The Role of Intermediate Abstractions in Understanding Science and Mathematics

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Abstract

Acquiring powerful abstractions -- i.e., representations that enable one to reason about key aspects of a domain in an economical and generic form -- should be a primary goal of learning. The most effective means for achieving this goal is not, we argue, the "top-down" approach of traditional curricula where students are first presented with an abstraction, such as **F=ma**, and then with examples of how it applies in a variety of contexts. Nor do we advocate the "bottom-up" approach proposed by situated cognition theorists. Instead, we argue for a "middle-out" approach where students are introduced to new domains via intermediate abstractions in the form of mechanistic causal models. These models serve as "conceptual eyeglasses" that unpack causal mechanisms implicit in abstractions such as **F=ma**. They are readily mappable to a variety of real-world contexts since their objects and operators are generic and causal. Intermediate abstractions thus give meaning to higher-order abstractions as well as to real-world situations, provide a link between the two, and a route to understanding.

Proponents of the theory of "situated cognition" argue that abstractions are an inherently impoverished and inert knowledge form, and that to be meaningful, knowledge must be contextualized in real-world situations. The following quotation exemplifies this position: "A situated theory of knowledge challenges the widely held belief that the abstraction of knowledge from situations is the key to transferability. An examination of the role of situations in structuring knowledge indicates that abstraction and explication provide an inherently impoverished and often misleading view of knowledge" (Brown, Collins, & Duguid, 1989).

In this paper we support the contrary position: Real-world situations are inherently complex and confusing -- The key to powerful cognition lies in acquiring knowledge in an abstract form, and in understanding the form and utility of abstraction. This position is consistent with the generally accepted view of mathematical and scientific knowledge, and this perspective is implicitly embedded within most existing math, science, and engineering curricula. The failure of many of these curricula, we argue, is not due to this belief in the importance of abstraction, rather, it is due to the way in which abstractions are introduced to students. In this paper, we will argue for the need to start curricula with "intermediate abstractions" in the form of models that (1) map readily to real-world contexts, (2) yet possess many of the properties of higher-order abstractions, and (3) unpack the meaning of higher-order abstractions, thereby providing a bridge to acquiring and using such abstractions. These intermediate abstractions serve as "conceptual eyeglasses" that enable students to make sense out of physical domains and mathematical formalisms.

The Meaning of Abstraction

If one looks up the word "abstraction" in the dictionary (The Second College Edition of the American Heritage Dictionary), one is presented with a variety of nuances for the term:

1. "Thought without reference to a specified instance". Abstract representations incorporate generic objects and operators. For instance, they reason about forces being applied to objects, rather than about kicks being applied to balls. Using such representations, one can reason about generic, abstract cases rather than about specific objects and situations. In this way, students can evolve knowledge in a form that can be mapped onto many different situations. The claim (supported by the research of Bassok & Holyoak, in press) is that such generic representations facilitate the transfer of knowledge to multiple contexts.

2. "The concentrated essence of a larger whole" One interpretation of this nuance is that abstract representations typically attempt to model only certain aspects of a domain. This is particularly true of abstractions, like F=ma and V=IR, employed in scientific disciplines. They enable one to achieve an economy of thought by simplifying the situation and thereby reducing the cognitive load. Yet, if one abstracts the key properties of the domain, powerful inferences can be made from such economical simplifications. Another interpretation of this nuance has to do with the form of the representation itself. For instance, V=IR is a more compact form for expressing the relationship between voltage, current, and resistance than is a dynamic, causal model of electrical circuit behavior. The quantitative law, V=IR, takes less space to represent than the causal model, and can be employed in algebraic, constraint-based reasoning. This form of reasoning, enabled by the form in which the law is represented, allows the efficient generation of certain inferences about the properties of any given circuit (see White & Frederiksen, 1987). Thus the phrase "concentrated essence" can refer to the economy of both the content and the form of what is being represented.

