Qualitative Reasoning about the Geometry of Fluid Flow

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Abstract: Understanding the interaction between dynamics and geometry is crucial to capturing commonsense physics. This paper presents a qualitative analysis of the direction of fluid flow. This analysis is dependent on qualitative descriptions of the surface geometry of rigid bodies in contact with the fluid and a pressure change in fluid. The key problem in designing an intelligent system to reason about fluid motion is how to partition the fluid at an appropriate level of representation. The basic idea of our approach is to incrementally generate the qualitatively different parts of fluid. We do this by dynamically analyzing the interaction of geometry and pressure disturbance. Using this technique, we can derive all possible fluid flows.

1 Introduction

Understanding the interaction between dynamics and geometry is crucial to capturing commonsense physics. Without spatial reasoning, dynamics cannot fully explain the physical world. For example, applying the same force to different points on an object can cause dramatically different behaviors. Without geometric information, these behaviors would be difficult to predict.

Unfortunately, the general spatial reasoning problem is intractable. Thus, recent research has focused on more constrained problems such as motion in limited domains [2,3], mechanical mechanisms [7,8] and fluid ontologies [1,6]. The studies dealing with mechanical mechanisms and motion focus only on rigid objects, ignoring the motion of fluid. In addition, the fluid ontology research is insufficient to fully explain fluid behavior. Two basic approaches to fluid ontology are *contained-stuff* ontology and *piece-of-stuff* ontology [1,6]. Neither of these approaches suffices to explain the geometry of fluid motion. Suppose we want to explain the motion of the gas in a piston when the valve in the middle of the right side is open and the pressure inside the piston is greater than the pressure outside. Since *contained-stuff* ontology treats the gas in the piston as one object, it is impossible to reason about different flow directions in the various parts of the piston. Similarly, it appears to be impossible to consider the motion of every piece of the gas. People do not seem to use either ontology to explain this problem. However, people can reason about the gas near the top surface moving downward to the right and the gas near the bottom surface moving upward to the right, etc.

This paper presents a technique for reasoning about the direction of fluid flow in two-dimensions using incremental generation of the *places* in space. We extend the work of [5,8,3] on places—from the rigid body domain onto the fluid domain. Since fluid motion is determined by the pressure difference and the geometry of surface contact with the fluid, we assume qualitative descriptions of these two terms as input. The theory predicts an equivalence class of places that are created based on the qualitative behavior of the fluid. In addition, it describes the flow in each of these places.

Section 2 presents the theory for reasoning about flow direction in qualitative and geometric terms, given a pressure change and surface geometry. The fluid are partitionied so each part has the same qualitative fluid motion. Section 3 explains how envisionments qualitatively simulate the behaviors of fluid in each part. In section 4 we summarize our results and discuss possible extensions to our theory.

2 A Qualitative Theory of Fluid Motion

The key problem in commonsense reasoning about fluid motion is how to partition the fluid at an appropriate level of representation. Our approach partitions the fluid into a set of *places* by reasoning about pressure wave propagation and geometry. These two factors determine the flow direction. We show how to decompose space incrementally into places.

2.1 Qualitative Direction

In spatial reasoning, the concept of direction is essential in describing the position, force, and motion [8]. We assume a single global reference frame. This reference frame can be translated but not rotated. In our theory, direction is represented by a qualitative vector [8] and qualitative vector arithmetic is used to compute the directions. In our 2D space representation, the first and second components of a qualitative vector represent the qualitative direction along the x-axis and y-axis, respectively. To represent the x-axis direction, we use "+" for "right" and "−" for "left" and "0" for center. For the y-axis, "+" is used for "up" and "−" for "down" and "0" for center. For example, (− −) indicates the vector lies to the lower left of some reference frame.

Definition 1 (Inversion) *Inversion*(v) is the qualitative vector v rotated by 180 degrees.

Definition 2 (Open-Half-Plane) *Open-Half-Plane*(v) are the vectors whose vector dot product with v is "+".

