

Using Knowledge Representation to Study Conceptual Change in Students for Teaching Physics

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Abstract

Our goal is to understand the development of physics concepts in students. We take the perspective that individuals construct their own understanding so as to 'fit' their experiences. This constructive activity results in conceptions about the physical world. The major challenges in physics instruction then are the tasks of identifying and inducing change in students' conceptions about the physical world. Our efforts to understand the nature of conceptual change are aided by knowledge representation techniques. We present examples in which some of the finer structure of conceptual change is represented which illustrate the potential of knowledge representation for studying conceptual change.

Introduction

Studies have found that students begin formal physics education with a system of physical *conceptions* that differ in deeply systematic ways from those of the physicist and present a significant obstacle to learning physics (Viennot, 1979; di Sessa, 1983; McDermott, 1984; Halloun and Hestenes, 1987). These *alternative* conceptions manifest themselves as *useful* commonsense beliefs about the world. Yet alternative conceptions are not addressed by standard instruction, either in physics classrooms or in introductory physics textbooks. Simply presenting students with the laws of Newtonian mechanics when they solve problems, for example, does not encourage conceptual learning because such statements do not make the intended sense in the context of their beliefs; instead the students interpret what is said in terms of their *existing* beliefs. As a result, most students depart from beginning physics courses without an understanding of the Newtonian *concepts* presented; their conceptual framework about how the world works is left essentially unchanged.¹ If physics instruction is to encourage the kind of learning which leads to new conceptual understanding, it must address the alternative conceptions that need to be changed. In our view, such conceptual change occurs only when new conceptions are *constructed* by individuals themselves. We believe students can make sense of Newtonian concepts if they experience situations which bring them to question their own conceptions and are then facilitated in their attempts to develop more viable replacements.

Identifying alternative conceptions has been the focus of a number of studies which have provided qualitative analyses of many of the conceptual difficulties students have in beginning physics (Minstrell and Stimpson, 1986; McClosky, 1983; McDermott, 1984; and Clement, 1982). The results of these analyses and others', as well as our own, have revealed many of the alternative conceptions associated with beginning mechanics. These results need to be integrated and organized so they can be used to study necessary conceptual changes in learning physics. We believe this requires representing alternative conceptions as structures comprised of salient features and relationships. We call these representations "conceptual maps". They represent in an explicit and pragmatic way, concepts, terms, features and their interrelationships that underly students' descriptions of the physical world. Successive conceptual maps represent in detail the changes students go through from "motion implies force", for example, to the Newtonian conception that "acceleration implies force". Such representations aid in identifying desired conceptual changes which can be associated with instructional techniques found to be effective for bringing about conceptual change. The adequacy of the maps is based ultimately on how well their representation of conceptions can effectively inform instruction.

Alternative Conceptions

Alternative conceptions make up students' fundamental beliefs about how the world works or how it is constituted which are quite different from those of the physicist. These beliefs apply to a variety of different

¹ This does not mean that these students necessarily get poor grades. Students are still able to solve the problems assigned without understanding of the underlying conceptions (Halloun and Hestenes, 1987).

situations. They are beliefs in an *explanatory* sense about causality. For example “motion implies force”² is a conception which leads students to suggest that,

- a.) there is a force (in addition to gravity) that propels a block down an inclined plane because it moves down the plane *or*
- b.) there is a force in the direction of a planet’s motion *or*
- c.) there is a force down on an object because it can move down.

Another example is the “materialistic conception” which leads students to endow things like electricity, light, and heat with matter-like properties (Reiner, Chi, Resnick, 1988).

Identifying Alternative Conceptions

In contrast to modeling problem-solving skills, studying conceptions is more difficult because there is a “level of indirection” between what we observe students doing and the conceptions that give rise to their behaviors. Conceptions must be *applied* to problem situations and thus can only be identified indirectly by analyzing student responses. This is in contrast to skills which are specified in terms of observable features of the problem situation (Anderson, Boyle, Corbett and Lewis, 1990) independently of what students may think.