3. "Meaningless or difficult to understand". Many students have had the experience of being presented with laws in physics class and of finding them difficult to comprehend. Yet, one of the primary goals of learning is to acquire such powerful abstractions that apply across a range of contexts (i) to generate explanations and (ii) to solve many different types of problems. In this paper, we argue that these abstractions need not be meaningless or difficult to understand, if they are introduced via "intermediate abstractions"

Examples of Intermediate Abstractions

Proponents of the theory of situated cognition argue for a "bottom-up" approach to education: Knowledge structures that are widely applicable (which they term generalizations[1]) should be gradually induced from exposure to many real-world instances (Brown, Collins, & Duguid, 1989). This is a viewpoint shared by many case-based reasoning proponents (e.g., Spiro, Coulson, Feltovich, & Anderson, 1989). Traditional curricula typically embody a more "top-down" approach to education: Students are first presented with general principles in the form of abstract formalisms, and are then given examples of how the principles apply in real-world situations. In contrast to both of these viewpoints, we argue for a "middle-out" approach to education. Students should first be presented with intermediate

[1]Generalizations, they argue (Brown, Collins, & Duguid, 1989), differ from abstractions in that generalizations are analogous to fables, whereas, abstractions are analogous to the moral of the fable.

abstractions, in the form of dynamic causal models, that share many properties of both the real-world and of higher-order abstractions. To foster their acquisition, these models can be made interactive and articulate by embedding them in a computer simulation. When internalized by students in the form of a mental model, they enable students to (i) interpret real-world situations and (ii) give meaning to higher-order abstractions like F=ma and V=IR. They provide a more efficient and effective route, we will argue, to understanding and utilizing powerful abstractions. In this section we present three examples of intermediate abstractions: one from mathematics, one from engineering, and one from physics.

Example #1: Understanding place-value notation and arithmetic: The first illustration presents an abstraction that is intermediate between concrete situations, such as working with Dienes blocks, and higher-order abstractions, such as working with arabic numbers and formal arithmetic. It incorporates a "bin model" for representing place value notation and the arithmetic operations of addition and subtraction. When embodied in a computer system, this model displays bins on the screen: a ones bin, a tens bin, a hundreds bin, and a thousands bin (see Figure 1). The students can give commands, such as ADD 386 or SUBTRACT 79, and the program will cause the appropriate numbers of icons to be added or subtracted from the appropriate bins in a manner analogous to the standard procedures for addition and subtraction. When a bin overflows, the process of carrying is animated. Similarly, when the student is subtracting and there are not enough icons in the appropriate bin, the model visually portrays the process of borrowing. While doing so, the computer explains what it is doing and why to the student using computer generated speech. When students worked with and began to assimilate this model, we found that their understanding of place notation and its relationship to the processes of addition and subtraction improved (Feurzeig & White, 1983). Once assimilated, the model was used as a foundation for introducing the standard arithmetic procedures that operate on arabic numbers, and for modelling alternative real-world situations. (See Resnick & Omanson (1987) for a discussion of Dienes blocks as a bridging abstraction for understanding arithmetic.)

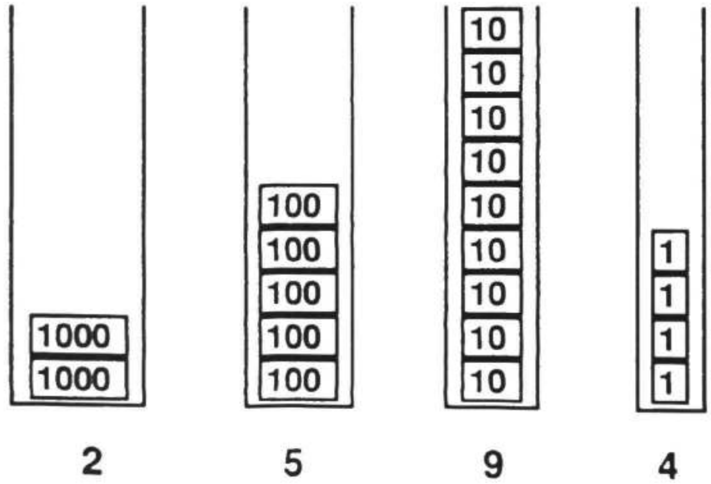

Figure 1. The bin model.