2.2 Rigid Object Representation

Rigid objects are represented by their surfaces in contact with the fluid. In our 2D space, a surface is represented as a line segment. For each surface, we represent the direction of the line segment as the position of one end-point relative to the other. (We define end-points as the points where the line segment meets the neighbors.) In general, the relative position can be defined for any two points:

Definition 3 (Relative-Position) *Relative-Position*(p1,p2) is the qualitative vector which represents the direction from point **p2** to point **p1**.

Consider a surface with end-points **p1** and **p2**, which are connected to other adjacent surfaces. The direction of the surface is represented by `Relative-Position(p2,p1)`.
Information about the relative position can be propagated using transitivity:

Law 1 (Transitivity of Relative-Position) For any points p1,p2, and p3, `Relative-Position (p1,p3)` is computed by adding given values `Relative-Position(p1,p2)` and `Relative-Position (p2,p3)`.

The surface normal represents the direction which points from the surface into fluid.

Definition 4 (Surface Normal) *Surface-Normal*(s) is the qualitative vector which represents the surface normal of the surface s.

Definition 5 (Surf-Rel-Pos) For any two adjacent surfaces s1 and s2 whose end-points are (p1,p2) and (p2,p3), *Surf-Rel-Pos*(s1,s2) represents `Relative-Position(p1,p3)`. This represents the relative direction of two adjacent surfaces.

2.3 Fluid

Unlike a solid, a fluid moves and deforms continuously as long as a pressure difference exists. Its shape is determined by the container. These properties of fluid make them difficult to individuate in a reasoning system. In general, people do not reason about the individual molecules of fluids, but rather they focus on the collection of molecules within fluids.

Pressure Wave Propagation (PWP): When a pressure disturbance occurs in a compressible fluid, the disturbance travels with a velocity of sound. For example, if we throw a stone into the pond in rest, we can see the circular wave-fronts on the surface are diverging from the source. If the disturbance is due to the lower pressure, then an *expansion* wave is propagated. If it is due to the higher pressure, then a *compression* wave occurs. The pressure wave moves from the source toward the wave-fronts and it is perpendicular to the wave-fronts. As the pressure wave is propagated through a still fluid, the fluid properties (i.e., pressure, temperature, and density and so on) change and it start to move. As a compression wave is propagated, the fluid molecules have a velocity which has the same direction as the wave propagation. On the other hand, when an expansion wave travels, the fluid has a velocity which has the opposite direction of the wave (i.e., toward the source of the disturbance). The induced velocity of the fluid by wave propagation is much slower than the wave propagation.

Definition 6 (Prop-Constraint) Suppose a surface `s` is in contact with the fluid. *Prop-Constraint* (s,d) is *true* when `PWP` is prevented in direction d near `s`.

Law 2 (Surface-Constraint) Suppose `s` is in contact with the fluid and its surface normal is sn. Then the pressure wave cannot propagate from the surface to the fluid. Thus for every d which belongs to `Open-Half-Plane`(sn), `Prop-Constraint`(s,d) is *true*.

Continuous Change (CN): We assume the flow is smooth and steady (*laminar*). When flow is not laminar but fluctuating and agitated (*turbulent*), it is impossible to explain the behavior. Even in fluid mechanics, no general analysis of fluid motion in turbulence yet exists and there may never be. People also have difficulty explaining the direction of the turbulent flow. In laminar flow, the changes of properties are continuous. To support laminar flow, we assume the surface is smooth and the changes in surface are not abrupt.

2.4 Place Generation

In FROB [3], given a geometric description of the surface, the places needed to envision the possible motions of a ball are generated by the constraints of geometry of the surface and gravity. Since the gravity constraint is the same everywhere, space can be divided without regard to the neighbors. However, since a direction of `PWP` can keep changing by the surface geometry of rigid body as it propagates, the place cannot be generated without considering interaction between these two. Even though two given spaces have the same geometry, they can be partitioned in completely different ways with different directions of pressure wave.