In order to identify students’ conceptions we must be able to make sense of their behaviors. We do this by applying a model of conceptual application, depicted in Figure 1, which is based on the hypothesis that when conceptions are applied in specific problem contexts, they are manifest as characteristic behaviors. For example, when two interacting objects have different sizes, students associate a larger force with the larger object. We would expect to see similar behaviors resulting from such a belief in different problem situations sharing this feature. But to be more convinced that we identify the underlying conception, we consider problem situations that include additional features, that is, we consider a variety of situations in which that conception might be applicable. This is because the specific

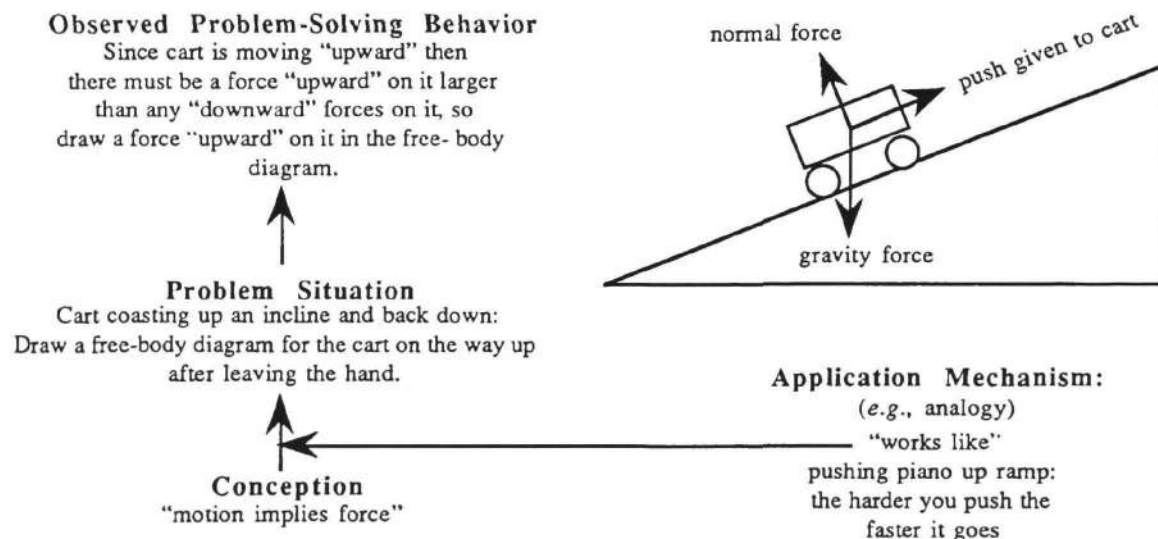


Figure 1: Model for Conceptual Application

A conception applied to a problem situation results in an observable problem-solving behavior. Note that in this example there is also evidence of force being “given to an object as if it were a property of the object.

behavior of associating a larger force with the larger of two interacting objects is probably the result of applying a conception like “force is a property of an object” in addition to a *specific* belief about different sized objects. By having students solve a number of different kinds of problems, we work backwards to determine whether a more

² We are aware that on some interpretations “motion implies force” is even true for Newton, *i.e.* the persistence of motion implies an internal force or *vis insita* (McGuire, 1990). However, the alternative conception we mean by “motion implies force” is different from this interpretation of Newton in that for students “force” has *not* been differentiated into ‘force of persistence’ and ‘force of acceleration’, although we would not be surprised to find the students making this same differentiation as they begin to form more Newtonian conceptions.

general conception underlies the belief. For example, students apply “motion implies force” to a wagon pulled by a donkey. But they also apply it to a body moving *without* apparent forces which is a very different kind of situation. This leads students to remark that the body is moving because force has been transferred to it from a body that originally set it in motion (similar to impetus).

The conceptual application model serves as a methodological framework for identifying conceptions. It is also explanatory of what we observe, enabling us to make sense of students’ behavior. However it is not a detailed psychological model in that it makes no claim as to the real mental or physical nature of conceptions nor does it specify the mechanisms by which they are applied. Rather, it is a model in the sense of a concrete construction which is used to catalogue, in a principled and organized way, the kinds of descriptions that students give in answers to questions and that can be inferred from their behavior in problem-solving situations.

