The Stream of Spatial Information: Spanning the Space of Spatial Relational Models

Paulina Friemann (friemanp@cs.uni-freiburg.de)
Cognitive Computation Lab, Georges-Köhler-Allee, University of Freiburg, 79110 Freiburg, Germany

Jelica Nejasmic (jelica.nejasmic@ph-ludwigsburg.de)
PH Ludwigsburg, 71634 Ludwigsburg, Germany

Marco Ragni (ragni@cs.uni-freiburg.de)
Cognitive Computation Lab, Georges-Köhler-Allee, University of Freiburg, 79110 Freiburg, Germany

Abstract

Given identical informational content, the order in which you receive spatial information may heavily influence the correctness of your mental representation. This can reveal important insights into the specifics of human spatial cognition and the way we integrate information. Despite its importance in everyday life, its causes and the mental processes involved still remain an open question. Most cognitive models so far have focused on modeling only answer distributions or just the most frequent answer given by all participants.

In this paper we take a rather radical approach: We turn to the individual spatial reasoner and focus our analyses on the stream of spatial information and related reaction times, i.e., how the spatial information is represented and cognitively processed. By spanning a space of 243 cognitive spatial models, some of which outperform the current state-of-the art models, it is possible to test the goodness of general principles underlying such models.

Keywords: Spatial Cognition; Reasoning; Continuity Effect; Cognitive Models

Introduction

Imagine that you are new to a city. It is a common experience that it is not very likely that you will have all spatial information available at the same time. Rather, you will receive it piece by piece. However, the way of how we receive spatial information can impact our mental representation, the time to understand the information, and possible conclusions we draw. But how do we process information mentally that we receive? How do we possibly integrate the spatial information into a mental representation? How difficult is it to process the information? What can existing cognitive approaches and computational models contribute?

Spatial relational information can be formulated by two objects and a relation: the first object is the object *to be located*, the relation gives information about how the objects are spatially connected, and the second object which is termed the *reference* object. Consider the following:

(1) The post office is to the left of the train station.
The train station is to the left of the main street.
The main street is to the left of the park.

Can you easily build a mental representation integrating this information at the same time? You should have no difficulty at all! Even receiving this information step-by-step, each new information nicely integrates with the most recent information. Such a problem is called a *continuous description*. Consider now the following description:

(2) The train station is to the left of the main street.
The park is to the right of the main street.
The post office is to the left of the train station.

This time, it might have taken more time and a bit more difficult to build a mental representation from the given assertion. While the information content was identical to before, the information could not be so easily integrated as in problem (1). This was mainly due to the last assertion that related the post office to the train station. Such problems are coined *semi-continuous*. Consider now this last description:

(3) The post office is to the left of the train station.
The park is to the right of the main street.
The train station is to the left of the main street.

Again, if you have received the assertions piece by piece, building an internal representation might have been again more difficult. While again the description has the same information content as all descriptions before, the first and the second assertion were unrelated, requiring to build two unrelated scenarios. Hence, such a problem description is coined *discontinuous*. All three problems allow for constructing an identical arrangement of the objects, namely

post office – train station – main street – park.

Such an arrangement of objects from the assertions is called a *model* of the assertions. These three problems have been investigated by psychologists in the so-called continuity effect (e.g., Ehrlich & Johnson-Laird, 1982; Knauff, Rauh, Schlieder, & Strube, 1998). But, why does the second and especially the third problem appear to be more difficult to be processed by humans? The *stream* of information makes the difference between problem descriptions. In continuous and semi-continuous descriptions, a common middle-term of two successive assertions exists. Since this is not the case in discontinuous orders, these assertions are more difficult to process and may even require to keep two distinct pieces of information in working memory.

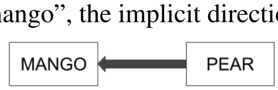
Because of the fine-grained nature of this effect, modeling the cognitive processes which underlie it can give insight into how exactly spatial information is processed in the mind. Several cognitive models have been proposed for spatial relational reasoning, among which an implementation in a cognitive architecture (Ragni, Fangmeier, & Brüssow, 2010), a model for reasoning with intervals (Schlieder & Berendt, 1998), and a stand-alone cognitive architecture (Schultheis & Barkowsky, 2011) (for a recent overview see Friemann & Ragni, 2018). To account for the continuity effect, a cognitive model needs to describe the nature of constructing a spatial model in great detail. This includes the introduction of a measure of difficulty, the *mental cost*, of a specific mental operation to account for the increase in reading times and the drop in accuracy. The cognitive models which satisfy these requirements are the *spatial reasoning as verbal reasoning* model (Krumnack, Bucher, Nejasmic, & Knauff, 2010) and *preferred inferences in reasoning with spatial mental models* (PRISM, Ragni & Knauff, 2013), which we now introduce.