In fluid motion, qualitatively different parts have different fluid directions since the pressure wave arrives from different directions. Thus *places* in our reasoning problem should be distinguished by difference of the direction of pressure wave in each part. Continuous interaction of pressure wave and geometry suggests our place generation should be incremental as the pressure wave propagates from the source of disturbance.

Definition 7 (Place) A *Place* is defined by its boundaries (i.e., left, right, up, and down) and the direction of the pressure wave and the direction of the induced velocity. Given a place `P`, *Pres-Wave*(`P`) returns the the direction of pressure wave in `P`. The boundaries represent the adjacent places or surfaces of a given place.

Definition 8 (Place) *Place*(`s`) maps from a surface `s` to the *place* in which `s` is a boundary.

Law 3 (Connectivity of Place) Given two adjacent surfaces `s1` and `s2`, `Place(s1)` and `Place(s2)` are also adjacent. Furthermore, since the relative direction between the two surfaces is `Surf-Rel-Pos (s1,s2)`, `Place(s1)` is oriented in the same direction.

As the law of *Connectivity of Place* shows, to represents the connectivity of places, the numeric information is not used. It is represented by the relative position between the places in qualitative terms. For example, in in Figure 1 `Place(s5)` is located to the right of and above `Place(s3)` since `Surf-Rel-Pos(s5,s3)` is $(-+)$.

In our approach, places are generated incrementally from the initial pressure change to the direction of `PWP`. For example, in Figure 1, when portal becomes open, the expansion wave is propagated from the outside to `s1` and `s2` first since they are close to the outside. Then places are generated around these surfaces. After that, places near `s3` and `s4` are generated as the pressure wave keeps traveling. Figure 1 graphically shows this incremental generation.

Since the surfaces of a rigid body can be the only explicit boundaries of the fluid from the input, our approach first generates the places near the surfaces by propagating the pressure wave across the pairs of adjacent surfaces. The places of the space which are not bounded by the surface are not generated at first since it is impossible to trace every point and give the boundary of the place. But as places around the surfaces are generated, the whole space can be covered by the property of fluid, `CC`. For example, in Figure 1, when places near `s1`, `s2`, `s3`, `s4`, `s5`,and `s6` are generated, by

Figure 1: Incremental Place Generation in Piston-Cylinder

An oval represents the a place generated. The locations of ovals represent the connectivity of places.

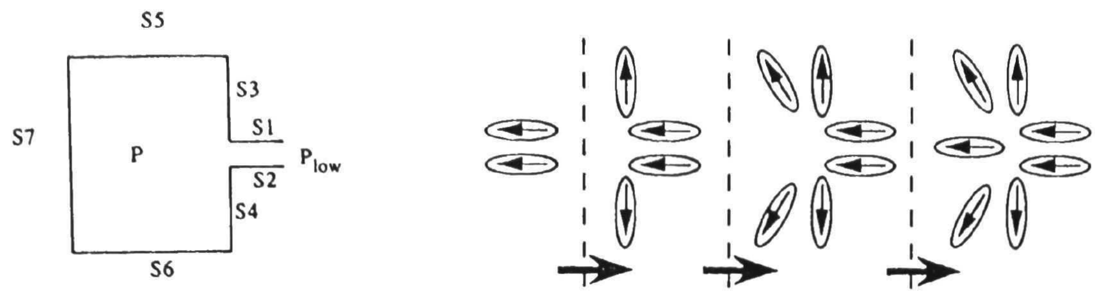
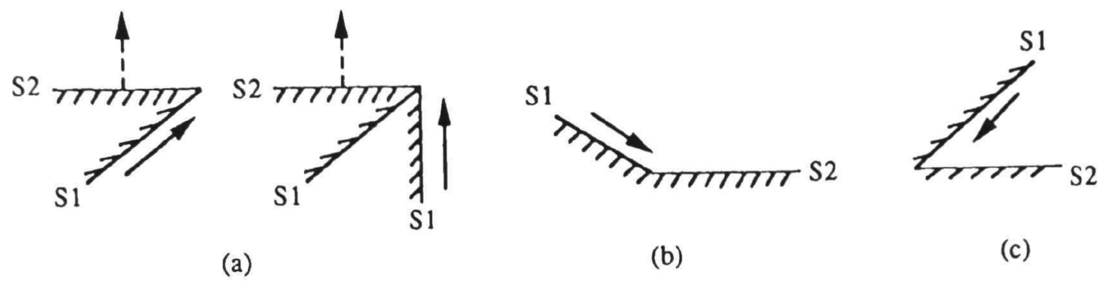