Conceptual Change

Conceptual change has come to play an important role in the history and philosophy of science in the last several decades (Kuhn, 1970; Feyerabend, 1988). These studies have begun to influence research in cognitive developmental psychology (Carey, 1985). Carey describes two possible senses of knowledge restructuring, *weak* and *strong*. According to Carey, knowledge restructuring in the weak sense involves a rearrangement of the relationships between existing *concepts* such as velocity, acceleration and force and the situations to which they apply. In weak restructuring, concepts are not changed, rather their applications are either extended, restricted, or rearranged. Knowledge restructuring in the strong sense involves *changes* in the concepts themselves, *i.e.* actual conceptual change. We agree with Carey’s provisos on conceptual change with some additional refinements that are a result of explicitly representing knowledge in conceptual maps.

An Example of Conceptual Change

The following is a description of our observations of a sequence of conceptual changes involving the relationship between force and motion, depicted in Figure 2, along with the method of inducing those changes (Minstrell, 1989). The method was originated by Minstrell almost ten years ago.

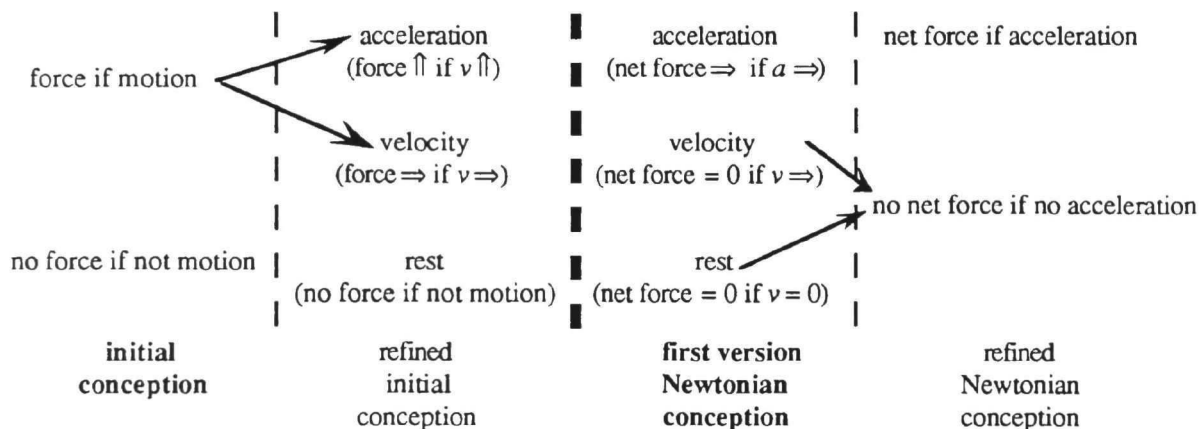


Figure 2: A Series of Conceptual Changes

The bold, vertical, dashed line at the center of the figure indicates a substantial conceptual change. The regular, vertical, dashed lines at either side indicate less substantial conceptual refinements (“ $\uparrow$ ” -- increases; “ $\Rightarrow$ ” -- remains constant).

Students typically come to us at the introductory college level with the conception, “motion implies force”, together with an undifferentiated view of motion. Even if students can *recite* Newton’s three laws of motion, their response to questioning and their problem-solving performance usually reveal conceptions which are not in accord with their statement of the laws. This situation is indicated by the leftmost column in Figure 2, labeled “initial conception”. Following a series of laboratory-based activities using microcomputer-based laboratory (MBL) equipment (Thornton, 1987), students figure out the interrelationships between various quantities used to describe the motion of particular objects from graphs of distance vs. time, velocity vs. time, and acceleration vs. time. They begin to discriminate different motions when reporting about falling bodies. For example, they refer to “speeding up” or “acceleration” instead of just “falling down”. This captures a distinction between the first and second columns in Figure 2.

This differentiation of motion does not, by itself, change students' conceptions about the *causes* of motion, but it does generate a conceptual division of motion. With respect to causation, this change provides a basis for elaborating or enriching their current conception. For example, students will say that to maintain a constant velocity a constant excess force is needed and if there is a changing velocity, then the excess force is changing as well. This situation is depicted in the second column from the left in Figure 2, labeled, "refined initial conception".

