Cognitive Processing Strategies for Complex Addition

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Abstract

Simple and complex addition problems were presented for true/false verification to 22 subjects across two times of measurement to test the general model for simple and complex addition proposed by Widaman, Cormier, & Geary (1985). Models fit to average RT data revealed that subjects were processing complex problems columnwise, beginning with the units column. Column sums seemed to be obtained through an incrementing process, and subjects exited problems as soon as a column error was encountered. Group level models were the same across complex problem types and for both times of measurement. However, individual level analyses suggested that nearly half of the subjects used a different processing strategy to obtain column sums for the second time of measurement. Results support the multi-staged model proposed by Widaman et al. (1985), but individual level results suggest that information processing models developed from group data may not represent the processing strategies used by all subjects, or the same subjects at different times.

Information processing models for cognitive addition in adults include counting based, direct memory access, and memory network models (Parkman & Groen, 1971; Ashcraft, 1982). Groen & Parkman (1972) found that the true sum of simple addition problems, and the smaller addend (MIN) of these problems predicted nearly equal amounts of reaction time (RT) variance. Groen & Parkman (1972) interpreted these results as suggesting that all simple addition facts were retrieved directly form memory, but a counting based (MIN) procedure was used with memory retrieval failure.

Ashcraft & Battaglia (1978) found that the true-sum-squared was the best predictor of simple addition RT. The authors concluded that there was a network representation for simple addition that resembled a square, printed addition table with entry nodes for the digits 0-9 on two adjacent sides. The point of intersection of the two entry nodes represented the correct sum. Entry nodes were assumed to be unevenly spaced; the network was "stretched" in the region of larger sums. Thus, larger sums resulted in longer vector distances and therefore longer RTs.

The network approach to cognitive arithmetic has successfully predicted performance in adults (Ashcraft & Stayzk, 1981), children (Ashcraft & Fierman, 1982), and for production tasks (Ashcraft, Fierman, & Battaglia, 1984). However, this model has been developed from simple addition and multiplication tasks (Stayzk, Ashcraft, & Hamann, 1982) and has not been generalized beyond complex addition tasks for sums greater than 30 (Ashcraft & Stazyk, 1981).

Widaman, Cormier, & Geary (1985) have proposed a general model for cognitive addition. The proposed model (for verification tasks) is multi-staged, and begins with encoding the type of problem, and then encoding the addends (columnwise) beginning with the units place. The units addends are summed either through a network of stored facts or procedurally. If there are more addends in the units column, the process recycles, and an additional addend is encoded and summed onto the sum of the first two addends. This process continues until all digits in the units column are summed. The obtained unit sum is compared with the stated sum, and a decision as to correctness is made. If the sum is incorrect the problem is terminated and a "no" response is given. If the units sum is correct, and there are more columns to be added, the process "loops back." The digits for the next column are encoded, then summed, etc. The looping is continued until all columns are completed, or until one column sum is incorrect, at which point the problem is self-terminated.

Widaman et al. (1985) verified this model for simple and complex problems, and found that complex (2-column) problems were done columnwise using a memory network to obtain column sums. Further, a carry operation was highly significant, and subjects tended to exit the problem as soon as an error was encountered. However, the most complex problems used by Widaman et al. (1985) contained 2 double-digit addends or 3 single-digit addends. The present study sought to replicate and extend the findings of Widaman et al. (1985) by testing their model with extremely complex problems. Further, to assess the reliability and generalizability of this model, the stability of processing models across two times of measurement and individual differences (Hunt & McLeod, 1978) in processing strategies were also examined.

Method

Thirty undergraduates were presented with 240 addition problems in a one hour session. This set consisted of 30 correct and 30 incorrect randomly selected: 1) two single-digit addends, 2) two double-digit addends, 3) two triple-digit addends, and 4) two four-digit addends. These problems were readministered to 22 of the 30 subjects one week later.

For incorrect problems, column sums differed from correct sums by ± 1, or ± 2. For complex problems the placement of the error was evenly distributed among the individual column sums, and each problem contained only one error. Similarly, a carry operation (column sum 10) was necessary for about one-half of the cases (per column), and approximately one-half of these cases contained an error. The presentation of problems was random except for the following constraints: 1) there were no more than four consecutive presentations of any one problem type, 2) there were no more than four consecutive presentations of correct or incorrect problems, and 3) same sums or addends were never presented consecutively.

Problems were presented for true/false verification in column form on an Apple II equipped with a cognitive testing station. Subjects pressed one response key to indicate if the presented problem was correct and a second key if the presented problem was incorrect. Reaction time and response accuracy were recorded.

Results and Disscussion

Models for addition were fit to average reaction time data using multiple regression techniques. Models fit to RT data included Parkman & Groen's (1971) five counting based models, Ashcraft's (1982) true-sum-squared parameter, and Widaman et al.'s, (1985) reinterpretation of Ashcraft's (1982) model. Specifically, Widaman et al., (1985) argued that Ashcraft's memory network would be better represented by the sum of the digits squared (SSQ), rather than the square of the summed digits, because each digit squared would represent the seperate vectors on the search through the addition table. For complex problems (2-4 column), models were fit according to the stages outlined by Widaman et al. (1985). Specifically, models were fit in columnwise fashion, with a carry parameter specified if the preceding column sum exceeded 9. A self-terminating parameter was also tested. The self-terminating parameter would represent an executive process (Sternberg, 1982) process, whereby subjects would "exit" the problem as soon as an incorrect column sum was encountered. Finally, the above (e.g. MIN) simple addition parameters were fit for each column sum.