Figure 2: Examples of Forward Propagation when Surface-Normal(s2) = (0+)



CC, a place whose Pres-Wave is (−0) is generated between them .

To propagate a pressure wave across the pairs of adjacent surfaces, the first step is to determine the propagation order between the adjacent surfaces. Given a newly generated place, our system checks how the pressure wave can propagate toward adjacent surface.

Definition 9 (Forward-Propagation?) Suppose s1 and s2 are two adjacent surfaces and their end-points are (p1,p2) and (p2,p3). If the Pres-Wave(Place(s1)) belongs to Open-Half-Plane(Relative-Position(p2,p1)), then *Forward-Propagation?*(s2, s1) is *true*. Otherwise, it is *false*.

As Figure 2 shows, when Forward-Propagation?(s2, s1) is true, we can infer the source of disturbance is not closer to s2. Thus the following law is introduced.

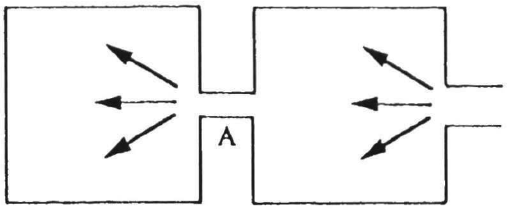
Law 4 (Forward Propagation) Suppose Forward-Propagation?(s2, s1) is *true* and the end-points of s1 and s2 are (p1,p2) and (p2,p3). Then s2 belongs to *next wave front* of s1 if Pres-Wave(Place(s1)) belongs to Open-Half-Plane(Surface-Normal(s2)) (*blocked*) (Figure 2a) or if s2 is not blocked and Pres-Wave(Place(s1)) belongs to Open-Half-Plane(Relative-Position (p3,p2)) (*further*) (Figure 2b). s2 belongs to *same wave front* of s1 if s2 is not blocked and Pres-Wave(Place(s1)) belongs to Open-Half-Plane(Inverse(Relative-Position(p3,p2))) (Figure 2c).

Law 5 (Further Propagation) Suppose Forward-Propagation?(s2, s1) is *true*. If s2 is *further* than s1 from the source, then Pres-Wave(Place(s2)) from the source belongs to Open-Half-Plane (Pres-Wave(Place(s1))).

This law shows if PWP is not blocked, then it smoothly changes the direction across the adjacent surfaces.

A pressure wave travels from the source of disturbance to the all direction unless it is blocked by the surface of the rigid body. Unless the direction of PWP is changed by any surface, then Pres-Wave of any point can be simply inferred as direction from the source to that point. However, as the wave travels, a new source can be generated by geometry of the surfaces. Figure 3 shows how a

Figure 3: New Source



new source is generated when wave arrives at point A from the source.

We identify the cases to generate the new source.

Law 6 (New Source) Suppose Forward-Propagation?(s2, s1) is *true* and the end-points of s1 and s2 are (p1,p2) and (p2,p3). Then there are two cases to generate a new source: (1) *Blocking* -If s2 is *blocked*, then new source is generated around p2. (2) *First-Moving* - If s2 is *further* than s1 from the source, then new source might be generated in Place(s1). Since the fluid in Place(s1) starts to move earlier than that of s2, this may cause a pressure disturbance.