Cognitive Theories, Models, & Complexity

Verbal Reasoning Model (Krumnack et al., 2010). The core assumption underlying the Verbal Model is that deduction processes does not necessarily require deduction-specific mechanisms to operate on internal representations. Instead, a simple order of object terms and some verbal cognitive mechanisms might guide the reasoning process. Following Polk and Newell (1995), cognitive processes in deductive reasoning might be based upon the same processes as language comprehension and generation. The model satisfies the criteria of verbal reasoning as outlined by Polk and Newell (1995). Verbal in that sense refers to transforming between verbal and semantic representations, that is constructing the queue (encoding) and "reading out" information that is not explicitly provided by verbal descriptions. It is assumed that reasoning is accomplished by applying well-trained linguistic processes. The approach does not obviate specific mechanisms but provides a more parsimonious explanation on how inferences can be drawn from given information without assuming additional mechanisms.

The computational model assumes the mental spatial structure to resemble a queue. In the same vein, each mental model has an *implicit direction*. This direction depends on the relation in the first premise and is contrary to the explicit direction in this relation. This can be understood as simulating an expectation on where the next object is about to appear, which can be easily understood by considering Table 3. For example, if the first premise was "The mango is to the left of the pear", the implicit direction would be to the right:

On the other hand, if the first premise was "The pear is to the right of the mango", the implicit direction is to the left:



PRISM (Ragni & Knauff, 2013). PRISM is an implementation of the theory of preferred mental models. The model simulates and explains how preferred models are constructed, inspected to find a putative conclusion, and then varied to find possible counter-examples. A spatial working memory structure is operationalized by a spatial array. A spatial focus inserts tokens into the array, inspects the array to find new spatial relations, and relocates tokens in the array to generate alternative models of the problem description, if necessary. The focus also introduces a general measure of difficulty based on the number of necessary focus operations (rather than the number of models).

Mental Costs and Complexity. The computational model PRISM was the first model to predict reasoning difficulty of spatial problems by assigning unit costs to the focus operations in a spatial working memory, a location where spatial models are built (Ragni & Knauff, 2013). By the numbers of operations PRISM is able to explain among others the continuity effect: as a successive insertion of the terms from left to right, do cost less than switches in the focus direction (semi-continuous case), which costs less than to generate, group, and insert different submodels (discontinuous case). The Verbal Model uses a similar cost measure.

The Order of Information Effect: Data

The order effect for human inferences has been reported in a number of articles (e.g., Ehrlich & Johnson-Laird, 1982; Knauff et al., 1998; Nejasmic, Bucher, & Knauff, 2015) and is explained with the effort to construct a mental representation of the assertions.

Table 1: Order of assertions in Knauff et al. (1998) and Nejasmic et al. (2015). Please note that $\sim$ represents the relation, which is 'left of' in the case of Experiment 1 in Nejasmic et al. (2015) and Knauff et al. (1998), and 'right of' in the case of Nejasmic et al. (2015).

Order	Assertions		
continuous	$A \sim B$	$B \sim C$	$C \sim D$
semi-continuous	$B \sim C$	$C \sim D$	$A \sim B$
discontinuous	$C \sim D$	$A \sim B$	$B \sim C$

Knauff et al. (1998) conducted an experiment, inspired by research of Ehrlich and Johnson-Laird (1982), to test effects on response times and error rates of continuous, semi-continuous, and discontinuous orders of spatial assertions (cp. Table 1) using the relation 'left of'.

The processing times and error rates are summarized in Table 2. While the continuous and semi-continuous order lead to a similar error rate of about 40%, reasoning about discontinuous orders of assertions was more difficult and lead to about 50% errors. Note that the processing time for the third assertion in discontinuous order compared to the other assertions is significantly higher.