After students have made this distinction explicitly in class, they are asked to *observe* the motion of an object which is clearly under the influence of a constant excess force and *are surprised* to find that it does not move with constant velocity, but with constant acceleration. Their surprise is partly indicative of a cognitive *disequilibrium* (See below). Upon further reflection, they realize that this invalidates their explanation for constant velocity which is probably now best represented by "velocity implies force". In classroom situations, where the teaching method is constructivist, discussion³ is encouraged in which students come to realize that a zero magnitude excess force is a condition for constant velocity which is more consistent with their new observation about acceleration. In arriving at this view they also shift their consideration of force from applied force to what the physicist calls *net force*. This conceptual change is represented as a shift from "refined initial conception" to "first version Newtonian conception" in Figure 2.

This series of laboratory-based activities leaves students short of what we would call a "refined Newtonian conception" in that the equivalence of constant velocity and rest has not been completely resolved. It is currently our hypothesis that differentiating between rest and zero instantaneous velocity underlies the change to a "refined Newtonian conception". While this differentiation is addressed in a purely kinematic context for the coin-toss problem during the students' MBL work, it should be brought up in the context of applying their new conception (first version Newtonian conception) to acceleration during a coin-toss. Another context in which the equivalence between constant velocity and rest might be addressed is in the consideration of inertial reference frames. This differentiation might clear the way for understanding *rest* as a particular state of constant velocity.

Assimilation, Accommodation and Disequilibrium

Changes in the knowledge state of a learner are often described as assimilations and accommodations. These terms were initially introduced in the context of learning by Piaget. We introduce assimilation, accommodation and especially disequilibrium into our discussion of conceptual change in order to describe, from the standpoint of learning and pedagogy, the necessary conditions for conceptual change. While we think of conceptual change in these Piagetian constructivist terms, we *do not* invoke the notion of Piagetian stages of cognitive development.

Assimilation is the recognition that an event (physical or mental) fits an existing conception (von Glasersfeld, 1987). This recognition process also involves a selective ignoring of discrepancies deemed not salient. Accommodation is a change in fundamental belief about how the world works, that is, a change in a *conception*. It is the construction of a new structure which *can* assimilate an event which could not be assimilated under previously held conceptions. Each of the three transitions in Figure 2 involves accommodation. Where the initial conception becomes refined and where the first version Newtonian conception becomes refined, we would say that "within conception" accommodation has occurred. In the former, for example, motion is differentiated into velocity and acceleration, but the conception that "motion implies force" remains essentially intact. "Conception-change" accommodation occurs at the vertical, bold, dashed line; *i.e.*, where the conception changes from the initial, everyday conception to a more Newtonian conception. The difference between "within conception" and "conception-change" accommodation is a difference in how fundamental the change in the students' knowledge structure is. It is not the same as the difference between Carey's weak and strong knowledge restructuring.

We believe that for accommodation to occur, the learner must become motivated to change by passing through a state of cognitive disequilibrium; sometimes profound, sometimes not, but always a disequilibrium. When an individual cannot fit an event into existing beliefs, a state of disequilibrium exists. The fact that certain conceptions may not change under normal instruction may be due to the failure of that instruction to disequilibrate students with respect to the conceptions they hold. If students can assimilate events (words, ideas, experiences) presented in the course of instruction, then there is no disequilibrium and no conceptual change. The point of instruction should be to induce conceptual change. It cannot accomplish this without *inducing* disequilibrium and *facilitating* accommodation.

It should be noted that disequilibrium is *not* contradiction. The latter refers to a logical inconsistency whereas disequilibrium is a conceptual incongruity. Disequilibrium is not a consequence of formal, truth-valued statements, but, rather, the surprise produced when an expected physical event does not occur. Conceptual change does not depend on contradiction, but on disequilibrium.⁴

³ It is important that this be a discussion between students, not between students and teacher.

