

MULTIPLE MODELS OF EVAPORATION PROCESSES

Allan Collins
Dedre Gentner

Bolt Beranek and Newman Inc.
Cambridge, Massachusetts 02238

We have been analyzing subjects' responses to eight difficult questions about evaporation processes in order to formalize the different kinds of mental models people use in reasoning about complex systems. In this analysis we have identified three different levels of mental models that subjects use to reason about evaporation: macroscopic functional models, microscopic aggregate models, and microscopic molecular models. Models at these three levels are closely interlinked: Each dependency in a functional model is supported by one or more aggregate models, and each aggregate model by one or more molecular models.

We have represented the models at the macroscopic and aggregate levels in terms of Forbus's (1982) Qualitative Process Theory as a series of qualitative proportionalities. The proportionalities form a causal chain linking one variable to another. In the macroscopic models the linked variables are summary variables (e.g., temperature, density) that characterize masses of elements as a whole. The aggregate models link the summary variables of the macroscopic models to aggregates over space or time of individual particles. The molecular models describe the interactions of the individual particles. They are represented in terms of the incremental qualitative analysis of deKleer (1977) and Forbus (1981).

Analysis of Subjects' Protocols

We will illustrate our analysis by comparing two subjects' responses to one of the questions about evaporation together with the correct answer to the question. We will present each subject's protocol, together with a brief description of the subject's reasoning. Then we will give our representation of models at the different levels. The question was "Why do you see your breath on a cold day?" The first subject's response was:

RS: I think again this is function of the water content of your breath that you are breathing out. On a colder day it makes what would normally be an invisible gaseous expansion of your breath (whatever), it makes it more dense. The cold temperature causes the water molecules to be more dense and that in turn makes it visible relative to the surrounding gases or relative to what your breath would be on a warmer day, when you don't get that cold effect causing the water content to be more dense. . . . So I guess I will stick with that original thinking process that it is the surrounding cold air - that the cold air surrounding your expired breath causes the breath itself (which has a high water content and well I guess carbon dioxide and whatever else a human being expels when you breathe out), causes the entire gaseous matter to become more dense and as a consequence become visible relative to the surrounding air.

At the macroscopic, functional level RS's argument is that cold air cools the breath, which causes it to be more dense, which in turn causes it to be more visible. Microscopically he suggests no mechanism for the cooling process, but in this and other answers he appears to believe a "moving crowd" model of increased density (that average distance between molecules depends on their speed) which in turn reflects a "billiard ball" model at the molecular level. He also implies that visibility at the aggregate level is a ratio of visible particles to volume of space, but does not indicate any molecular model of particle visibility.

The second subject's response to this same question was:

PC: The reason is because the air that you breathe or rather the air that you should breathe out, comes from your body and is hot air. The air which surrounds your body, because it is a cold day, will be cold air. When the hot air that you breathe meets with the cold air of the atmosphere, it will tend to vaporize almost like steam from a kettle, which of course, can be seen. Thus unlike on a hot day, when there is hot air around you and the hot air that you breathe are the same temperature, roughly, you cannot see your breath because the steam will not be formed, but on a cold day because of the variation in the temperatures and the vaporization of your breath, you can see when you are breathing. This phenomenon would not occur on a hot day because of the similarity in temperature.

At the macroscopic level PC's argument is that the vaporization rate of water in your breath depends on the temperature difference between the breath and the air. In turn the amount of steam formed depends on the vaporization rate and the visibility of the breath depends on the amount of steam formed. PC here and elsewhere equates steam with water vapor. No aggregate or molecular models are explicit in PC's answer, though he implicitly believes that vapor holds together in space and that visibility depends on the ratio of visible to invisible particles.

The actual process that leads to seeing your breath on a cold day goes as follows: The cold air cools the water vapor in the breath, which leads to a high condensation rate of the water vapor, which leads in turn to a large amount of condensed water. It is this liquid water that is visible. At the aggregate level the cooling of the water vapor is a heat-exchange process, based on "billiard ball" collisions at the molecular level. Condensation is an aggregation of water molecules around a nucleus at the aggregate level, based on dipole electrical attraction at the molecular level. The amount of water in the breath depends on condensation and dispersion at the aggregate level, which depends on the billiard ball model of molecular interaction. Finally, the visibility of the condensed water depends on the ratio of visible particles to volume of space at the aggregate level, which depends on the absorption and re-emission of photons at the molecular level.

