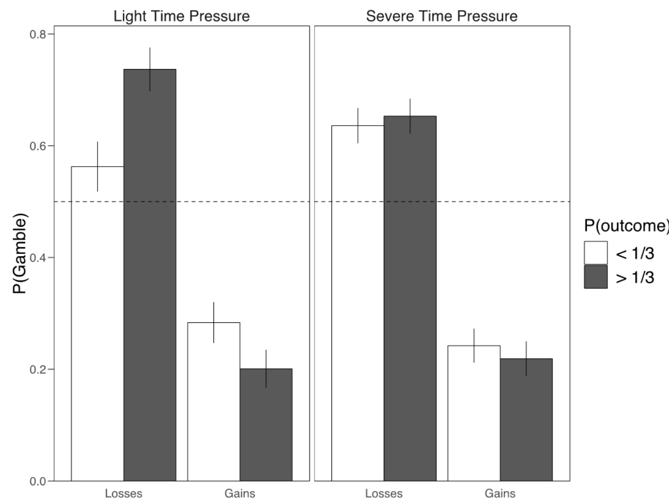


Figure 3: The effect of time pressure on risky decision-making. Under the light time pressure condition (LTP; 5 sec) participants showed more of a fourfold pattern compared to when under the severe time pressure condition (STP; 1.5 sec).

to another (see Fig. 3).

However, it remains unclear if this change in preference is a result of a reduction in the participant's ability to comprehend the gambles offered and correctly respond. It is possible that time limitations would lead to a nonspecific increase in choice randomness, as opposed to the proposed reduction in cognitive resources used. To test this alternative account, we compared the percentage of correct responses to the catch trials in the strong time pressure condition to test if it was significantly higher than chance. Using an Exact Binomial test, we were able to confirm that participants were capable of responding to the catch trials well above chance (Accuracy = 0.91, $p \leq 2.2 \times 10^{-16}$). This is to be expected as those participants who did not respond accurately in general—either due to lack of attention or understanding—were excluded from the analyses.

Finally, individual differences in processing speed were found to be related to risk preference in the predicted direction. Under light time pressure (LTP condition), individual differences in processing speed interacted with both framing of the problem ($\beta = 0.29$, SE = 0.13, $p = 0.02$) and the interaction between outcome probability and the framing of the problem ($\beta = -0.51$, SE = 024, $p = 0.03$). As processing speed increased, the extent to which participants exhibited a fourfold pattern also increased. Put another way, as processing speed decreased they were less likely to endorse a fourfold pattern (see Fig. 4). Moreover, these changes in risk preferences were not likely due to random performance on the task as processing speed and catch trials accuracy was not correlated ($r = -0.0081$, $p = 0.94$). Similarly, under severe time pressure (STP condition), both the two-way interaction between processing speed and the framing of the problem ($\beta = 0.24$, SE = 0.09, $p = 0.01$) and three-way interaction

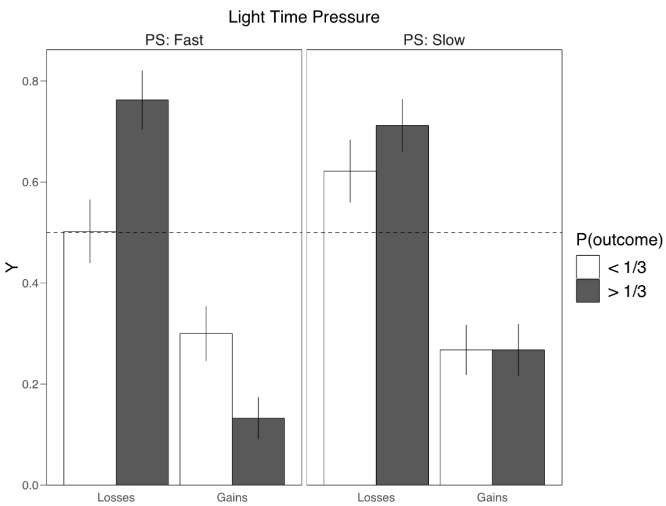


Figure 4: Under the light time pressure condition individual differences in processing speed (PS) predicted the extent to which participants endorsed a fourfold pattern. Processing speed conditions were assigned based on a median split.

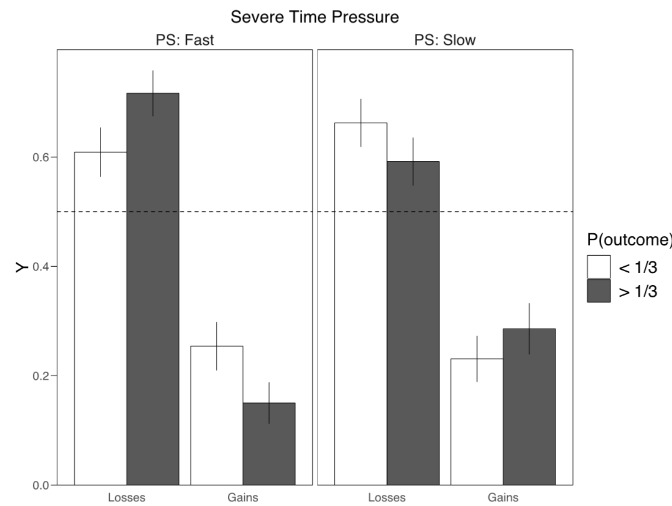


Figure 5: Under the severe time pressure condition individual differences in processing speed predicted the extent to which participants endorsed a fourfold pattern. For ease of exposition, processing speed scores (PS) were split based on the median.

between outcome probability, framing, and processing speed ($\beta = -0.30$, $SE = 0.12$, $p = 0.01$) were statistically significant (see Fig. 5).