Once the next surface to be propagated is chosen and a new source is identified, Pres-Wave(Place (s2)) is determined by the relationship between s1 and s2, say, whether s2 is in the next wave front or the same wave front and a new source might be generated. In the case of *Blocking*, Relative-Position (p3,p2) becomes Pres-Wave(Place (s2)) since a new source of disturbance is generated near p2. In case of *First-Moving*, PWP by the original and the newly generated source should be considered. If a pressure wave can arrive at s2 without the blocking by surface, the relative direction from the source of disturbance to the surface s2 can be computed by adding that of s1(i.e., Pres-Wave(Place(s1))) and Relative-Position(p3,p2) by the law of *Transitivity of Relative-Position*. Since our approach is based on the qualitative information, every possible inference is made. Thus Relative-Position(p3,p2) and Pres-Wave(Place(s1)) + Relative-Position(p3,p2) are possible directions for Place(s2). In the case of *Same wave front*, at least the parts of s2 which are closer to s1 have the same Pres-Wave as s1. Thus places of s1 and s2 are *merged*. As we mentioned previously, during generation of the places based on this, new places may be generated by the CC.

2.5 Inferring Propagation in Backward Direction

Since our approach propagates the pressure wave across the connected surfaces and divides the space based on the places near the surface, it may not suffice given a more complicated geometry. For example, in Figure 4 when Place(s1) is generated, a new place cannot be generated any more since there is no surface adjacent to s1 in the forward direction of PWP. By the same reason, new place cannot be generated after Place(s4).

However, by the reverse inferencing of the forward PWP, this problem can be solved. As we can expect, this reverse inference may bring out ambiguities. Since our technique does not use any metric information, several possibilities can be introduced. But using some constraints due to the characteristics of fluid, we can eliminate many ambiguities.

Definition 10 (Backward-Propagation?) Suppose s1 and s2 are two adjacent surfaces and their end-points are (p1,p2) and (p2,p3). If the Pres-Wave(Place(s1)) belongs to Open-Half-Plane (Inverse(Relative-Position(p2,p1))), then *Backward-Propagation?*(s2,s1) is *true*.

If Backward-Propagation?(s2,s1) is *true*, then Forward-Propagation?(s1, s2) is *true*. Thus s1 belongs to the next wave front or the same wave front of s2. We identified how to infer whether s2 belongs to previous or same wave front of s1 from the geometric analysis for all possible cases.

Figure 4: Complex Geometry

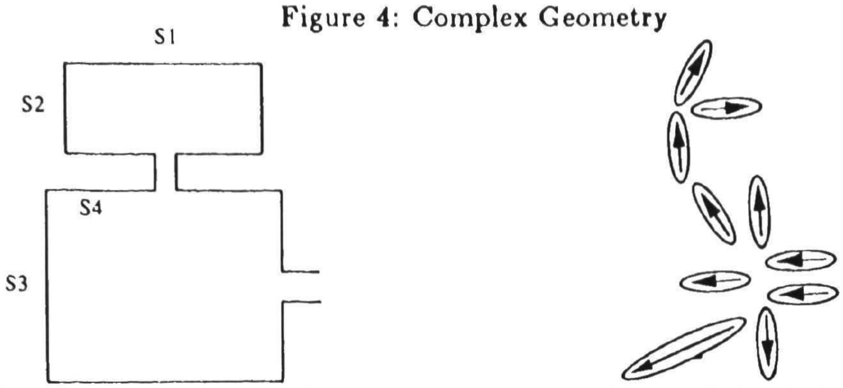
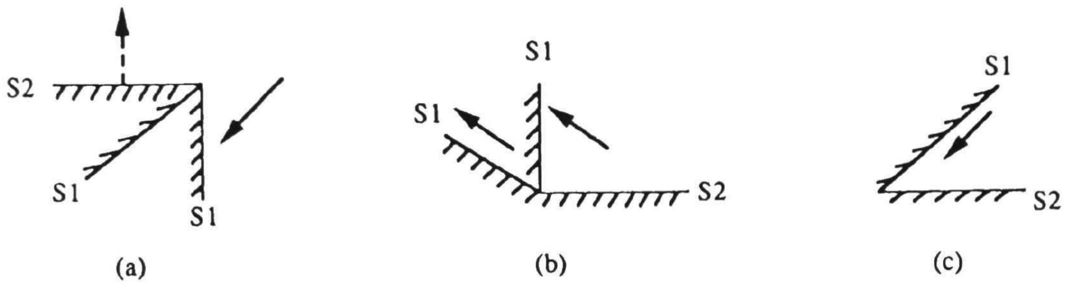