Nejasmic et al. (2015) investigated underlying cognitive processes in two experiments using a random presentation of the 72 problems of the three premise orders *continuous*, *semi-continuous* and *discontinuous*. Each premise was presented

Table 2: The four-term-problems in the experiment of Knauff et al. (1998) with reading-times (RT in seconds) and error rates (in percentage correct). Participants were presented with interval relations.

	Assertion			
Order	RT 1	RT 2	RT 3	Error rates
continuous	13.0	11.2	10.9	39.7
semi-continuous	13.6	11.0	11.9	40.1
discontinuous	12.4	13.9	19.5	50.0

sequentially (in a self-paced manner and only one premise visible at a time). The premise described the spatial relation between four small, equal-sized, and disyllabic objects (tools, fruits, or vegetables) for example: "The mango is left of the pear, the pear is to the left of the kiwi, the kiwi is to the left of the apple."

The instruction was to imagine the arrangement described by the premises (in the example: mango – pear – kiwi – apple). Subsequently participants were asked to define the correct arrangement by typing the initial letters of the named objects using the computer keyboard. After the last letter was entered, the trial finished automatically. The next trial started not before the participant hit the "return" key. The program recorded (a) premise reading times (respective time from stimulus onset to key press calling up the next premise), (b) the number of correct responses, and (c) corresponding response times (time from request onset till enter of the last letter).

Experiment 1 and 2 differ mainly in the used relation resulting in different working direction. In Experiment 1 the relation 'left of' was used suggesting a working direction from left to right. In contrast, Experiment 2 used the relation 'right of' resulting in a working direction from right to left. The position of new named objects is leftmost (see Table 3).

Table 3: Example premises and models for a continuous order

	Experiment 1		Experiment 2	
	Premise	Model	Premise	Model
1	A left of B	AB	D right of C	CD
2	B left of C	ABC	C right of B	BCD
3	C left of D	ABCD	B right of A	ABCD

Results from the first experiment are in line with previous findings concerning the continuity effect. Participants need more time to process unrelated information and more errors occur in the discontinuous condition. In the second experiment the continuity effect was presumably counteracted by the working direction. Although processing third premises in the discontinuous condition took the most time, there was an overall and consistent increase of reading times over all conditions. It was expected that reasoners find it more difficult to work in the culturally nonpreferred right-to-left direction, but in the case that the continuity effect results from the integration of two separate models when confronted with discontinuously presented information, the working direction should not matter. So, results support the assumption that one preliminary model is constructed and modified in cases of discontinuity.

Results and Discussion on Aggregated Data

The Kendall rank correlation coefficient τ_b with the mean reaction times for Experiment 1 and 2 of Nejasmic et al. (2015) and the reported data in Knauff et al. (1998) was calculated. We removed all reading times which were outside the 1.5 interquartile range. The results can be found in Table 4.

Table 4: Correlations and significance level for PRISM and the Verbal Model on the aggregated experimental data.

	PRISM		Verbal Model	
	r_{τ_b}	p	r_{τ_b}	p
Nejasmic et al.: Exp 1	.800	.007	0.730	.018
Nejasmic et al.: Exp 2	.033	1	0.225	.501
Knauff et al.	.730	.182	.609	.044

For Experiment 1 from Nejasmic et al. (2015), PRISM had a better correlation than the Verbal Model. The same procedure was done with the data from (Knauff et al., 1998) (Experiment 3 in Table 4), which used the same setting as Experiment 1. PRISM and the Verbal Model correlated significantly with the data.

For Experiment 2 however, the correlations dropped strongly. This indicates that the process to generate a mental model are different from relational descriptions from left to right than building directions from right to left.

As outlined above, many cognitive models have focused on explaining aggregate data. But, how good are these models in predicting each individual reasoner? And, are there other models that can predict individual reasoner better? To further investigate the performance of the models, we turn to the individual reasoners.

To approach this challenge, there are two possibilities: creating cognitive models which are *adaptable to*, or creating cognitive models *designed for* individuals.

The remainder of this paper will investigate the second option. Taking features of models from the literature and insights from psychological experiments, we will span a large space of possible cognitive computational models for spatial relational reasoning.

