BELIEF MAINTENANCE WITH UNCERTAINTY

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ABSTRACT

A framework for representing and reasoning with uncertain information is described. A network knowledge structure is used which makes the reasons for believing or not believing a proposition explicit. These reasons, or endorsements, are quantified by a measure of belief and certainty. Heuristics are integrated with the knowledge structure to collect, and evaluate the endorsements.

1. INTRODUCTION

The research reported in this paper pursues the problem of developing representational and inference mechanisms which are capable of dealing with incomplete, inaccurate, and uncertain information. The direction taken is based on the assumption that methods which deal effectively with uncertainty must play an integral role in both models of human reasoning, and flexible computational reasoning systems.

Most formal reasoning systems combine both the extent of belief and certainty of belief into a single truth value, whether binary or multi-valued (McCarthy 1980; McDermott & Doyle 1980; Reiter 1978a; Zadeh 1983). In many cases, this compression is justified, but consider the proposition RICK LIKES MATH. The extent of belief in this proposition may be high whether it is quite certain (Rick has taken, and enjoyed a wide range of math courses) or quite uncertain (Rick has only taken a single math course).

Recently Cohen (1983) has formulated a model of reasoning which maintains that reasons for believing or disbelieving propositions can be collected, providing a more comprehensive description of belief. Our approach is to employ a knowledge structure such that these reasons, or endorsements, are made explicit. The endorsements for propositions can be quantified by a measure of belief and certainty. In addition, a network of endorsements among propositions may be used to: (1) determine how supportive a body of evidence for a particular hypothesis is and (2) represent evidential relationships such as conflicts between decisions (Craddock 1986).

The algorithms which compute the belief and certainty of a proposition may be formulated to operate uniformly on all supporting knowledge, or the algorithms may be subject to heuristics which emphasize the importance of selected portions of the supporting knowledge. In the development of heuristic methods we have been guided by the approach taken by Kahneman and Tversky (1982a, b) Their model indicates that humans employ a set of basic heuristics which aid in making decisions in conditions of uncertainty. These heuristics enable humans to constrain problem domains such that the uncertainty becomes manageable but still useful. In addition humans can also employ heuristics to determine complex evidential relationships between different sources of evidence.

2. BELIEF AND CERTAINTY

The development of our reasoning system requires the definition of a network containing nodes which are propositions with associated belief and certainties. Interconnections in the network represent the support that one proposition offers another. First, let P be a set of cognitive units $P = \{n_1 \dots n_m\}$. Each of these cognitive units may represent a proposition such as I LIKE MATH or relationships among objects or concepts. Each n_i has associated with it a belief strength b_i, which is a measure of the extent to which the cognitive unit is believable. The believability of n_i is a measure of the strength of the supporting evidence for n_i, not a measure of its incidence of occurrence or its possibility of occurrence. The belief strength b_i can be defined as follows: A cognitive unit n_i is believable if there is an endorsement for n_i or if the endorsements supporting n_i are stronger than those against it. As $-1 \leq b_i \leq 1$, we may view the cognitive units as statements in fuzzy propositional logic in which a belief of -1 indicates n_i is false and a belief of +1 indicates n_i is true.

In addition, each n_i has associated with it a certainty, c_i of the assignment of the value b_i, where $0 \leq c_i \leq 1$. The certainty of a belief value is defined as a measure of the reliability or trustworthiness of the evidence which was used to calculate a particular belief (Hamburger, 1985). Thus each cognitive unit represents two distinct aspects of the *Rational* $R_i = (b_i, c_i)$ of n_i.