Example #2: Understanding how electrical circuits work: The next example is a generic model of a transport mechanism that represents the aggregate behavior of particles (see Frederiksen & White, in press; White & Frederiksen, in press). It is more abstract than a model of the movement of individual particles, and less abstract than models based on steady-state principles such as V=IR. In this model, there are objects that contain "stuff" (where stuff can represent, for instance, collections of electrically charged particles). If two such objects are connected together, and if they contain different amounts of stuff, then stuff will flow from one to the other (see Figure 2). In each time increment, the flow of stuff between adjacent objects depends on the "stuff gradient" (i.e., the difference in the amount of stuff). The explanation for why stuff flows is reductionistic and is based upon the motion of particles and their interactions (e.g., attraction & repulsion). The aggregate transport model provides an introductory representation for many physical processes such as (1) the distribution of electrical charge, (2) the diffusion of gases, and (3) the flow of heat. If one creates a model of a physical system using this transport process, such as a model of an electrical circuit, one can see how the steady-state laws (such as Ohm's law and Kirchhoff's laws) are emergent properties of a system that incorporates this process (see Figure 3). The model thus links the behavior of individual particles with the steady-state system equations. It is interesting to conjecture that there may be a few such physical-process models that can potentially play a crucial role in understanding physical systems. These process models may be key because (1) they facilitate the understanding of a wide range of systems, (2) they are relatively easy to interpret since they can be explained in terms of concepts and processes, like attraction and repulsion, derived from common experiences with the every-day world, and (3) they provide a mechanistic origin for the steady-state laws that are so useful in predicting the behavior of physical systems.

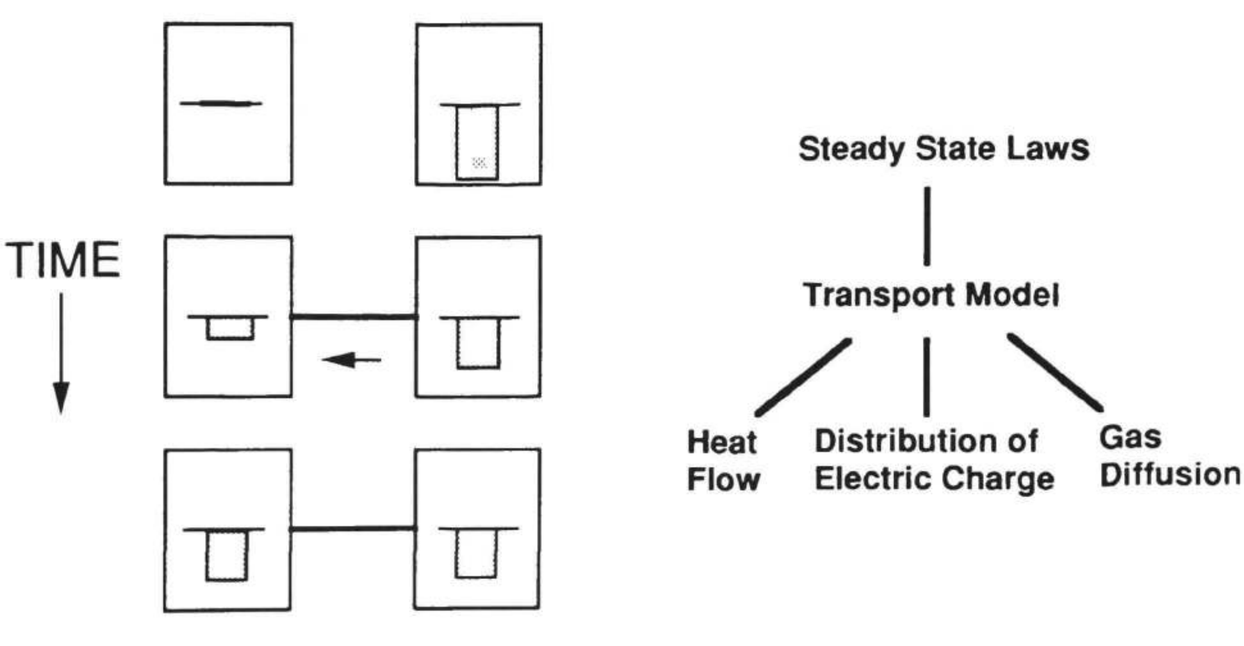

Figure 2. The transport model.