## General Discussion

The results presented here show that both situational and personal factors which limit cognitive resources contribute to changes in participants' risk preferences. Under little time constraints, participants produced a fourfold pattern of risk preferences—consistent with prospect theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992). However, as predicted by a recently-proposed sample-based variant of expected utility theory, SbEU (Nobandegani et al., 2018), limitations in available cognitive resources—induced either through time pressure or measured by individual differences in processing speed—lead participants to go from showing a fourfold pattern to a framing effect.

While, descriptive models like prospect theory describe the risk preferences when selecting between gambles, this provide no account for how these preferences evolve over time or how limiting cognitive resources affects preferences. Thus, our results surprisingly reveal that the ostensibly irrational framing effect, fourfold pattern, and the demonstrated transition between the two, can all be explained as resulting from rational use of limited cognitive resources.

Interestingly, Stanovich and West (1998) demonstrated that performance on classic reasoning and judgment tasks and relationships to measures of academic achievement, correlates within individuals. Taken together with the results presented here, there is mounting evidence that the use of heuristics and biases may reflect the rational use of limited processing resources, thus suggesting that future models of choice should take into consideration individuals' cognitive abilities (or limitations). However, more work needs to be done to identify which specific cognitive capacities (e.g., processing speed or working memory) significantly contribute to the use of certain heuristics, thus providing an opportunity to better understand the cognitive mechanisms required for the performance of a given task.

Recent empirical work has also found time pressure to produce a similar pattern of results—severe time pressure lead to stronger framing effects—but failed to observe a fourfold pattern; instead they found individual preferences to reflect a weaker framing effect under light time pressure (Guo et al., 2017). However, the results were interpreted to arise from using a fast, intuitive systemas opposed to a slow, deliberative system.

Some have suggested that heuristics and biases are more than merely a result of flaws in human reasoning but are adaptive strategies to deal with conditions of limited time, knowledge or computational capacities (Simon, 1956; Todd & Gigerenzer, 2012) or take advantage of the structure of information in the environment (Todd & Gigerenzer, 2012). In fact, both experimental work (Goldstein & Gigerenzer, 2002), and theoretical work (e.g., Nobandegani & Shultz, 2019) has shown that fast and frugal algorithms can outperform standard integrative algorithms when knowledge is limited. Our results are in accordance with this compromise between normative and heuristic views of cognition as we show that biases like the framing effect can be explained as a strategic use of limited cognitive resources.

While previous work has also interpreted the framing effect as being a result of quick and intuitive thinking, these explanations make appeal to dual-process theory (De Martino, Kumaran, Seymour, & Dolan, 2006; Guo et al., 2017; Kahneman & Frederick, 2005; Sloman, 1996; Stanovich & West, 1998). Surprisingly, here we show that a rational *single-process* model can account for the observed results: an apparent framing effect can arise from limiting the number of samples in a resource-rational, sample-based expected utility model, SbEU (Nobandegani et al., 2018). A single-process framework is favorable over dual-process models as it provides a more parsimonious account of the observed effect.

Interestingly, unlike dual process theory would suggest, our results reveal that even when using a slow, deliberative system one can produce ostensibly irrational behavior. Concretely, according to our findings, deliberation takes us from one ostensibly irrational bias (framing effect) to another (the fourfold pattern of risk preferences)—and, as our work suggests, all of this can be understood as the optimal use of limited cognitive resources.

## References


Crump, M. J., McDonnell, J. V., & Gureckis, T. M. (2013). Evaluating amazon's mechanical turk as a tool for experimental behavioral research. *PloS One*, *8*(3), e57410.

De Martino, B., Kumaran, D., Seymour, B., & Dolan, R. J. (2006). Frames, biases, and rational decision-making in the human brain. *Science*, *313*(5787), 684–687.

Di Mauro, C., & Maffioletti, A. (2004). Attitudes to risk and attitudes to uncertainty: experimental evidence. *Applied Economics*, *36*(4), 357–372.

Fehr-Duda, H., Bruhin, A., Epper, T., & Schubert, R. (2010). Rationality on the rise: Why relative risk aversion increases with stake size. *Journal of Risk and Uncertainty*, *40*(2), 147–180.

Gigerenzer, G., & Selten, R. (2002). *Bounded rationality: The adaptive toolbox*. MIT press.

Goldstein, D. G., & Gigerenzer, G. (2002). Models of ecological rationality: the recognition heuristic. *Psychological Review*, *109*(1), 75.