Any cognitive unit may endorse another. For example, a cognitive unit representing I LIKE COMPUTING may be endorsed by I LIKE ALGEBRA, I LIKE PHYSICS, and the negative endorsement I HAVE TROUBLE WITH TECHNICAL MANUALS. Each endorsement has associated with it a numeric value corresponding to the extent of the support between the units. If n_i endorses n_j then the support node s_{ij} is the *support for the endorsement* where $-1 \leq s_{ij} \leq 1$. If $-1 \leq s_{ij} < 0$ then the endorsement n_i for n_j is said to be inhibitory and if $0 \leq s_{ij} \leq 1$ then it is said to be excitatory. A diagrammatic version is supplied in figure 1. The support nodes for the endorsement, s_{ij} may be endorsed by other cognitive units. For example: I LIKE PSYCHOLOGY may endorse I LIKE COMPUTING, but the support for the endorsement may be contingent on COMPUTATION MAY MODEL COGNITION. (see figure 2). If the belief in COMPUTATION MAY MODEL COGNITION is false then the support for the endorsement will decrease.

If we consider $P = \{n_1 \dots n_m\}$ as the set of propositional nodes of the network, then we can define $S' = \{s_{ij} \mid n_i, n_j \in P, s_{ij} \neq 0\}$ as a subset of support nodes such that n_i endorses n_j with support s_{ij}, and $T = \{t_{s_{ij}} \mid s_{ij} \in S', n_k \in P\}$ as the other subset of support nodes such that n_k endorses s_{ij} with support $t_{s_{ij}}$. We can then define the network $N = <P, S>$ where $S = S' \cup T$ is a finit set of support nodes representing the arcs and P is a finit set of propositions.

3. COMPUTING BELIEF AND CERTAINTY VALUES

We wish to develop ways of computing the values of *Rational* R_j for a proposition n_j on the basis of the endorsements available for that node. The first important observation is that the strength of endorsement between two nodes n_i and n_j is not only dependent on s_{ij}. This strength must be computed with consideration of b_i. We can compute this endorsement strength as $e_{ij} = b_i s_{ij}$. The second observation is that the belief strength of a node may be computed from the beliefs and certainties of its endorsements. Nodes which do not have endorsements, and in fact any node in the system, can be provided with an *Intuition* represented as $I_i = (b'_i, c'_i)$. This structure appears much the same as the Rational structure except that its values are never computed, but they remain available to take part in the computation of other beliefs. Intuitive values correspond to the usual direct assignment of belief and certainty to a proposition from which other beliefs and certainties are to be determined.

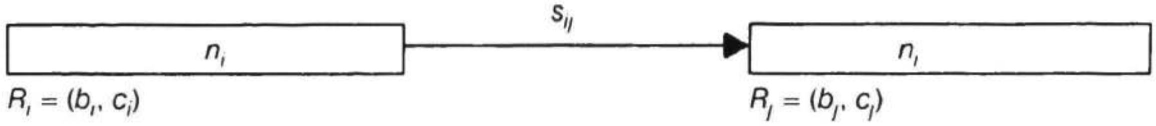

Figure 1. : Nodes of a network representing cognitive units with belief, certainty, and endorsement structure.

The computation of a belief value for a node is largely dependent on the manner in which its endorsements interact. For example, the final belief in a node is a function of the summary of the evidence and arguments. In such instances, belief depends on: (1) measuring the varying contributions of the individual endorsements and (2) measuring the effects of interaction among the different endorsements. This interaction among a set of endorsements $\{n_1 \ldots n_m\}$ for n_j depends on the relative importance of each endorsement defined as

$$r_{ij} = \frac{s_{ij}}{\sum_{k=1}^{m} |s_{kj}|} \quad [1]$$

and the relative certainty defined as

$$rc_i = \frac{c_i}{c^*} \quad [2]$$

where c^* is max $\{c_i \ldots c_m\}$. In this manner we can define a measure of belief using a formula such as

$$b_j = \sum_{k=1}^{m} rc_k \times r_{kj} \times b_k \quad [3]$$

The endorsements with the greatest relative importance and greatest relative certainty have the most impact on the final belief. The interaction of endorsements is analogous to a tug-of-war where the different endorsements tug and pull against one another until an equilibrium is reached.