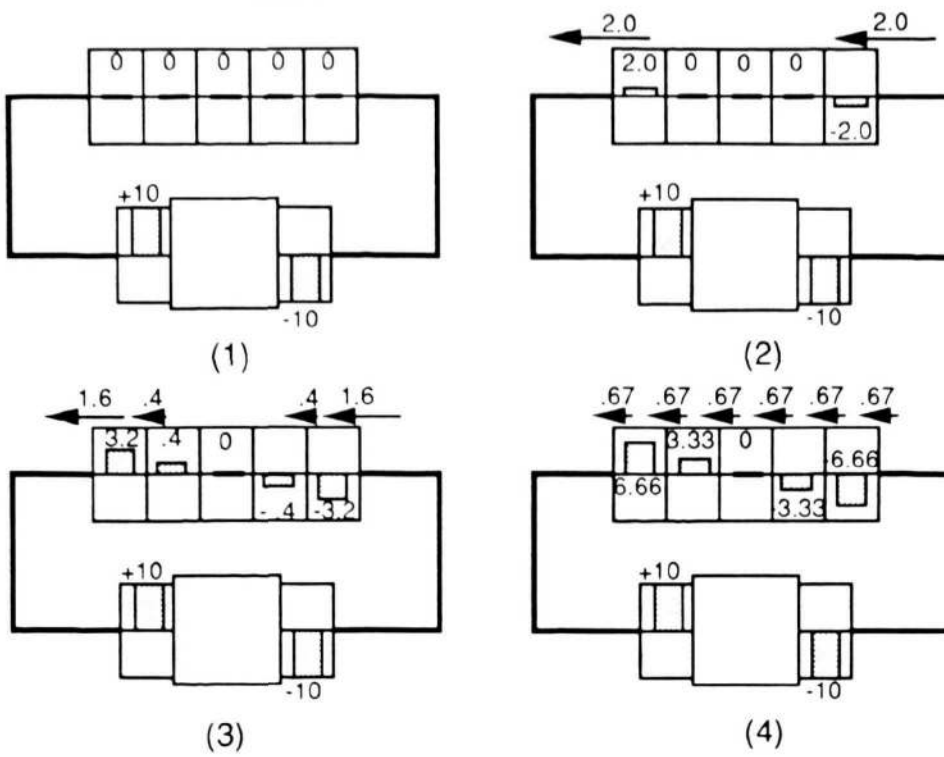

Figure 3. An electrical circuit with a resistor and a battery.

At steady state, current flows are equal and charge densities form a stable gradient.

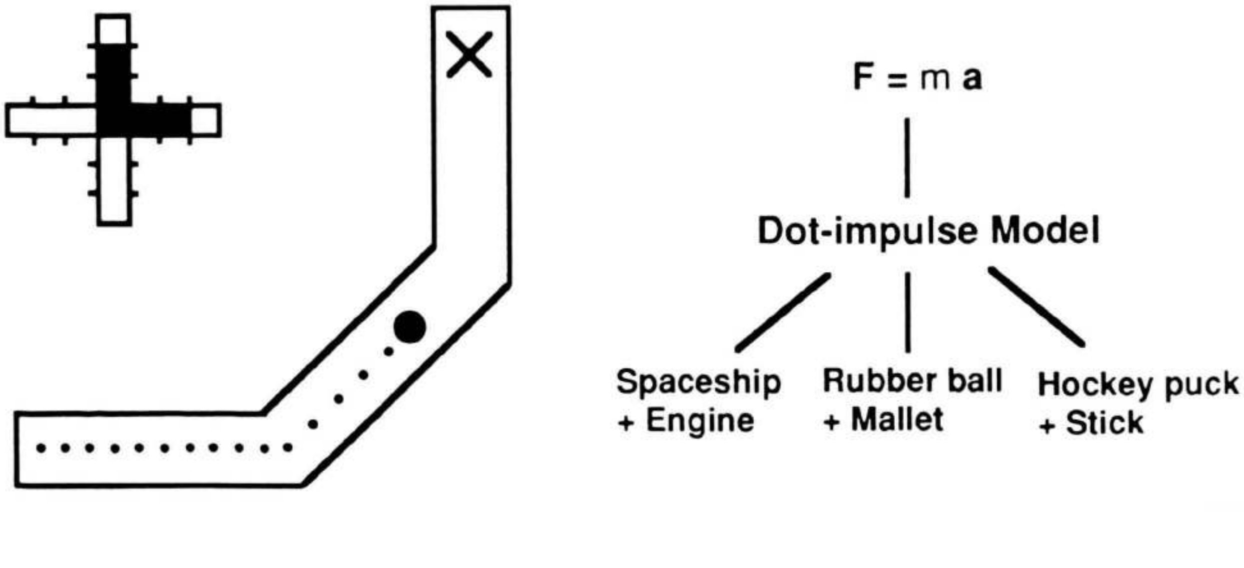

Figure 4. The dot-impulse model.