Figure 5: Examples of Backward-Propagation?((0+),s1)



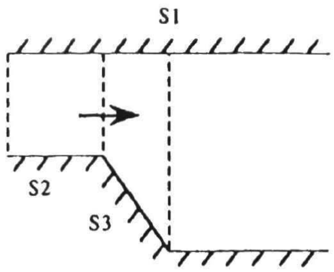
Law 7 (Backward Propagation) If BackwardPropagation?(s2,s1) is *true*, the relationship between s1 and s2 in propagation of a pressure wave is: (1) *same wave front* - Pres-Wave(Place(s1)) belongs to Open-Half-Plane(Inverse(Surface-Normal(s2))); In addition, if Relative-Position (p1,p2) is equal to the Pres-Wave(Place(s1)), then this geometry can be the reverse of the *Blocking* in forwarding propagation (Figure 5a), (2) *previous wave front* - unless s2 is *same wave front* (Figure 5b).

In case of *same wave front*, Pres-Wave(Place(s2)) is computed by adding Pres-Wave(Place(s1)) and Relative-Position(p3,p2). For the reverse of the *Blocking*, Pres-Wave(Place(s2)) can be any *d* which belongs to Open-Half-Plane(Surface-Normal(s1)) and is not constrained can be Pres-Wave(Place(s2)). In the case of *previous wave front*, we cannot compute the possible directions but can give constraints which filter the illegal ones.

Law 8 (Source-Constraint) Suppose Backward-Propagation?(s2, s1) is *true* and the end-points of s1 and s2 are (p1,p2) and (p2,p3). Then, (1) pressure wave cannot propagate from p2 to s2. Thus for every *d* which belongs to Open-Half-Plane(Relative-Position(p3,p2)), Prop-Constraint(s2, *d*) is *true*. (2) pressure wave cannot propagate from s1 to s2. Thus by the law of *Further Propagation*, for every *d* which belongs to Open-Half-Plane(Inverse(Pres-Wave(Place(s1)))), Prop-Constraint (s2,*d*) is *true*.

By the inferring in reverse order, it is not easy to determine the direction of a possible pressure wave; there may be several possibilities. When there are several possibilities for one problem, people tend to eliminate inadequate ones by constraints to get the final solutions. By giving the *Surface* and *Source* constraints, we can filter out the illegal ones. For example, in Figure 5b, if we apply these two constraints, the only possible directions for Pres-Wave(Place(s2)) are (−0) and (−+). In case of c, no direction is left after filtering out, which means Pres-Wave(Place(s1)) cannot have the direction of (−+). Like in c, even if the illegal places generated by reverse inference by the lack of information, they can be filtered out later.

Figure 6: Flow and Surface Interaction



3 Envisioning Flow direction

Given an external disturbance of pressure, the space of interest is incrementally divided by connected places. Our system computes every possible combination of the places. Then the fluid in each place starts to move in the direction of a pressure wave if it is a compression wave or in the opposite direction of a pressure if it is an expansion wave. By the connectivity of the places, we can predict the next place where the fluid will go. For example, if the induced velocity of a place is (+−), then the fluid in that place will flow into the places where are located to the right or down from the place.

When the moving flow comes to the place, then the induced force by the interaction of the flow and surface of the rigid body may be applied to the flow. Figure 6 shows an example of this. Arrows represent the direction of the induced velocity. When the fluid in Place(s2) arrives at Place(s3) with the velocity (+0), it keeps going to that direction. However, the area near the surface s3 change and has less molecule of the fluid compared to the other parts of Place(s3).