⁴ For example, while people who believe in heliocentrism would find the retrograde motion of the planets a contradiction of geocentric beliefs this was not the case for those who held the geocentric view. For them, the retrograde motion of the

Representing Conceptual Knowledge for Instruction

A knowledge representation scheme that we believe can capture the kinds of differences illustrated in Figure 2 includes a subcategorization hierarchy (Korner, 1970) represented as a network of nodes and links between them which can be instantiated in a computer (Brachman, 1979; Nirenburg and Monarch *et al.*, 1988), as depicted in Figure 3.⁵ The nodes at the highest level represent the most abstract categories such as object, attribute, relation and situation (*e.g.* state, process), becoming less abstract traveling down the hierarchy. Links are relations between categories. In the domain of mechanics, the primary kind (*i.e.* subcategory) of object considered is “physical object” and example kinds of “situation” are “motion”, “constant velocity” and “accelerated motion” (the latter two are subcategories of “motion”). Examples of

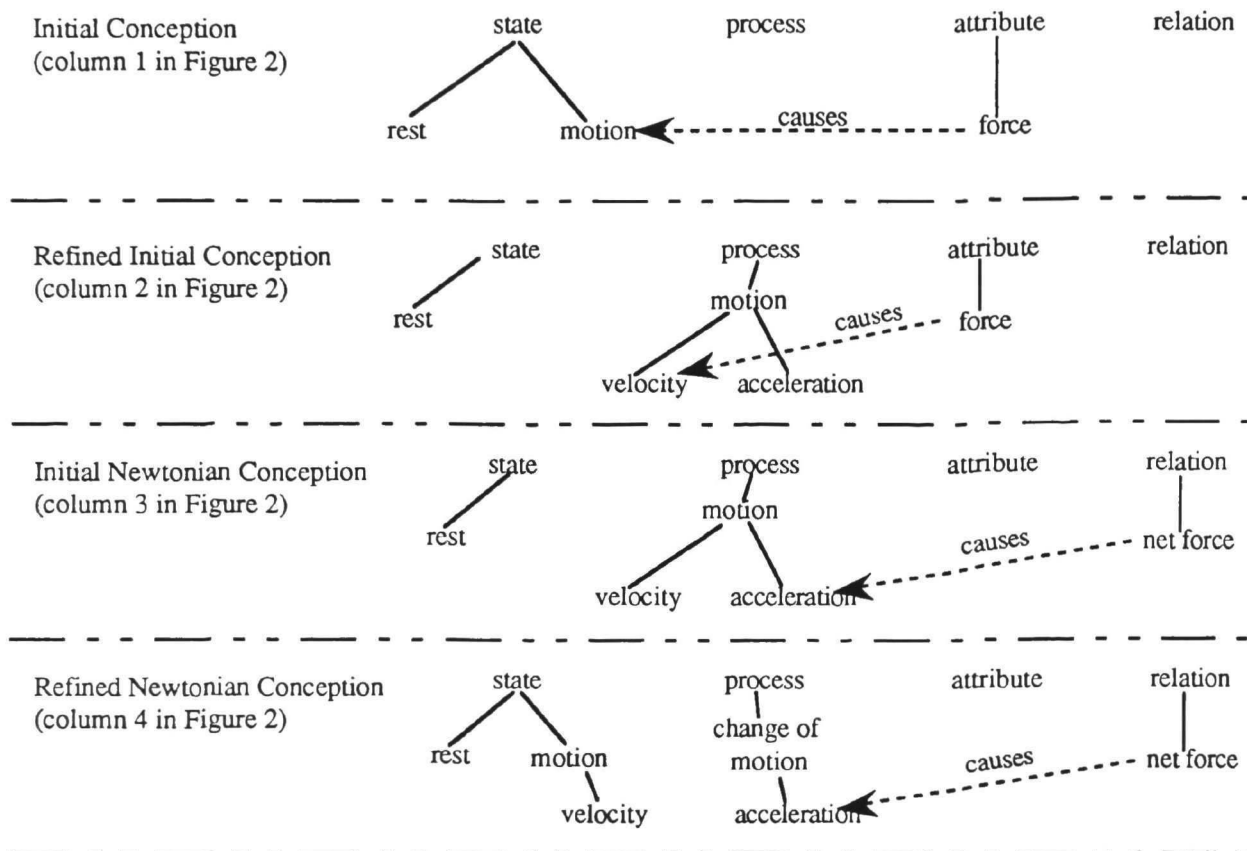


Figure 3: A Series of Conceptual Maps
A network representation of the force and motion conceptual changes which are depicted in Figure 2.