The error rate was 5.8 percent and 5.6 percent for the first (T1) and the second (T2) time of measurement, respectively. Less than one percent of the RT data were deleted as outliers (exceeding 2.58 S.D. of individual RT means).

Parkman & Groen's (1971) MIN (smallest digit) model provided the best fit for simple addition RTs. The MIN model with the truth parameter denoting an intercept difference between true and false problems provided the best fit for both T1 and T2. Consistent with previous findings (Groen & Parkman, 1972; Ashcraft, 1982; Widaman, et al., 1984), false problems were processed more slowly than were true problems. Across sessions parameter estimates decreased, suggesting that subjects were becoming more efficient as a function of practice. Overall, these results suggest that subjects used an incremenenting model for simple addition problems.

Results revealed that similar columnwise parameters provided the best fit to the RT data for all (2-4 column) complex problems, across sessions. The specific column sums were best predicted by the sum of the column (unitsum, tensum, etc.), followed by column MIN, and the Widamam, et al. (1985: sum of digits squared: SSQ) parameter, respectively. Parameter estimates, again, decreased across times of measurement, suggesting that subjects were becoming more efficient. The column sum parameter suggests that subjects were using some type of incrementing process to obtain column sums, however the speed of the incrementation makes it unlikely that it was language based (Landauer, 1962).

The carry/self-terminating (carryst, car10st, carhunst) parameters represented a carry operation if the preceeding column sum was correct and exceeded 9 otherwise the problem was self-terminated and a response "no" executed. The carry parameter by itself was highly significant, and the addition of the self-terminating portion increased the explained RT variance an average of 2.8 percent for T1, and 4.1 percent for T2. Accordingly, subjects were exiting complex problems as soon as an error was encountered, and were using this executive process more efficiently during the second time of measurement.

Individual RTs were fit to the three models best representing the group level data. For simple addition, the MIN, SUM, and true-sum-squared models, along with the truth parameter, were used. For complex problems, columnwise SUM, MIN, and SSQ (sum-digits-squared), with the carry/self-terminating and truth parameters, were used.

For simple addition (1- column) problems the group data suggested the MIN was the processing strategy used. Indeed, the majority (13, and 12, for T1 and T2, respectively) of individual RT data were best fit by the MIN parameter. However, Ashcraft's (1982) true-sum-squared provided the best representation of RT data for 9 and 7 (of 22) subjects for T1, and T2, respectively. In all, 11 subjects maintained the same strategy for T1 and T2. Five of the remaining subjects switched to the more mature true-sum-squared strategy for T2. The remaining subjects switched from a memory network strategy to an incrementing (SUM or MIN: Groen & Parkman, 1972) strategy. For simple addition the processing strategies appeared stable for about 50 percent of the subjects. The remaining subjects appeared to use a different strategy for the second session.

For complex problems (2-4 column), the columnwise SUM with carry/self-terminating and truth parameters provided the best fit for the group data. For 2-column problems there were equal numbers of individuals using the columnwise SUM and MIN proceudres for both T1 and T2 (8 and 9 persons, respectively), with about half as many subjects using the SSQ algorithm. In all, 12 of 22 subjects used the same procedure across sessions, but there was no clear pattern for those subjects switching procedures.

Strategies for 3-column problems showed the greatest stability with 15 of 22 subjects showing no change across times of measurement. However, unlike 2-column problems, subjects seem to use the columnwise SUM (12 and 13 persons) or SSQ (8 and 8 persons) strategy for T1 and T2, with only a few subjects using the MIN strategy. For T2, equal numbers of subjects switched to the SUM or SSQ strategy. Finally, the most complex problems showed the least stability across sessions, with slightly less than one-half of the subjects utilizing the same strategy for T1 and T2. The columnwise MIN strategy provided the best fit for 11 subjects and the columnwise SUM for 9 subjects at T1. There was a clear shift to the columnwise SUM strategy for T2, with this strategy being favored over MIN for T2 for most subjects (14 and 4 persons, respectively).

Individual level results suggest that about 50 percent of the subjects used the same columnwise processing strategies across T1 and T2. Patterns of change suggest that about one-half of those changing use a more mature algorithm (e.g. counting to memory network) for T2. Overall, these results suggest that algorithms estimated for group data represented individual level data fairly well, but did not reflect algorithms used for almost half of the subjects, and did not reflect individual changes in processing stategies across sessions.

In conclusion, both group level and individual level data support the model proposed by Widaman et al. (1985). For complex addition subjects used a columnwise (e.g. Poltrock & Schwartz, 1984) processing strategy beginning with the units column. The proposed executive (Sternberg, 1982), self-terminating strategy was supported. Subjects exited incorrect problems as soon as a column error was encountered and were becoming more efficient in the use of this metacognitive strategy across sessions.

The finding that the MIN algorithm best represented simple addition performance does not support Ashcraft's (1982) memory network model. The use of an incrementing stategy (Groen & Parkman, 1972) may be due to the fact that simple problems were presented randomly with more complex problems. Thus, the use of an incrementing strategy for complex problems may have led to the use of an incrementing strategy for simple problems.

For complex problems, columnwise algorithms used suggested that subjects were using an incrementing stategy to obtain column sums; however, because of the speed of the incrementation it is unlikely that it is language based (Landauer, 1962). Alternately, subjects may have been moving slowly through a memory network, because of the complexity of the multi-column problems (Polltrock & Schwartz, 1984). However, these results suggest that subjects were using a procedural, rather than a memory based algorithm to complete complex problems.

Overall, the Widaman et al. (1985) model seems to reflect subjects processing stategies accurately for complex addition. The results from the individual level analyses suggest that subjects were using similar processes across sessions, but computational algorithms seemed to vary.

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