Generating the Space of Spatial Reasoning Models

To investigate the goodness of the general assumptions, we looked at a whole family of potential models. This approach is driven by the idea that individual participants may not use the same strategy and their flow of information processing may differ. Hence, rather than constructing a certain model, we identified features in which potential models can differ. These are inspired by proposed cognitive models for spatial relational reasoning in the literature. PRISM, for example, proposes a mental model manipulation device, called focus, which acts just like a foveal area for mental models. The Verbal Model assumes that a spatial mental model has an implicit direction, which can offer an explanation for the better performance in modeling the right-to-left task from Experiment 2 (Krumnack, Bucher, Nejasmic, Nebel, & Knauff, 2011). As for the discontinuous case, the Verbal Model does not offer a solution for the presentation of discontinuous information, as in the connection of two formerly unrelated chunks of information. *PRISM* on the other hand offers a solution in the form of constructing two unrelated mental models, and integrating them group-wise when connecting information is presented.

We chose 8 partly interdependent features to span the space of investigated models, leading to 243 possible cognitive models:

Mental Spatial Structure

The main difference between the Verbal Model and PRISM is the underlying spatial representation structure. PRISM assumes a grid-like structure in the human mind, with a mental focus which inspects one object at a time, can move through the mental representation object by object with an unary cost in each direction, and is persistent throughout the whole task (Ragni & Knauff, 2013). The Verbal Model on the other hand proposes a queue-like structure, meaning that there exists an implicit direction in the mental model, which is dependent on the relation in the first premise (Krumnack et al., 2010). The question whether a mental model has an implicit direction is the focus of the first three main features, leading to the first $2^3 + 1$ possibilities:

Implicit Model Direction Inspired the Verbal Model, models can have a queue-like mental spatial structure with an implicit direction. Moving through this queue in the implicit direction is assumed to be computationally cheap, while moving against this direction is costly. The opposite assumption would be a grid-like mental array similar as is used in PRISM.

Persistency of Direction In the Verbal Model, the implicit direction depends on the relation in the first premise. For the relation 'left of', the direction of the queue would be to the right and vice versa. We added this dependency as a possible feature, as well as the possibility of a reversed dependency, i.e. for the relation 'left of', the direction of the queue would be to the left as well.

Preliminary Integration Following the research in Nejasmic et al. (2015), it seems likely that when reading discontinuous information, such as "a is left of b, c is left of d", reasoners build a preliminary, connected model instead of a second, disjunct model. Therefore, we introduced this idea as another feature for models which assume an implicit direction: Construct a temporary model with the discontinuous information inserted into the mental model in direction of the queue.

Focus

Moving through the mental model is, in PRISM and the Verbal Model, assumed to require some mental operation. Following the terminology in PRISM, we introduce this idea as the so called *focus*, a device which is able to move through the mental model object by object.

If including the focus into the cognitive model, we can further differentiate between different types of foci. For example, while PRISM has a persistent mental focus throughout the whole task, the Verbal Model implicitly introduces a focus-like notion which resets with each premise. The idea is that when a premise contains an object which is already in the queue, the model has to move through the queue from the position of this object. In a sense, this could be described as a focus with the ability to *jump*. The focus feature adds another $2^3 + 1$ possibilities, as a model which assumes a focus can have any of the three mentioned focus features:

Jumping Focus As in the Verbal Model, when reading a premise, the focus can jump to the addressed object which is already existent in the mental model. After this jumping, the focus then has to move one by one.

Access Tail In a queue-structure, like it is assumed in the Verbal Model, the first element, the start of the queue, can be easily accessed. One could assume that the last element, the tail of a mental model, can be accessed just as easily.

Find Reference Object When a premise is read, the object which is already in the mental model has to be found to determine the positioning of the new object. However, if both items already exist in the queue, the relative positioning of the objects *in* the model have to be compared against the new premise. If the focus position is on one of the objects, it could be that for determining the relation between the objects, the focus now only has to move to the other object. However, taking into account the difference between the object *to be located* and the *reference* object, it is possible that the focus has to first move to the *reference* object and then to the object *to be located* to determine the relation between these objects.

Processing the Relation 'right of'

The experiments in Nejasmic et al. (2015) indicated that processing the sentence "a is right of b" is more difficult (at least for speakers of a language which is written from left to right (Krumnack et al., 2011)) than the 'left of'-relation. While the queue-structure in the Verbal Model can account for this fact, we introduced two features to allow a model with a direction-neutral spatial structure, like the one used in PRISM, to show this asymmetry. This feature space comprises 3 possibilities.