Since the support of an endorsement can be endorsed we can also deal with situations in which only one of several endorsements is adequate to allow a cognitive unit to be believed. For example, the statement I CAN TAKE A GRADUATE COURSE IN COMPUTING is endorsed by I AM A COMPUTING GRADUATE, or I AM AN ELECTRICAL ENGINEERING GRADUATE, or I AM A COMPUTING UNDERGRADUATE WITH GOOD MARKS The three endorsements are mutually exclusive; only one of them need be true. In this example a cognitive unit n_j endorsed by $\{n_1 \ldots n_m\}$ will be given an endorsement of $b^* s_{kj}$ where $b^* = \max\ \{b_1 \ldots b_m\}$. The node n_k will inhibit all the other endorsements by giving an inhibitory endorsement to their respective supports for the endorsement. A more complete description is available in Craddock (1986).

The certainty of a belief is calculated as a function of the agreement of the individual endorsement strengths with the final belief value calculated from them. Thus, belief must be calculated before certainty. The importance of the agreement is once again measured as a function of the relative support of the individual endorsements and their relative certainties. As these values increase so does the uncertainty associated with disagreement. Where $\{n_1 \ldots n_m\}$ are the endorsing nodes for n_j, this effect can be modelled in formulas such as:

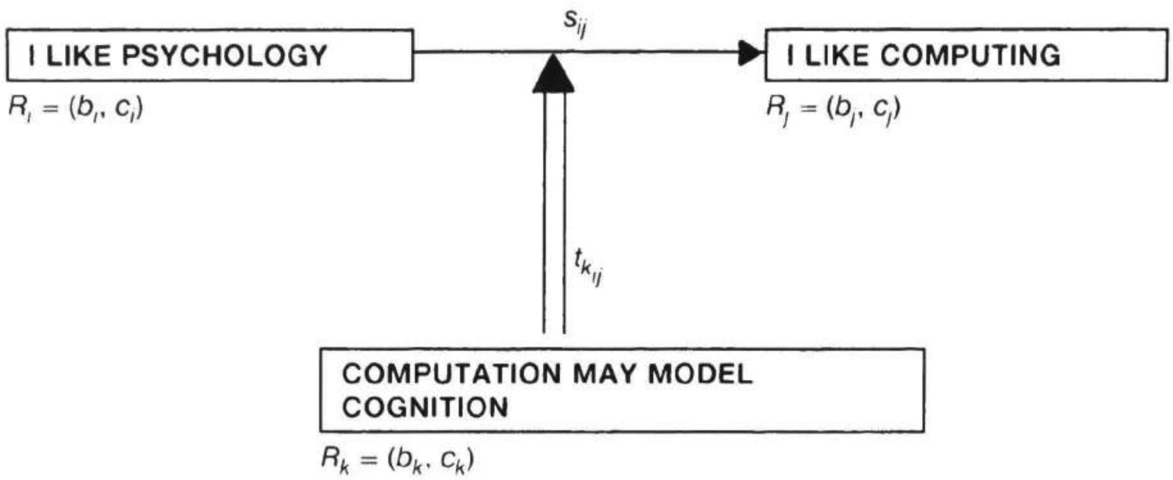

Figure 2. : An example of an endorsement which influences the strength of endorsement between two other nodes.

$$c_j = 1 - \left[\sum_{k \neq i,\, k=0}^{I} |(b_k - b_i)| \times r_{ki} \times rc_k \right] \qquad [4]$$

4. CONTRADICTIONS

Rational contradictions among endorsements are defined as follows: If A is compelling evidence against n_i but B is equally compelling evidence for n_i then the endorsements for n_i are inconsistent. In addition to a rational contraction an *intuitive contradiction* can also be defined: If the intuitive belief, b'_i is not equal to the rational belief, b_i then the two beliefs are inconsistent. If we define a threshold of intuitive contradiction, T_I then (1) if $|b'_i - b_i| > T_I$ then assign b_i to I'_i and recalculate the rational beliefs of all the nodes endorsed by n_i such that $s_{ij} \neq 0$ or (2) if $|I_i - R_i| < T_I$ then assume a state of equilibrium has been reached and do not recalculate the rational beliefs of any of the nodes endorsed by n_i.