Compared to the pressure disturbances in previous section, the influence of disturbance of flow and geometry is small and local since as soon as it happens the flow in that place changes the direction by the induced force. Thus its disturbance is diminished. But even if the induced force is applied to the moving flow, the flow does not immediately change the direction to the direction of the applied force since the flow already has the momentum. Since this effect is local to the flow in that place, it generates a *local* place inside the place. Its effect assumes to be limited inside of the place. Even though we can infer this region exists inside of the place, it seems to be impossible to explicitly give their boundary since its effect keeps diminishing and the interaction between the flow close to that region and that disturbance keeps changing.
Since the local place is also generated by pressure change, two kinds are possible:

Definition 11 (N-Local-Place) Suppose P represents Place(s). If the pressure adjacent to s is lower than the other part of inside of P, then *N-Local-Place*(P,s) is generated near s.

Definition 12 (P-Local-Place) Suppose P represents Place(s). If the pressure adjacent to s is higher than the other part of inside of P, then *P-Local-Place*(P,s) is generated near s.
We identified the interaction between flow and geometry as follows:

Law 9 (Pulling) Suppose P represents Place(s) and the flow with the velocity v is entering the place P. If Surface-Normal(s) belongs to Open-Half-Plane(v), then N-Local-Place(P,s) is formed. This N-Local-Place gives the force in direction of Inverse(Surface-Normal(s)) to the flow in P (i.e., it pulls the flow into the surface).

Law 10 (Pushing) Suppose P represents Place(s) and the flow with the velocity v is entering the place P. If Surface-Normal(s) belongs to Open-Half-Plane (Inverse(v)), then P-Local-Place(P,s) is formed. This P-Local-Place gives the force in direction of Surface-Normal(s) to the flow in P (i.e., it pushes the flow into the surface).
Pushing happens since the fluid molecules hits the wall and those collisions increase the pressure near the wall.

Thus fluid direction in each place can be envisioned by starting from its original place and traveling into the adjacent places with possible changing of its direction due to the surface interaction. The flow will be stop if it goes to equilibrium after moving.

4 Discussion

This paper presents a theory of geometry of fluid flow in two-dimensional space. Given qualitative descriptions of geometry and a pressure change in fluid, we can determine the possible directions of fluid motion. The interaction between the surface geometry of rigid body and pressure wave propagation is identified in our theory. This idea is being implemented.

We have only dealt with the velocity change of fluid here. We plan to expand our theory to have a complete theory for reasoning about fluid. Reasoning about the other important properties of fluid, such as pressure, temperature, and density so on is left as future work. What we hope to analyze eventually is a real system, such as internal combustion engine, which should be explained by tightly integrating dynamics and kinematic of rigid bodies and fluid. Our theory for analyzing the directions of fluid flow is one step towards that goal.

5 Acknowledgements

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References

[1] Collins, J. "Reasoning About Fluids Via Molecular Collections", Proceedings of AAAI-87, July, 1987

[2] deKleer, J. "Qualitative and Quantitative Knowledge in Classical Mechanics", TR-352,MIT AI Lab, Cambridge,MA., 1975

[3] Forbus, K. "A Study of Qualitative and Geometric Knowledge in Reasoning about Motion", TR-615,MIT AI Lab, Cambridge,MA, 1981

[4] Forbus, K. "Qualitative Process Theory", *Artificial Intelligence*, **24**, 1984

[5] Forbus, K., Faltings, B., and Nielsen, P. "Qualitative Mechanics: A Framework", UIUCDCS-R-87-1352, University of Illinois, 1987

[6] Hayes, P. "Naive Physics 1: Ontology for Liquid" in Hobbs, J. and Moore, B. (Eds.), *Formal Theories of the Commonsense World.* Ablex Publishing Corporation, 1985

[7] Joskowicz, L. "Shape and Function in Mechanical Devices" Proceedings of AAAI-87, July, 1987

[8] Nielsen, P. "A Qualitative Approach to Mechanical Constraint" Proceedings of AAAI-88, August, 1988

[9] White, F. *Fluid Mechanics*, McGraw-Hill Book Co., 1979