ANALOGICAL REASONING PATTERNS IN EXPERT PROBLEM SOLVING

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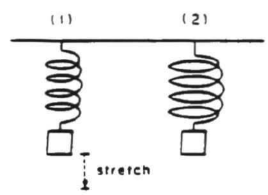
Spontaneous analogies have been observed to play a significant role in the problem solutions of scientifically trained subjects [1,2]. In some cases analogies can even lead to the construction of a new mental model for understanding a problem domain. This paper describes a number of different analogical reasoning patterns that have been observed in thinking aloud protocols from expert problem solvers. The purpose of the present study is to identify, classify, and label the critical subprocesses involved in such analogical solutions. In this study each of ten subjects were given a number of problems, including the following one:

FIGURE 1



<u>Spring Coils Problem</u>

A weight is hung on a spring. The original spring is replaced with a spring made of the same kind of wire, with the same number of coils, but with coils that are twice as wide in diameter. Will the spring stretch from its natural length, more, less, or the same amount under the same weight? (Assume the mass of the spring is negligible compared to the mass of the weight.) Why do you think so?

Subjects were advanced doctoral students and professors in technical fields who had reputations for being creative problem solvers. Seven of the ten subjects generated spontaneous analogies in solving this problem. A spontaneous analogy occurs when the subject, without being prompted, shifts to consider a situation B which differs in a significant way from the original problem situation A, and then tries to apply findings from B to A. In solutions by analogy the two contexts being compared are often perceptually different but they are still seen to be functionally or structurally similar in some way. For example, five subjects attempted to relate the problem to the analogy of a bending rod, as in the transcript excerpt below taken from video tape.

S1: (Draws bending rod in drawing G2-B of fig.2.) My intuition about that [the rod] is that if you.. doubled the length and hung some weight on it, that.. it, would bend considerably further... it would seem that that means that um, in the original problem, the spring in picture 2 [the wider spring] is going to hang farther.

Here S1 generates an analogy by drawing the picture of an analogous problem involving bending rods instead of stretching springs. This analogy has in fact led him to the correct answer, and provides a plausible but only partial justification for it.

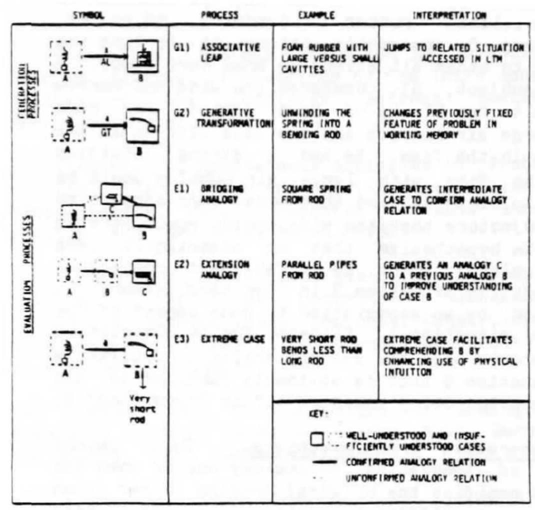


FIG. 2

<u>ANALOGICAL REASONING PATTERNS OBSERVED IN EXPERT PROBLEM SOLVING</u>

Analysis of more complex expert protocols however. makes it apparent that analogical reasoning is not a simple, one-step process, but involves a number of different processes, shown below.

(P1) <u>Generating the Analogy</u>. Given the original conception A of an incompletely understood situation, the analogous conception, B, is generated, or "comes to mind";

(P2) <u>Confirming the Analogy Relation</u>. The analogy relation between A and B must be "confirmed";

(P3) <u>Comprehending the Analogous Case</u>. Conception B must become well understood, or at least predictive;

(P4) <u>Transferring Findings</u>. The subject transfers conclusions or methods from B back to A.

Table 1

The last three processes can occur in any order. Analogies are often proposed tentatively, and processes (P2) and (P3) especially, can be quite time consuming. We have also been somewhat surprised to find that <u>there appear to be not one, but several ways of carrying out each of the above processes</u>. Some of the most important of these sub-processes are shown in fig.2. The figure provides a basic typology of analogical reasoning patterns that have been observed across different subjects. This paper gives an example and brief explanation of each pattern.

ANALOGY GENERATION PROCESSES

Associative leaps. The subject using an associative leap jumps to an analogous situation that differs in many ways from the original problem. A second subject, S2, generated evidence for several associative leaps in the spring problem when he said: "I feel as though I'm reasoning in circles and I think I'll make a deliberate effort to break out of the circle somehow...like rubber bands, molecules, polyesters.." apparently attempting to link the problem to other situations he knew more about. A third subject, S3, compared the wide and narrow springs to two blocks of foam rubber, one made with large air bubbles and one made with small air bubbles in the foam. He had a strong intuition that the foam with large air bubbles would be easier to compress, and this added some support to his conjecture that the wide spring would stretch more. We hypothesize that an associative leap takes place when an established conceptual framework for situation B in long term memory is activated by an association to some aspect of the original situation A. Evidence for an associative leap occurs when the subject shifts to consider a new situation B that is obviously familiar to him or refers to "being reminded of" or "recalling" B.

Generative transformations. This second method of generating an analogy occurs when the subject modifies the original problem rather than recalling a different analogous situation from memory. In other words, the subject transforms the problem by changing an aspect of it which was previously assumed to be fixed. For example, S2 refers to the rod as an "unwound spring". In this case the unwinding of the spring is considered a transformation because the subject is modifying a feature of the spring (its shape) that would ordinarily be held fixed in the problem.