Revert When reading "a is right of b", insert b to the right of a first, only to break up that connection and insert it on the left.

Revert only the First Premise Revert only on the first premise, after then the insertion to the left is automatically correct. A model which has the revert-feature, does not have the feature of reverting only the first premise, as the latter is included in the former.

These features result in the following equation for the space of cognitive models:

$$\underbrace{(2^3+1)}_{implicit\ direction} \cdot \underbrace{(2^3+1)}_{focus} \cdot \underbrace{3}_{reverting} = 243 \quad (1)$$

Results and Discussion

Best Models for all Participants

Table 5: Correlations r_{τ_b} for individual data from Experiment 1 and all generated models.

	Median	Max	PRISM	Verbal Model
Exp. 1	.197	.22	.22	.218
Exp. 2	-.023	.059	-.05	-.05

To examine the goodness of fit of the generated models for the whole group of participants, we calculated the Kendall rank correlation coefficient τ_b for each model and normalized reading time of participants in the two experiments. The process for normalization was to first correct the reading times of each participant in each condition for outliers, and second to divide the reading times of a specific participant by her maximum reading time. This was done to account for individual processing speed differences and resulted in reading times between 0 and 1 for each trial without losing the relative speed differences of a specific reasoner across trials and conditions. Results from the correlation can be seen in Table 5.

In Experiment 1, PRISM was among the best models, correlating significantly with the normalized reading times ($p < .001$), as did the Verbal Model ($p < .001$). Again, Experiment 2 was much harder to predict for all models. However, also the close to significant correlation of the Verbal Model with the aggregated data disappeared when calculating the correlation with each individual reasoner. It even showed a significant negative correlation ($p = .001$). The correlation coefficient for PRISM was not significant ($p = .051$). Calculating the correlation for both experiments, the models which performed best ($r_{\tau_b} = .171$, $p < .001$) had the following configurations:

The models assume a mental spatial structure that is, contrary to the Verbal Model, persistent in its direction: a rightwards directed queue turned out to perform quite well. Contrary to the results from Nejasmic et al. (2015), models with *no* preliminary integration of features performed better on the two experiments combined. This indicates that this feature needs more investigation in terms of cognitive modeling and psychological investigation. A spatial focus structure with the ability to jump turned out to give the highest performance. The presence of the features considering the access of the last element (tail) and finding the reference object, in the configurations, seems to be, at least within this analysis paradigm, irrelevant.

This indicates several things, among which: (i) that PRISM and the Verbal Model are good models to reproduce the left-to-right tasks, (ii) that for the right-to-left relations, there exist models which can approximate the individual data points better than the models from the literature, and (iii) that restricting cognitive model of spatial reasoning to use only a single model for all participants might soon hit an insurmountable upper bound.

Best Models for Individual Participants

To explore further the idea that individual reasoners may use different strategies, operations or structures, we again calculated the Kendall τ_b coefficient, but this time we allowed for each participant to be assigned the cognitive model which fits best. With this, the median correlation was $r_{\tau_b} = .25$, with a maximum of $r_{\tau_b} = .489$ ($p < .001$).

The previously for the population identified best models only occurred in 42.9% of participants of Experiment 1, and in 14.3% of participants in Experiment 2. The percentage, to which features are present in the individual models, can be seen in Tables 7 and 8.

Table 7: Percentage to which main structural features are present in the best models for the individual reasoner.

	Direction			Preliminary	
	No Direction	Left	Right	Integration	Focus
Exp. 1	16.6%	21.9%	61.6%	52.4%	63.2%
Exp. 2	0%	42.9%	57.1%	28.6%	92.9%

Table 8: Percentage to which secondary features are present in the best models for individual reasoner. The percentages are conditional in the case of focus features, because they only apply if the focus is present.

	Focus				
	Jumping	Tail Access	Find Ref.	Revert	Revert First
Exp. 1	55.7%	50.0%	50.0%	30.8%	65.4%
Exp. 2	65.3%	50.0%	53.8%	17.9%	28.6%

Table 6: Best cost assignment for individual reading time prediction. Possible cost assignments were in the interval between 0 and 1, in increments of 0.1. This assignment yielded a median correlation of $r_{\tau_b} = 0.302$.