relations are “takes place in” and “causes”, while “mass” is an example of an attribute. The four networks in Figure

planets was explainable by epicycles and thus not seen as contradicting the belief that all heavenly bodies revolve around the earth. While the identification of retrograde motion did not contradict geocentric beliefs, it did create enough cognitive dissonance to require the introduction of epicycles, *i.e.* a “within-conception” accommodation. When epicycles became too unwieldy, this contributed, along with other factors, to further disequilibrium and finally to “the Copernican Revolution”. Even more telling of the difference between contradiction and disequilibrium is that what appears a contradictory event to some may not disequilibrate others. This frequently happens in traditional instruction. Carefully planned activities fail to lead students to the conclusions teachers intend. Students frequently fail to be disequilibrated by the experience of seeing a material at constant temperature while its surroundings change temperature during heating and cooling through state changes.

⁵ It should be noted that the diagrams in Figure 3 are schematic, showing only enough detail to illustrate our points. Considerably more detail, we believe, is necessary for capturing students’ knowledge relating to force and motion for instructional purposes.

3 correspond to the columns in Figure 2. In the first network there is no subcategorization of "motion" and "force" is causally linked with this undifferentiated "motion". In the second network, "force" is causally linked to "velocity" which is now a part of a differentiated "motion." In the third network the notion of force has become "net force", a relation between objects instead of an attribute of an object and it is causally linked to "acceleration." In the fourth network "motion" becomes a state like "rest" while "change of motion" is a process -- thus "velocity" is "motion" as a state while "acceleration" is a change of motion.

Conceptual Maps

We call a network representation of the knowledge state of a student a "conceptual map". It enables us to give precise and explicit specifications of the elements and relationships associated with conceptual change. The nodes in the network represent what we call individual *concepts* (e.g. "force") while conceptions can be expressed as propositions (e.g. "motion implies force") which are combinations of concepts. The proposition, "motion implies force", indicates a causal relationship between the concepts of force and motion that is explicitly captured in the network. Though conceptions are mental entities, we are able to *refer* to them through propositions represented explicitly in the map. Since conceptions, like "motion implies force", are the basis for understanding how the world works, they are represented by (causal) relationships between nodes relatively high up in the network hierarchy. Conceptual change in general gives rise to different kinds of changes in the network. For example, a conception may be "enriched" by the differentiation of an existing concept that is part of the conception. This differentiation is represented as the generation of subcategories in conceptual maps (within-conception accommodation). For example, in the first column of Figure 2, motion is undifferentiated, yet in the second column it becomes differentiated leading to an enriched conception.

Our scheme for representing students' knowledge is unique in that it enables us to bring together a constructivist (Glaserfeld, 1984) learning framework and knowledge representation techniques. The representation makes explicit the categories by which students discriminate different situations and the conceptual apparatus they use to identify and describe those situations. There are recent indications that others interested in science education are beginning to use knowledge representation techniques (Nersessian, 1989). Skill models give a precise way of modeling the student's skill knowledge, and can be tested by model tracing, which involves simulating problem-solving behavior. Likewise, conceptual maps will provide a concise way of modeling conceptual knowledge since they can be used to trace a student's conceptual learning. Accommodation can be represented as a sequence of two or more conceptual maps tracing a conceptual change in which conceptions are altered either by enriching them through new subcategories or changing relationships between categories. Thus, an accommodation can be tracked going from a network representing motion as a process requiring force to a more Newtonian network in which acceleration is a process requiring force.⁶

There are two important points to note here. The first concerns abstract categories like "process" and "state", as depicted in Figure 3, while the other concerns the problem 'tracking' concepts across conceptual changes. With regard to the first point, we are not recommending that beginning students learn to distinguish the differences between certain abstract categories, like "process" and "state", and to be able to classify certain physical concepts under them. What we *are* recommending is that teachers recognize the categorial affiliations of student concepts so that they can better understand their students' learning needs.

