

Why are Situations Hard?

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Abstract

An ecological model of human information processing is introduced which characterizes *intuition* as a state oracle providing information for particular types of situations for which attunements to constraints have been developed. The consequences of this model are examined showing among other things that: for a cognitive task with a fixed problem space difficulty can only be reduced by introducing metaphor, difficulty of translation is minimum for a situationally equivalent metaphor, a situationally equivalent metaphor preserves and reflects extrinsic information about the situation, any situation containing a subcategory isomorphic to a problem situation can be made into a metaphor by supplying instructions, these characteristics can be exploited by an algorithm which chooses a metaphor in such a way that attunements are substituted for problem constraints and instructions are used as an "error term".

Introduction

Despite twenty years of experience and widespread commercial success cognitive principles underlying the effectiveness of direct manipulation and visualization interfaces remain a mystery. Advertising brochures and users glibly describe them as *intuitive*, *direct*, or user-friendly but neither psychologists nor computer scientists can agree on exactly what these terms mean.

Lewis (1991,1992) has proposed an ecological model based on *interactive situations* which operationalizes "directness" and "intuition". Cognition is presumed to operate on situations

involving objects in relations rather than propositions about them. The dynamics of these mental situations are governed by attunements to pervasive regularities in our environment such as object constancy. These attunements are presumed to be automatic processes. This "mental model" is animated by initiating actions which are either imagined or perceived. The mental events which follow, unfold in accordance with our attunements to the constraints affecting the situation. By modeling courses of events the mental model makes the resultant states available to cognition. The novelty of this approach to mental models lies in incorporating the decomposition of situations into states of affairs and constraints borrowed from situation theory (Barwise & Perry, 1983) and the characterization of cognitive tasks as search of a problem space (Newell & Simon 1972). This synthesis allows a unified treatment of task difficulty and metaphor as allocation of processing problems. The model and its consequences provide a framework for automating the design of cognitively efficient scientific and problem visualizations and the design and evaluation of graphical user interfaces.

Intuition is presumed to describe the effects of attunements (mental constraints) developed in response to the regularity of certain events in our environment. Attunements are associated with particular *types* of situations and to allow us to imagine/update states of these situations automatically. This process of automatically updating states is referred to as *envisioning*. A related process of *inspection*, makes this state information available for conscious processing. Together, envisioning and inspection form a cycle which acts as a "state oracle" by supplying information about changes in state at essentially no cost.

The model attributes the difficulty of cognitive

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Table 1: Problem Isomorphs				
	Form	Tower of Hanoi	Monster Globe Move	Monster Globe Change
anchored (ordinal)	a_i	disk size	globe size	monster size
unanchored (nominal)	u_j	disk location	globe location	globe size
Rule 1	$a_i > a_j \wedge u_k . a_i \wedge u_k . a_j$ $\rightarrow \neg \Delta u_l . a_i$	By Attunement	A monster may only pass its largest globe	If monsters hold globes of the same size, only the largest can change
Rule 2	$a_i > a_j \wedge u_k . a_i \wedge u_l . a_j$ $\rightarrow \neg \Delta u_k . a_i$	A larger disk may not be moved on top of a smaller disk	A monster may not pass its globe to a monster holding a larger globe	A monster may not change its globe to a size held by a larger monster