In the task displayed above, the student is controlling the motion of the large dot in order to make it navigate the track and stop on the target X. The small dots indicate the history of the large dot's motion, and the cross at the upper left indicates the large dot's orthogonal velocity components.

Example #3: Understanding Newtonian mechanics: This final example describes an intermediate abstraction that has proven useful in helping middle school students understand force and motion (see White & Horwitz, 1988; White; 1988). When presented with the computer embodiment of this causal model, students engage in tasks such as attempting to control the motion of an object by applying impulses to it via a joystick (see Figure 4). The students are not asked to think of this object as a spaceship with a rocket engine, nor as a billiard ball controlled by a stick. Instead they are asked to think of it as a generic object (which is simply referred to as the "dot") being controlled by impulses (i.e., forces that act for a short time). The students' task is to determine the physical laws underlying this model. They formulate principles such as "whenever you apply an impulse to the dot, it changes its speed". These qualitative laws form the foundation for understanding more abstract laws such as **F=ma**. Further, the generic quality of the students' laws plays a role in enabling them to transfer and apply the principles underlying this model to real world contexts (see White, 1988).

Properties of Intermediate Abstractions

These three examples of intermediate abstractions share some common properties that we conjecture are crucial to their success in fostering understanding:

1. Useful: They model and generate explanations relevant to key aspects of a domain. For instance, example #1 presents a model of place value notation and its implications for the design of arithmetic procedures. Example #2, when instantiated in the context of electrical circuits, presents a model for the concept of voltage drop, and for how voltages and currents change when the conductivity of devices within circuits change. Example #3 presents a model for forces and how they affect the motion of objects. These models thus enable one to predict, envision, and explain the behavior of systems. They can thereby foster the acquisition of difficult domain concepts and processes that are crucial to domain understanding.

2. Transferable: The objects and actions in intermediate abstractions are represented in a decontextualized form -- they embody generic objects and forces. For instance, the tens icon in example #1 is just a generic icon which, in the course of problem solving, can be instantiated to represent anything from ten jelly beans to a ten dollar bill. Similarly, the "stuff" in example #2 can be thought of as generic stuff, and can be instantiated to represent anything from electrical charge to gas molecules. Further, the "dot" in example #3 is just a generic object which can map onto anything from a spaceship to a hockey puck. Presenting generic icons, operators, and processes might play a key role in enabling students to readily map these intermediate abstractions to different real-world situations, and to thereby give higher-order abstractions meaning in many contexts.

3. Meaningful: Intermediate abstractions are meaningful and relatively easy to understand because they build on intuitive notions of causality and mechanism. They parse the behavior of a system into a sequence of discrete causal events, and introduce a sense of mechanism. Further, they are grounded in primitive abstractions (akin to diSessa's (1983) phenomenological primitives). That is, they are constructed from conceptual and process abstractions, like *resistance* and *balancing*, which students derived as children from experiences with the everyday world. For instance, in example #1, adding more objects causes a container to overflow. In example #2, a change in connectivity or amount of stuff causes stuff to change location. And, in example #3, impulses cause changes in objects' velocities. All of these are easy to understand, and link abstractions, like V=IR and F=ma, to mechanistic causal phenomena. Intermediate abstractions thus provide a bridge that gives meaning to higher-order abstractions (such as arabic numbers and arithmetic, steady-state circuit laws, and Newton's laws of motion). They also provide a causal theory for interpreting real-world situations.

Acquiring Intermediate Abstractions

Despite the fact that intermediate abstractions incorporate primitive abstractions, they are, nonetheless, a more abstract and sophisticated knowledge form. They include graphical representations, such as the cross shown in Figure 4 (which represents the orthogonal velocity components of the dot), and principles for operating on those representations to predict system behavior. When reified in a computer microworld, they become formal models whose properties can be discovered and debated.