It is hypothesized that a generative transformation occurs when the subject focuses on an internal representation of the existing problem situation A in working memory and changes an aspect of it to create a new but closely related situation B . Thus a generative transformation usually leads to the construction of a new situation B rather than activating an already constructed framework in long term memory.

This subject also generated another analogy via a transformation below while thinking about moving the weight along the spring wire:

S2: Hmmm, what if I imagined moving the weight along the spring? Now what if I recoiled the spring and made the spring twice as long...instead of twice as wide?...uhhh..it seems to me pretty clear that the spring that's twice as long is going to stretch more.

The analogy to the thought experiment of comparing springs of different lengths suggests to him that a wider spring may stretch more than a narrow spring. Notice that the analogy was generated from the rather playful transformation of sliding the weight up and down along the spring wire.

Evidence for a third method, generating an analogy via an abstract principle, has been observed on occasion, but only infrequently [1,2].

ANALOGY EVALUATION PROCESSES

Another finding that has surprised us is the fact that rather than simply generating a single analogy, some subjects generate chains of several analogies. Two types of chains are shown as processes E1 and E2 in fig.2. These are used to evaluate analogies. Processes used to critique and evaluate analogies are at least as important in expert problem solving as processes used to generate them.

Bridging analogies. Determining a match between key relationships in cases A and B is the first and most obvious method for confirming an analogy relation [4,5,6]. However, another interesting process in the form of a "bridging analogy" may also be used. For example, S2 was concerned about the apparent lack of a match between the non-constant slope in the bending rod and the constant slope of a stretched spring. In order to evaluate the bending rod analogy, he constructed the intermediate, bridging example of a spring with square coils as shown in drawing E1-C of fig.2. This allowed him to recognize that restoring forces in the spring come from twisting in the wire as well as bending-- a major breakthrough in his solution which corresponds to the way in which engineering specialists view springs. His discussion of the square spring is evidence for a cognitive bridging analogy, C, which helps him decide whether conceptual frameworks A and B are truly analogous. In this case the square spring analogy eventually aquired the role of a mental model which gave him a new understanding of how springs work.

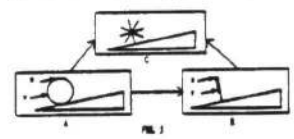


FIG. 3

In a question about whether one can exert a more effective force on a wheel at the top or at the axle (in pushing on the wheel of a covered wagon, for example) several subjects compared the wheel to a lever hinged on the ground (fig.3B). Pushing higher up on the lever would allow it to move a larger weight, they reasoned. Another example of a bridge, which helped one subject to confirm the appropriateness of this analogy is the spoked wheel without a rim shown in fig.3C.

Although physicists usually analyze the wheel problem directly in terms of torques, mathematicians often do not. The reader may be interested in conjecturing about how one mathematician, S4, solved this problem via an analogy to a pulley.

Extension analogies. The diagram for process E2 in fig.2 shows an extension analogy proposed by S1 in the form of two parallel pipes. S1 was hoping to predict whether the radius/stretch relationship in the spring was linear or quadratic or cubic, and his understanding of the bending rod analogy was not sufficient to help him. So he generated a further analogy to the bending rod. In this analogy two pipes are fixed at the left side and held together in such a way that when the weight is applied to the right side, the upper pipe is stretched and the lower pipe is compressed. His analysis of this thought experiment was part of an attempt to model the bending rod in more detail and determine its length/deflection relationship so that this information could in turn be used in analyzing the spring. In such an extension analogy, a second analogous case is used to understand the first analogous case. Thus analogies can be used recursively to understand and evaluate a previous analogous case.

Extreme cases. Aiding in understanding an analogous case is also one of the uses of extreme cases. For example, S2 generated the extreme case of a very short rod in order to confirm his prior prediction that a short rod would bend less than a long rod (process E3 in fig.2). Other methods of understanding an analogous case are to use a specific fact recalled from memory, a physical intuition, or an analysis in terms of abstract principles [2].

SUMMARY

Fig.2 illustrates several alternative subprocesses for acheiving processes P1, P2, and P3 in Table 1. Together, these subprocesses constitute a collection of intuitive heuristics used by experts in solving problems via analogy. Few of these subprocesses are described by subjects explicitly as they occur (they do not have names for them.) Rather, they must be inferred from patterns in the content of the subject's investigations. Reasoning patterns G1 and G2 in fig.2 are analogy generation patterns. Pattern E1, the bridging analogy, is a method for evaluating an analogy relation. Patterns E2, the extension analogy, and E3, extreme case analysis, are methods for evaluating and improving one's understanding of the analogous case. These reasoning patterns form a set of non-deductive problem solving strategies which: (1) are quite different from traditional problem solving procedures; (2) are associated with imagery reports; and (3) are capable of generating new insights and recognitions of previously undiscovered causal factors in a problem solution [3].

Various "compound solutions" combining two or more of the basic processes shown in fig.2 have also been observed. Our current hypothesis is that most observable chains of reasoning using spontaneous analogies are describable as recursive combinations of these basic patterns.

In the cases of the square spring and the parallel pipes, the novelty of these cases argues that they were at least in part invented by the subject rather than recalled directly from memory. Thus, the analogies observed do not always consist of familiar cases recalled from long term memory; the analogies can also consist of invented cases constructed in working memory. Furthermore, in the square spring and parallel pipes cases, the analogy is used as a mental model which allows the subject to understand the problem situation in a new way. This type of mental model construction appears to be important in the development of creative problem solutions and may play an important role in the development of new explanatory models in science [6].

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