tasks to two factors: 1) the *intrinsic difficulty* associated with the size and complexity of the problem space and 2) the *extrinsic difficulty* associated with the controlled processing needed to update states and supply constraints in searching that space. Attunement to constraints makes cognitive tasks easier by eliminating some "illegal" events from the problem space. Where problem constraints and attunements perfectly coincide the frame problem is resolved and difficulty is limited to that of searching legal states. This model of cognitive difficulty can be illustrated using the Tower of Hanoi and two of its isomorphs. Subjects find the Monster-Globe problems much more difficult. (Hayes & Simon, 1977), for example, reports differences in average solution times of less than two minutes for the three disk Tower of Hanoi problem, and half an hour for the corresponding Monster-Globe (change) problem. The Monster-Globe (move) problem is of intermediate (14 min). (Kotovsky, Hayes, & Simon, 1985) difficulty. The Monster-Globe change problem is the most difficult because it violates object constancy, a basic attunement which plays a primary role in theories of psychology ranging from cognitive development to perception. Searching its problem space requires the use of limited working memory resources to determine the changes in state resulting from actions because events do not follow environmental constraints to which we are attuned. The Monster-Globe move problem relates states through the movement of objects, to which we are attuned and therefore eliminates the need to use controlled processing and intermediate storage to update states. The problem space made available through these attunements,

however, is substantially larger than the official one because we can envision globes being moved among any of the monsters, while the problem rules constrain these movements. Because rule 1 requires information about the initial state of a move and rule 2 requires information about its terminating state, both states and the linking action must be referenced to apply the problem rules. In the Tower of Hanoi rule 1 is subsumed by attunements and violation of rule 2 is determinable by inspection alone, because of the illegal state which results. As a consequence we are mentally constrained to ignore movements of disks from the bottom of stacks (rule 1) and can judge legality by inspecting the terminating state (rule 2) without additional reliance on working memory. This reduces the problem to a controlled search of a space of 50 states and 75 transitions in which each of the 36 prohibited moves are ruled out by inspection for the illegal "larger on top of smaller" state at a path of length of 1.

As these examples illustrate, cognition is conceived to be a heterogeneous mixture of automatically updating models and resource consuming rules. A commonsense interpretation of this dichotomy is that cognitive tasks are *direct, intuitive*, and easy to the extent that they do not require instructions. The model is analogous to a computer with a limited capacity general purpose processor and a high capacity specialized one. The most efficient program for such a machine will be one which balances the costs of translating data for specialized processing with the savings it offers. This paper examines the consequences of treating human information processing in the same way.

Consequences

The difficulty of interacting with a problem situation is dominated by the rules, f , a user must actively supply. The relation between an interactive situation, S , problem constraints, C , attuned constraints, A , instructed constraints, f , and the problem situation, $C \circ S$, the basis situation, $A \circ S$, and user's situation, $f \circ A \circ S$, they define can be expressed as:

$$C \circ S \cong f \circ A \circ S$$

"The official problem space appears to the user as a situation in which some disallowed events are not imagined ($A \circ S$) but others can only be eliminated by consciously applying rules ($f \circ A \circ S$)". The extrinsic difficulty of the problem will depend on the complexity of the rules, f , which must be composed with the attuned constraints, A , to bring the user's constrained situation, $f \circ A \circ S$, into agreement with the constrained situation, $C \circ S$, which defines the problem space. Assuming that difficulty measures exist for controlled processing and that attunements are specifiable and indexable by situation-types:

- (1) *The difficulty of a cognitive task involving an interactive situation can only be reduced by introducing metaphor.*

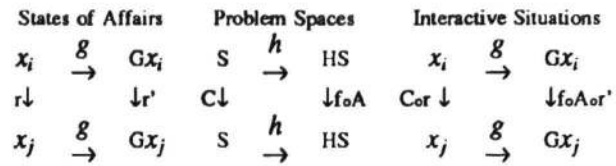
This follows from the definitions. The intrinsic difficulty of a task cannot be reduced without altering the task. The only avenue to reducing difficulty is therefore to reduce extrinsic difficulty. The extrinsic difficulty of a task is determined by the constraints in f which must be supplied using controlled processing. The constraints in f are those needed for composition with attunements, A , in order to match C . The attunements, A , are in turn determined by S . The only way to reduce extrinsic difficulty is therefore to introduce a new but equivalent situation S' . Introducing S' , however, requires defining a translation, M between S and S' . If the task is incompletely characterized (there may be additional goals or constraints associated with the objects or relations of S) then resource consumption associated with M must be considered as well. Assuming M to require controlled processing, there exists a measure of its difficulty, $D(M)$. The difficulty of a task can therefore be reduced iff there exists an interactive metaphor, M .