When students first interact with computer microworlds that embody these intermediate abstractions, they typically induce principles that are overly situation specific. For instance, when exposed to the dot-impulse model of example #3, they formulate rules such as "If the dot is moving to the right at one unit of speed and you want it to stop, apply an impulse to the left". In order to help students to internalize these models in a more useful form, instructional techniques need to be developed that center around interacting with the microworlds. For instance, in research with the dot-impulse model (see White & Horwitz, 1988), we developed techniques that include getting students to evaluate alternative laws proposed for the microworld. The laws vary in correctness, generality, and parsimony in order to foster discussions about the properties of a good scientific law. Such techniques help students to evolve mental models in a generally applicable form. They also foster an awareness about the form of scientific knowledge. Only after students have assimilated the intermediate abstraction are they asked to consider its application to real-world situations: Does it apply in a given context and is it useful? And, only after they have internalized the model do they use it to derive higher-order abstractions. The model thus forms the "conceptual eyeglasses" for interpreting both the real-world and higher-order abstractions.

Conclusions

Intermediate abstractions, we have argued, can enable students to understand physical domains and mathematical formalisms. They provide a medium for exploring the form and utility of abstractions: What are models, how do they evolve, and why are they useful? The challenge for cognitive scientists and instructional designers is (1) to determine the properties that effective intermediate abstractions must possess, (2) to create models that possess these properties, and (3) to devise interesting projects for students that require the acquisition and use of these models. Such projects could include designing electrical circuits, or constructing a theory of force and motion. Thus, intermediate abstractions, when appropriately designed and embodied as interactive articulate microworlds, can provide tools not only for developing understanding, but also for introducing students to the practices of engineering and scientific inquiry.

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Learning From Examples: The Effect of Different Conceptual Roles

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Many studies of category learning have emphasized a single role of the concept that is learned --namely, the concept as a mechanism for classifying objects and discriminating them from members of other categories. Recently, researchers have noted that concepts have many purposes besides classification--prediction, communication, explanation, goal attainment, and so on. This paper presents a study that varied the roles of concepts during a classification learning task. Specifically, one group of subjects (the discrimination group) was given standard instructions to learn about pairs of categories. A second group of subjects (the goal group) was given these instructions but also informed about the functions of the categories. The results of the study suggest that the two groups formed different concepts, even though they saw the same examples of the categories. The concepts of the discrimination group were based on those features in the examples that had predictive value--features with high cue and category validity. In contrast, the concepts of the goal group were based on predictive features and features that were important to the function of the category (called "core" features). Relative to the discrimination group, the goal group placed less emphasis on predictiveness. The results are discussed in terms of their implications for standard classification tasks in psychology and explanation-based and similarity-based approaches in machine learning.

INTRODUCTION

In many category learning tasks, the experimenter presents examples of two or more categories, and subjects learn concepts that allow them to discriminate members of one category from those of others. These tasks provide insight into the general nature of people's concepts (e.g., Reed, 1972; Rosch, Simpson, & Miller, 1976; Medin, Wattenmaker, & Michalski, 1987; Nosofsky, Clark, & Shin, 1989).This paradigm is similar to the similarity-based learning paradigm in machine learning. In similarity-based learning, a program examines a number of examples of different categories and creates generalized descriptions (concepts) of those categories. The descriptions enable the program to identify new category members (see Dietterich & Michalski, 1983; Fisher & Langley, 1985, for overviews of these systems).

There are at least two problems with this approach, however. First, it focuses people and programs on forming concepts that emphasize only one of many roles that concepts can have (i.e., classification or discrimination). The importance of the concept is for accurately classifying category members. However, concepts must represent information about a category other than that used to identify its members. Otherwise, why have concepts? In the real world, people don't form concepts solely to use them to identify objects. Object classification is just one of many roles of a concept. Other roles of concepts include using them to attain goals, construct explanations, make predictions, and so on (Schank, Collins, & Hunter, 1986; Matheus, Rendell, Medin, & Goldstone, 1989). Second, the approach focuses people and programs on forming concepts that are based only on information that is explicit in the training examples (but see Stepp & Michalski, 1986). As a result, it ignores the effect of background knowledge on concept formation. In the real world, people inductively learn about categories by integrating background knowledge with the information provided in the examples. Some of this background knowledge includes people's basic, common sense theories of the world (Murphy & Medin, 1985; Medin & Wattenmaker, 1986; Murphy & Wisniewski, in press; Pazzani & Schulenburg, 1989).