Table 1
Multiple Models of Why You See Your Breath on a Cold Day

	<u>Macroscopic Model</u>	<u>Aggregate Model</u>	<u>Molecular Model</u>
RS	Temp (B) \propto_Q Temp (A)	-	-
	Density (B) \propto_Q -Temp (B)	Moving Crowd Model	Billiard Ball Model
	Visibility (B) \propto_Q Density (B)	Visibility Model?	Reflectance Model?
PC	Vaporization-rate (B) \propto_Q Temp (B) -Temp (A)	-	-
	Amount-of (S) \propto_Q Vaporization-rate (B)	Container Model?	Billiard Ball Model?
	Visibility (B) \propto_Q Amount-of (S)	Visibility Model?	Reflectance Model?
CA	Temp (V) \propto_Q Temp (A)	Heat Exchange Model	Billiard Ball Model
	Condensation-rate (V) \propto_Q Temp (V)	Aggregation-on-nucleii Model	Dipole Attraction
	Amount-of (W) \propto_Q Condensation-rate (V)	Container Model	Billiard Ball Model
	Visibility (B) \propto_Q Amount-of (W)	Visibility Model	Absorption & Re-emission

A=air, B=breath, S=steam, V=water vapor, W=water

Table 1 summarizes the three answers: that of subjects RS and PC as well as the correct answer (CA). The macroscopic view in the first column specifies the qualitative proportionalities (Q-props) that form the functional models for the three answers. These Q-props are the relations used to describe a process history in Forbus's (1982) theory. They summarize the dependencies referred to in the three paragraphs above as the macroscopic models of the process. The individuals referred to in the Q-props are specified at the bottom of the table. Where a dash appears in the Aggregate Model or Molecular Model columns, it is because it is impossible to surmise what model the subject was using. A question mark indicates uncertainty whether the subject's answer was based on a particular model.

Aggregate Models

Table 2 shows two of the aggregate models referred to in Table 1. Each attempts to define the constraints operating on the aggregations of particles interacting over time and/or space to produce the corresponding Q-prop at the macroscopic level.

Table 2

Aggregate Models of Evaporation Processes

<u>Moving Crowd Model of Gases</u>	<u>Visibility Model of Suspensions</u>
$Density(M) \propto_Q \sum_{i=1}^n \sum_{j=1}^n Distance(m_i, m_j)$	$Visibility(M) \propto_Q \sum_{i=1}^n Volume(v_i) / Volume(S)$
$Distance_t(m_i, m_j) \propto_Q Speed_{t-x, t}(m_i) + Speed_{t-x, t}(m_j)$ where m_i and m_j collided at $t-x$	$\sum_{i=1}^n Volume(v_i) \propto_Q Amount-of(V)$ where M = suspension of particles v_i = visible particle in M S^i = space occupied by M V = visible matter in M
$Speed(m_i) \propto_Q Temperature(M)$ where M = unbounded mass of a gas m = molecule of gas M	

RS clearly believes that density of molecules in a gaseous state depends on the temperature of the gas, the second Q-prop for RS in Table 1. From this and other answers this appears to be supported by a moving crowd model of gasses: *The faster any particle is moving in an unbounded gas, the more distance it puts between itself and other particles. We have represented this model as a set of Q-props relating entities at the macroscopic level to aggregations of particles at the aggregate level.* The first Q-prop states that the density of a gas (at the macroscopic level) is negatively proportional to the distance between each pair of molecules. The second Q-prop states that the distance between any pair of molecules that collide at some time is proportional to their speed after they collide. The third Q-prop states that the speed of any molecule is proportional to the temperature of the gas. Thus the Moving Crowd Model relates the density of a gas to its temperature in terms of the steady state behavior of aggregates of molecules.

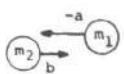
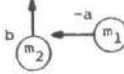
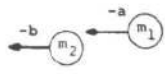
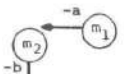
The second model shown represents the visibility of a mass of particles. The first Q-prop in the model states that the visibility of the particles is proportional to the ratio between the volume of the visible particles aggregated together and the volume of the space in which the mass is suspended. The second Q-prop states that the volume of the individual particles is proportional to the amount of the visible stuff in the mass (i.e., the number of the visible particles). The visibility model then relates the visibility of a mass of particles to the amount of visible material in the mass.

These models exemplify how we have tried to capture understanding at the aggregate level. Each model relies on mappings between functional quantities at the macroscopic level (e.g., temperature of a gas) and aggregate quantities at the microscopic level (e.g., average speed of the particles in the gas). By mapping down to the aggregate level people can "understand" a macroscopic dependency in terms of a set of dependencies at the aggregate level.