$M: f \circ A \circ S \rightarrow f' \circ A' \circ S'$ such that

$D(M) + D(f') < D(f)$ if translation is required

$D(f') < D(f)$ if translation is not required

The three possibilities for the translation between a problem situation and a possible interactive metaphor are shown in the commutative diagrams below:



Here x_i are objects in the interactive situation, r and r' are the relations within the two situations, C is the problem constraints in the first situation, A is the attunements to the interactive metaphor, f is the instructed constraints for the interactive metaphor, and G and H are the mappings between the states and the problem spaces respectively of the two situations.

We will show that isomorphism between states of affairs (diagrams 1 and 3) is needed to minimize $D(M)$ the difficulty of translating between situations while isomorphism between problem spaces (diagrams 2 and 3) is needed to satisfy the definition of interactive metaphor. The useful result is that a less constrained state equivalent situation can generally be transformed into a situationally equivalent metaphor by supplying instructions while a problem space equivalent situation cannot.

These distinctions and the role of instructions in creating metaphors can be illustrated in a simple example using the 2-blocks, mouse in the maze, and 2-lights situations shown on the next page. Allowed state transitions are indicated by arrows. The basis situation for the mouse in the maze is an interactive metaphor for the 2 blocks situation because they have isomorphic problem spaces. One mapping is:

$$\begin{aligned}
 F = \{ & \text{mouse-in-left-chamber} \rightarrow \text{white-block-ontop} \\
 & \wedge \neg \text{black-block-ontop}, \\
 & \text{mouse-in-middle-chamber} \rightarrow \neg \text{white-} \\
 & \text{block-ontop} \wedge \neg \text{black-block-ontop} \\
 & \text{mouse-in-right-chamber} \rightarrow \text{black-block-} \\
 & \text{ontop} \wedge \neg \text{white-block-ontop} \}
 \end{aligned}$$

The mouse in the maze situation is not isomorphic to the two blocks problem situation, however, because they are not isomorphic in their constituents $\langle\langle r_1, \text{black-block, white-block} \rangle\rangle$ vs. $\langle\langle r_2, \text{mouse, maze} \rangle\rangle$ or $\langle\langle r_3, \text{mouse, chamber-left, chamber-middle, chamber-right} \rangle\rangle$.