As regards the second point, we believe that overlaps in the situations to which the concepts are applied and the stability of many of the concepts from network to network will allow us to identify earlier and later versions of the same concepts. While this view regarding the traceability of conceptual change is similar to others (Carey, 1985; Keil, 1989), we believe our conceptual maps, or networks, will specify much more explicitly what changes and what remains the same in conceptual change. Conceptual networks will not only represent alternative conceptions but Newtonian conceptions as well. We will be able to represent all manner of intermediate conceptions in different maps from a student's initial alternative conceptions to the aimed for Newtonian conceptions.

The Knowledge Framework

The framework for representing knowledge encompasses conceptual maps for pedagogically relevant knowledge states and for the set of conceptual changes which link them. The maps are built by analyzing data, acquired from student interviews and problem-solving tasks, into conceptions and category distinctions or concepts. Representing this analyzed data involves the identification of pedagogically relevant knowledge by an examination of

⁶ In a Newtonian framework, motion as constant velocity is a *state* like rest. The preservation of such states do not require external forces (McGuire, 1990). Only accelerated motion as a change of motion or change of state is a *process* requiring external force. Such "conception-change" accommodation may require changes in ontological categorization (Korner, 1970). In this case the conceptual change is represented in part as a differentiation of motion, where the differentiated categories are each classified under different ontological categories, process on the one hand, state on the other (Koyre, 1968; Kuhn, 1970; Feyereabend, 1988)

student conceptions in comparison with Newtonian conceptions. As is illustrated in Figures 3, some of the salient differences are causal linkages and hierarchical relations of physics concepts to various ontological categories. Conceptual changes occur in the context of knowledge elements that appear to be invariant from student to student.⁷ These invariant elements include abstract ontological categories like “state” and “process” on the one hand and concrete elements such as prototypical scenarios which people have in common (for example, students typically refer to the relation between the “gas” pedal in a car and the motion of the car). These concrete knowledge elements are very important because they are what students often argue from to support their pre-Newtonian conceptions. Having students carefully examine these elements can be the basis for students to begin conceptual change (Minstrell, 1989).

Thus, conceptual maps are comprised of ontological categories and concrete elements, as well as physics concepts such as “motion”, “velocity”, “force”, *etc.* Conceptions (both alternative and physicists’) which can change are situated against the background of the stable framework of ontological and concrete elements. The fact that there are stable elements in a conceptual map constrains the number of possible variations and makes the task of mapping a student’s conceptions less difficult than might be imagined.

Once conceptual maps of student ideas have been built, desired conceptual changes must be determined. Such maps together with maps of goal conceptions can be used to draw connections between alternative conceptions and goal conceptions, for example, implied by the sequence of conceptual changes depicted in Figure 3. These *may* reveal whether there are other knowledge-state elements or structures which must be further divided as preconditions as these conceptual changes, such as differentiating the concept of motion described above.⁸

Conclusion

Pedagogically, conceptual maps enable instruction to focus on explicitly depicted aspects of students’ understanding through the kinds of distinctions students make when they think about the physical world. That is, the maps organize and make explicit the essential content of experiences intended to disequilibrate students. Thus, analogous to problem features that comprise the conditional sides of rules used to model skills associated with specific problem-solving actions, portions of conceptual maps can be considered to represent features of student knowledge states associated with conceptual change techniques that effect changes in those knowledge states. Conceptual maps, as we have described them, enable us to monitor and evaluate student learning by providing a more precise way of representing conceptual change. The more salient detail that is included in the networks, the more accurate will be the representations of students’ knowledge on which instruction will be based and, we believe, the more effective will be instruction.

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⁷ For other views which share the intuition of this assumption see Smith *et. al.* (1985) and Keil (1989).

⁸ Often it is the case that new forms of representation can reveal unanticipated, valuable insights. Boyle (private communication) has indicated that representing the skills necessary for solving proof problems in geometry revealed a previously unnoticed proof in a pedagogically important geometry problem. Similar surprises have occurred in physics where new phenomena have been predicted from new representations, *i.e.* new particles predicted by applying group theoretical representations.

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