A Molecular Model

We can illustrate a molecular model by an expert "billiard ball" model of molecular interaction. Our analysis, shown in Table 3, is an extension to colliding balls of the incremental qualitative analysis of deKleer (1977) for rolling balls and Forbus (1980) for bouncing balls. All possible collisions of two balls of equal mass are summarized by the four cases shown and their combinations.

Table 3
Billiard Ball Model of Molecular Interaction

Case 1		Case 3																															
																																	
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When two balls collide, the ball moving faster initially is defined to be m_1 . The orientation of the X axis is defined by the direction of m_1 (negative in the X direction). The result of the impact is defined for four critical directions m_2 may be moving with respect to the X axis: left, right, up, down. Other possible trajectories of m_2 are additive combinations of two of these cases: e.g., up and left, up and right, etc.

The point of impact on m_2 is defined by an impact angle, measured from the center of m_2 as the origin, one side parallel to the X axis and the other side defined by the contact point. We have defined the result of impact for each of three critical angles: 0 degrees where for Case 1 the two balls meet head on, 45 degrees where the line from the center of m_2 to m_1 has a slope of 45 degrees, and 90 degrees where the two balls just barely touch each other. Other possible impact angles have values intermediate between these three angles: These are the critical angles for inferring what will happen when two balls collide.

Let us explain the table in terms of what happens in Case 1. The two balls come toward each other each with a component of velocity in the x direction, but none in the y direction. Therefore, the entry velocity for m_1 has $-a$ for its x-component (minus because it is headed in a negative direction) and 0 for its y-component. Similarly, the entry velocity of m_2 has b for its x-component and 0 for its y-component. If the two particles meet head on (i.e., their impact angle=0 degrees), they exchange momentum. Thus, m_1 goes off to the right with velocity b and m_2 goes off to the left with velocity $-a$.

If the two particles meet at a 45 degree angle, then there is a range of possible outcomes. The two boundary conditions ($|b|=|a|$ and $b=0$) for that range of outcomes are shown: b cannot be greater than a because m_1 is arbitrarily defined as the faster moving ball. When $b=a$, after the balls collide m_1 goes straight up with a velocity of a and m_2 goes straight down with a velocity of a . When $b=0$ (i.e., m_2 is stationary), m_1 goes off at a 135 degree angle with its x and y-components of velocity each $1/2 a$ and m_2 goes off at 225 degrees with the same components of velocity. As b increases from 0 to a , the angle at which m_1 goes off moves from 135 degrees to 90 degrees, its

x-component of velocity in absolute terms decreases from $1/2 a$ to 0, and its y-component of velocity increases from $1/2 a$ to a . Similarly, for m_2 the exit angle changes from 270 degrees to 225 degrees as b increases, the x-component of velocity decreases from $1/2 a$ to 0 and the y-component of velocity increases from $1/2 a$ to a .

Naive models of billiard ball interaction are not this sophisticated, but they can be represented in similar terms. For example, a naive model might not assume momentum transfer in a head-on collision. A naive person might assume rather that the input speed for each particle is the same as its output speed in such a collision. Furthermore, naive people may not know what happens in some of the cases, or have only approximate bounds on what will happen. A more qualitative representation related to this one would give the output angles and whether the velocity is zero or not. This may come closer to the way people intuit particle interaction. But we think the parsing into cases, the combining of cases, and interpolating values corresponds to the way people think about particle interaction at a molecular level.

Conclusion

When we looked in detail at people's reasoning about evaporation, we found that they reason at three distinct levels: (a) in terms of macroscopic variables like temperature, density, or volume, (b) in terms of aggregates of particles that behave in a similar way, and (c) in terms of individual particles and their interactions. We have tried to show how people's models at these different levels can be represented in terms of the Qualitative Process Theory of Forbus (1982) and the Incremental Qualitative Analysis of deKleer (deKleer, 1977). In particular we would argue that in principle each step in a macroscopic functional model is supported by one or more aggregate models, and in turn each aggregate model is supported by one or more molecular models.

Our study perhaps raises more questions than it answers. One important question is how many different kinds of models people have at each level of analysis. Our guess is that there are many such models, since they reflect knowledge that subjects learn throughout their lifetimes. The commonality between subjects will be in the levels at which such models are constructed and the internal language in which they are constructed.

References

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