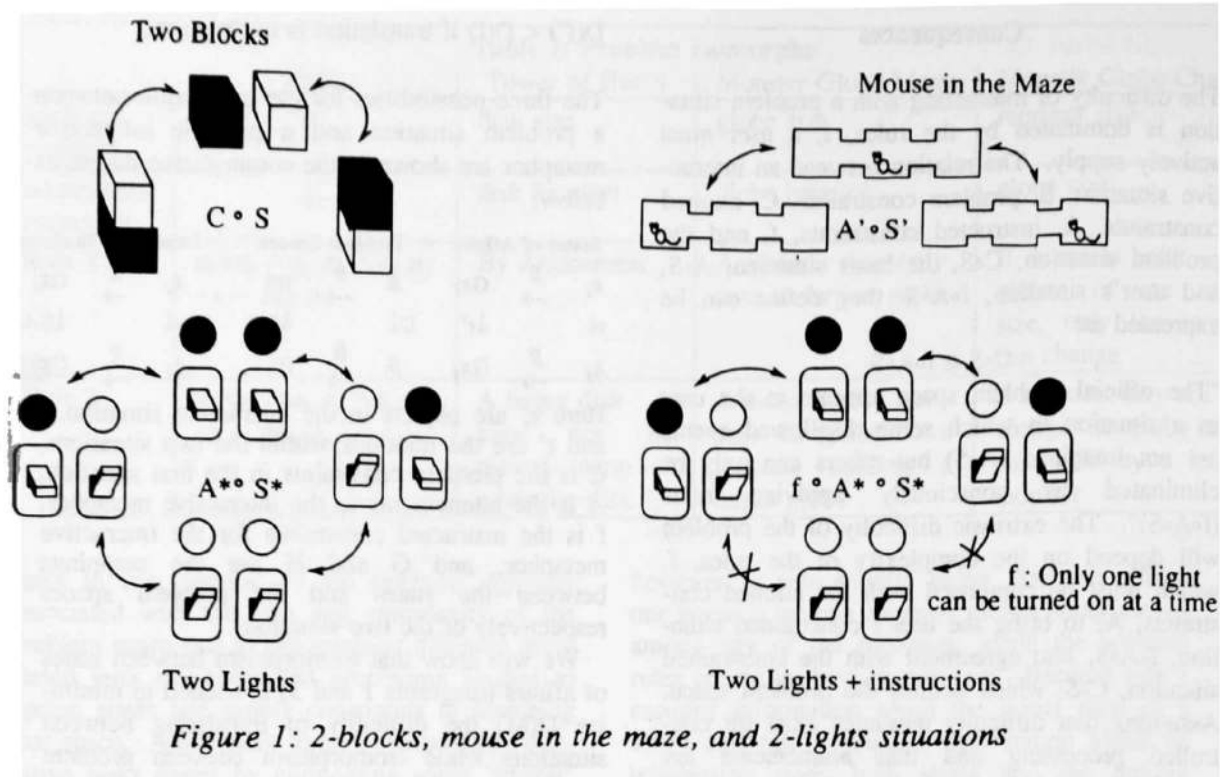


Figure 1: 2-blocks, mouse in the maze, and 2-lights situations

To verify this claim note that there is no mapping of constituents which preserves the equivalence of constraints (e.g., H is not decomposable into constituent mappings).

In its basis situation, states of the 2 lights are isomorphic to those of the 2 blocks but it is not an interactive metaphor for the two blocks situation because the 2-lights problem space includes a state in which both lights are on (or alternately both are off) while the constraints governing the 2-blocks situation prohibit both the black-block and the white block being on-top (of each other) at the same time. Although the representations appear dissimilar, by choosing an appropriate f (both lights cannot be turned on because the circuit has a limited capacity) we produce:

$G: C_{blocks} \text{ 2-blocks} \rightarrow f \circ C_{lights} \text{ 2-lights}$

which is now an isomorphic situation. To verify this claim, note that if we assign as constituent mappings:

b: black-block \rightarrow left-light,
 w: white-block \rightarrow right-light
 r: stack \rightarrow turn-on
 then

$G = b \circ w \circ r$

As an example of an additional property of equivalent situations consider a new goal in the 2-blocks situation, "move the black block". In the 2-light situation this is translated by G as turn on/off the right-light. In the mouse in the maze situation there is no unique translation under H .

This example highlights properties of the three forms of equivalence which are important for exploiting interactive metaphor.

(2) *The difficulty, $D(M)$ of translating between an interactive situation, S , and its interactive metaphor, S' is minimum if they are isomorphic in states of affairs.*

Assuming situational mental representation, M must translate the objects and relations of S to objects and relations of S' and vice versa. The number of mapping rules in M must therefore be at least as great as $|x|$ the cardinality of objects plus $|r|$ the cardinality of relations in the states of affairs of S . If S and S' are isomorphic in states of affairs then there is a one-to-one correspondence between objects and relations in the two situations and only $|x| + |r|$ rules are required. If S and S' are only problem space isomorphic then M may require up to $|S| \times (|x| + |r|)$ translations (one for each object and relation in each state of affairs). To construct such an isomorphism let S' be a situation having one of the numerals $1..|S|$ as the object in each of its states and identity as its relation. Randomly assign a state of affairs in S to each state of S' and construct a set of constraints, C' , to match C . S' is now problem space equivalent but will require a separate set of translations for objects and relations for every state. This is true for the mouse in the maze and the two blocks situation, as well, although only three states were involved. We are presently

investigating a less extreme instance of the disruption of translation by state inequivalence. Subjects required to translate between the state equivalent Tic-Tac-TOH and Monster Globe (change) problems have averaged 25.4 translations in a fifteen minute session while only 10 translations are achieved between the state inequivalent Tower of Hanoi and Monster Globe (change) problems.