Intuitive contradictions are useful for recognizing changes in belief through a knowledge base when endorsements are added and removed. In addition they can be used to control cycles which may force more global interpretations on input propositions. When cycles exist within a network

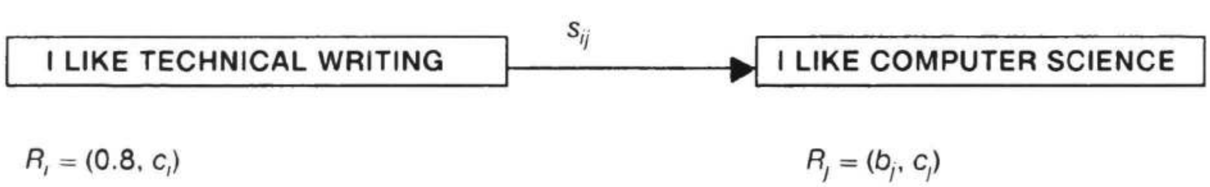

Figure 3. : An example of an endorsement which may have net positive or net negative support.

$N = <P, S>$, belief and certainty values will only be calculated for nodes in a partial network $N' = P', S'$, where $P' \subseteq P$, and $S' \subseteq S \cap (P X P')$, where there exists a node $n_i \in P - P'$ such that there is an elementary path between n_i to P' and $|I_i - R_i| > T_i$.

5. CONCLUSIONS

The model discussed in this paper seeks to develop representational and inference mechanisms capable of dealing with incomplete, inaccurate, and uncertain information. To this end, a connectionist model is proposed and heuristics are developed to collect and evaluate the endorsements for propositions in the network of beliefs. At the same time it is intended that the model represent at least some of the processes used in human reasoning.

As the model is intended to represent belief maintenance with uncertainty it differs from existing connectionist models (Anderson 1982; Rumelhart and McClelland 1982; McClelland and Rumelhart 1983) in several important respects. First, the uncertainty of a proposition is represented numerically, as the values of R_i, and non-numerically, as the structure of endorsements. Second, once the endorsements have been collected they are subject to reasoning and natural heuristics to compute numeric values as depicted in formulae [1] to [4].

Kahneman and Tversky (1982a, b) have shown that their heuristics; availability, representativeness, and adjustment and anchoring, can help describe human decision making under conditions of uncertainty. Once these heuristics are recognized as part of human reasoning it no longer appears illogical in the sense of being erratic, but rather more pragmatic and difficult to specify in terms of the logic inference mechanisms of traditional logic. Kahneman and Tversky (ibid.) provide numerous examples in which subjects reach decisions which run counter to those reached by mathematical theories. While the heuristics proposed in this paper are by no means as exhaustive nor the formulae necessarily optimal, they do illustrate how heuristics might be incorporated into the decision making model in a straight forward fashion (Craddock 1986).

In contrast, most connectionist models ignore, or do not explicitly deal with the non-numeric representation of uncertainty, depending on numeric values alone which provide no evidence as to how they were calculated, what they actually represent, or how reliable they are. Of major issue is the belief that numerical values, blindly tallied, are an inadequate representation of reasoning. Symbolic structures of support are necessary to specify how and why numeric beliefs are calculated. The availability of an endorsement structure allows the model to not only provide numerical information but also a description of its own reasoning process. The advantages of having a model whose reasoning can be readily understood are numerous and imperative if the justifications for a decision are to be made clear.

A further difference is that the proposed model can represent varying degrees of interaction between sources of evidence while models such as MYCIN (Shortcliffe, 1975) must make the assumption that all evidence is conditionally independent and that hypotheses are mutually exclusive. For example, the dynamic strengths of endorsement (see figure 2) allow us to represent evidence which is disjunctive, that is, strong belief may be propagated on the basis of only one of many supports.

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