Initialization	Insert	Group	Break Links	Move with Dir.	Move against Dir.	Tail Access	New Start	Jump
0.7	0.1	0.1	0.1	0.8	0.5	0.8	0.3	0.8

Alternative Cost Measure

To examine the adequacy of the unary cost measure, we performed a search on the assignment between model actions and mental costs. This was done using Python's `scipy` library for scientific computing[1]. Using a random search algorithm, we explored the space of cost assignments in the interval between 0 and 1, in increments of 0.1. The goal is to find values for the costs, such that the correlation is maximized. For each assignment, we calculated the Kendall τ_b correlation between the predicted costs of each model and each participant's outlier corrected reading times of Experiments 1 and 2 from Nejasmic et al. (2015). We then selected the best model for the individual participants and took the mean of their correlations as the utility for the optimization. The best cost assignment can be taken from Table 6.

Using this method, the best configuration we found achieved a median correlation of $r_{\tau_b} = .302$. The most expensive actions in the assignment were the jumping movement, the access of the tail, and the movement in direction of the queue, or in any direction if there is no implicit direction. Initialization of a model is also costly. The direction *against* the implicit direction was chosen to be less costly than moving *with* the direction. Inserting a new object is not expensive in this assignment, as were breaking connections and setting a new starting node (as was assumed in Krumnack et al., 2011). Similarly, the cost of grouping objects into chunks, which was set to have a cost of n-1 with n being the number of objects in Ragni and Knauff (2013), was also assigned a low cost.

General Discussion

In this paper, we analyzed 243 cognitive models of spatial relational reasoning on their capability to predict *individual* reading times from studies on the continuity effect. These models comprised configurations of features from successful cognitive models from the literature and psychological experiments. While many configurations performed well on aggregated data and a model building direction from left ro right, none of them, including the cognitive models from the literature we based this study on, were able to correctly predict reading times for a direction from right to left. We then followed the notion that different people might use different strategies, and investigated whether assigning a specific cognitive model to individual reasoners would greatly improve performance. While we reached a better correlation using this method, it was still in question why the correlation did not increase even further. We thus challenged the unary cost measure proposed in Ragni and Knauff (2013). Using a search algorithm, we investigated whether a different cost assignment would lead to better predictions for the individual. While the fit got better, it still demonstrates that the individual variety is not yet captured. Especially Experiment 2, which explored a presentation of spatial information using the relation 'right of' revealed a low correlation, on the individual, but also on the aggregated level.

We explored the space of possible cognitive models for spatial relational reasoning using features present in cognitive models from the literature. However, this space did not yield a model which was able to predict reading times across tasks robustly. This can be due to several issues: (i) the core assumption of these models, that we build an abstract spatial representation (a mental model) is wrong, (ii) the true mental processes in our brain when processing spatial relational information differ from those assumed in the models of the literature, or (iii) the assumption of a sequential processing of spatial information has to be revised. The construction of a mental model nonetheless is a notion which is broadly accepted (Johnson-Laird, 2004; Ragni & Knauff, 2013). If the mental processes of model construction differ from those presumed by the state-of-the-art cognitive models, it stands to reason what other processes could be taking place. The sequential processing is common to most cognitive spatial models (Friemann & Ragni, 2018). Modeling of individual data is limited, as individual data, and especially reaction time, is noisy. However, if cognitive models fit averaged data well, but are not able to capture any single individual in the experiment, the meaning of cognitive modeling and goodness-of-fit needs to be reevaluated.

Conclusion

It seems we are still far from understanding the way our mind integrates spatial information. This study challenged common assumptions and practices from the area of cognitive modeling for spatial reasoning. These customs are found to be insufficient when applying them to the modeling of *whole* empirical data sets instead of the aggregated data. There is still much to be learned about the way we process streams of information, what mental operations are performed, and in how far we can generalize conclusions from the aggregated data to the individual human mind.

Acknowledgements

This paper was supported by DFG grants RA 1934/3-1, RA 1934/2-1 and RA 1934/4-1 to MR. We also thank Sara Todorovikj for assistance and helpful comments.

[1] `www.scipy.org/`

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