(3) *If an interactive situation, S, and its metaphor, S', are state equivalent, the metaphor will reflect and preserve extrinsic information about the situation.*

If S and S' are equivalent situations then M is a full, faithful, and representative functor and must reflect and preserve goals or constraints defined over considered relations in S. The translation of the added goal between the 2-blocks and 2-lights problem is an example of this property. For constructing interactive metaphors this means that as long as they remain state equivalent they do not need to be complete.

(4) *Any Situation S' with $|S'| \geq |S|$, and containing CoS as a problem space isomorphic subcategory of A'S' can be made an interactive metaphor for S simply by supplying instructions f.*

This is trivially true. In the worst case the constraints in C can simply be transferred to f. Its specialization is more useful.

(a) *Any Situation S' with $|S'| \geq |S|$, and containing CoS as a situation isomorphic subcategory of A'S' can be made a state equivalent metaphor for S simply by supplying instructions f.*

This specialization shows that if we reverse the normal process of seeking metaphors in exact graph matches and instead simply look for situations with equivalent states we can generally bring them into agreement and enjoy the full advantages of situational equivalence simply by generating instructions.

(5) *An interactive metaphor $f \circ A \circ S'$ which minimizes the difficulty of an interactive situation, S, can be found by an algorithm A.*

This follows from the existence of a measure D of the difficulty of controlled processing and the indexing of attuned constraints by situation-types. If a metaphor preserving extrinsic constraints is desired, the criterion to be minimized is $c=D(f')$

+ $D(M)$, otherwise it is $c=D(f')$ (consequence 1). The algorithm performs an exhaustive search of the taxonomy of situation-types examining every combination of situation-type and each replication of relation-type and object role within combinations of situation-types for each problem space isomorphic subcategory of $A \circ S'$. f, M, and c are determined and the metaphor (f, S') with minimum c is retained. At conclusion, if $c' < D(f)$ the algorithm returns $C' \circ S'$, a description of an interactive graphic and f, the constraints defining a set of instructions to be supplied to a user. The description, $C' \circ S'$, is a partially determined interactive situation resembling the abstract visual objects used in scientific visualization. Conceptually the algorithm chooses S' so as to shift constraints from f where they require controlled processing to A' where they do not.

(6) *An incremental algorithm, A_m approximates the results of A by re-writing relation-types.*

A more tractable algorithm considers only those situations, S', having relations isomorphic to S as candidate metaphors. If $c= D(f') + D(M)$ only state equivalent situations are considered minimizing D(M) by consequence (4). The rewrite method conducts an incremental search for relations which when substituted into S will cause some constraint expression in the new version of C to match a constraint expression in the taxonomy of attunements. When a match is found, the relation is rewritten $S \rightarrow S'$, $C \rightarrow C'$, the constraint is marked in C', and f is updated. When no further matches can be found the method terminates returning $C'S'$ a description of the metaphor and its behavior, f, a list of instructions generated from the unmarked constraints, and M the map, $S \rightarrow S'$.

Concluding Remarks

In this paper we have examined some consequences of adopting an ecological model human cognition. Although ecological models are commonly believed to be less precise and tractable than their conventional counterparts we have shown that some aspects of cognition, such as intuition, which are difficult to quantify in conventional models can be handled under ecological assumptions. The problem space principle (that difficulty of cognitive tasks is fixed) is habitually made in cognitive science.

Without abandoning this partial truth (conservation of difficulty), we have shown that the extrinsic difficulty of tasks is not fixed and can be systematically manipulated through representation.

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