Griffiths, T. L., Lieder, F., & Goodman, N. D. (2015). Rational use of cognitive resources: Levels of analysis between the computational and the algorithmic. *Topics in Cognitive Science*, *7*(2), 217–229.

Guo, L., Trueblood, J. S., & Diederich, A. (2017). Thinking fast increases framing effects in risky decision making. *Psychological Science*, *28*(4), 530–543.

Icard, T. (2014). Toward boundedly rational analysis. In *Proceedings of the* 36$^{th}$ *Annual Conference of the Cognitive Science Society* (pp. 2391-2396). Austin, TX: Cognitive Science Society.

Kahneman, D., & Frederick, S. (2014). A model of heuristic judgment. *The Cambridge Handbook of Thinking and Reasoning*.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decisions under risk. *Econometrica*, *47*, 263–291.

Levin, I. P., & Gaeth, G. J. (1988). How consumers are affected by the framing of attribute information before and after consuming the product. *Journal of Consumer Research*, *15*(3), 374–378.

Lieder, F., Griffiths, T. L., & Hsu, M. (2018). Overrepresentation of extreme events in decision making reflects rational use of cognitive resources. *Psychological Review*, *125*(1), 1.

Loke, W. H., & Lau, S. L. L. (1992). Effects of framing and mathematical experience on judgments. *Bulletin of the Psychonomic Society*, *30*(5), 393–395.

Markowitz, H. (1952). The utility of wealth. *Journal of political Economy*, *60*(2), 151–158.

Mathias, S. R., Knowles, E. E., Barrett, J., Leach, O., Buccheri, S., Beetham, T., ... Glahn, D. C. (2017). The processing-speed impairment in psychosis is more than just accelerated aging. *Schizophrenia Bulletin*, *43*(4), 814–823.

McNeil, B. J., Pauker, S. G., Sox Jr, H. C., & Tversky, A. (1982). On the elicitation of preferences for alternative therapies. *Preference, Belief, and Similarity*, *583*.

Miller, J. G. (1960). Information input overload and psychopathology. *American Journal of Psychiatry*, *116*(8), 695–704.

Moxey, A., OConnell, D., McGettigan, P., & Henry, D. (2003). Describing treatment effects to patients. *Journal of General Internal Medicine*, *18*(11), 948–959.

Nobandegani, A. S. (2017). The Minimalist Mind: On Mininality in Learning, Reasoning, Action, & Imagination. McGill University, PhD Dissertation.

Nobandegani, A. S., da Silva Castanheira, K., Otto, A. R., & Shultz, T. R. (2018). Over-representation of extreme events in decision-making: A rational metacognitive account. In *Proceedings of the* 40$^{th}$ *Annual Conference of the Cognitive Science Society* (pp. 2391-2396). Austin, TX: Cognitive Science Society.

Nobandegani, A. S., & Shultz, T. R. (2019). Toward a formal science of heuristics. In *Proceedings of the* 41$^{st}$ *Annual Conference of the Cognitive Science Society.* Austin, TX: Cognitive Science Society.

Salthouse, T. A. (1985). Speed of behavior and its implications for cognition. In *Handbook of the Psychology of Aging,* 2$^{nd}$ *Ed. (pp. 400-426).* New York, NY.

Scholten, M., & Read, D. (2014). Prospect theory and the forgotten fourfold pattern of risk preferences. *Journal of Risk and Uncertainty*, *48*(1), 67–83.

Shadlen, M. N., & Shohamy, D. (2016). Decision making and sequential sampling from memory. *Neuron*, *90*(5), 927–939.

Simon, H. A. (1956). Rational choice and the structure of the environment. *Psychological Review*, *63*(2), 129.

Sloman, S. A. (1996). The empirical case for two systems of reasoning. *Psychological Bulletin*, *119*(1), 3.

Stanovich, K. E., & West, R. F. (1998). Individual differences in rational thought. *Journal of Experimental Psychology: General*, *127*(2), 161.

Starmer, C. (2000). Developments in non-expected utility theory: The hunt for a descriptive theory of choice under risk. *Journal of Economic Literature*, *38*(2), 332–382.

Todd, P. M., & Gigerenzer, G. (2012). *Ecological rationality: Intelligence in the world*. OUP USA.

Tversky, A., & Kahneman, D. (1981). The framing of decisions and the psychology of choice. *Science*, *211*(4481), 453–458.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, *5*(4), 297–323.

Tymula, A., Belmaker, L. A. R., Roy, A. K., Ruderman, L., Manson, K., Glimcher, P. W., & Levy, I. (2012). Adolescents risk-taking behavior is driven by tolerance to ambiguity. *Proceedings of the National Academy of Sciences*, *109*(42), 17135–17140.

von Neumann, J., & Morgenstern, O. (1944). *Theory of games and economic behavior (commemorative edition)*. Princeton University Press.

Vul, E., Goodman, N., Griffiths, T. L., & Tenenbaum, J. B. (2014). One and done? optimal decisions from very few samples. *Cognitive Science*, *38*(4), 599–637.

Zur, H. B., & Breznitz, S. J. (1981). The effect of time pressure on risky choice behavior. *Acta Psychologica*, *47*(2), 89–104.