Thinking With a Mouse

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Abstract

Isomorphic problems are to cognition what optical illusions are to perception. By drawing attention to anomalies such as problems which are identical in form but vary widely in difficulty they highlight cognitive processes normally hidden among the minutiae of our theories. Results are reported from an experiment in which subjects solved a three disk Tower of Hanoi problem and its Monster Globe change isomorph using direct manipulation tableaus or paper and pencil. Subjects using direct manipulation were found to solve the Monster Globe problem in half the time taken by paper and pencil subjects. An explanation revolving around attunement to environmental constraints is advanced to account for this difference.

Introduction

Scientific visualization, direct manipulation interfaces, and other visual media rely on intuition rather than logical inference to convey meaning. The mental processes involved seem to rely more on our ability to follow the course of events in the world than to reason about them. To account for the central role intuition appears to play even in formal pursuits such as mathematics, we need to acknowledge the significance of our ability to apprehend and affect events and states of the world directly. Non-inferential cognition is often ascribed to *mental models*, but when examined more closely (Rouse & Morris 1986) the term fragments, seeming to mean all things to all people. Mental models tend to be general rather than specific. So, for example, if I understand the behavior of one pulley system, I am likely to understand the behavior of others. They often involve analogy, for example, thinking about electrical circuits as a form of fluid flow (Gentner 1983). They seem implicated in common sense reasoning and understanding of physical dynamics. They seem closely allied and integrated with other more tractable forms of cognition such as a circuit designer who might treat a circuit as both an instance of fluid flow and an electrical entity subject to Ohm's and Kirchoff's Laws. Conventionally the presence of elements of propositional inference such as mention of electrical laws is taken as evidence that a cognitive process is entirely propositional ignoring the possibility of heterogeneous reasoning involving both mental simulation and inference. The contradiction between purely propositional thought and data is revealed by isomorphic problems which are logically equivalent yet diverge widely in difficulty. Kotovsky, Hayes, and Simon (1985) propose a five prong explanation attributing effects of: rule learning, external memory, rule application, spatial memory load and real world knowledge to the accretion of sufficient influence to shift problem difficulty by orders of magnitude. According to this account data reside in the world and rules in the head. Propositions about states of affairs can be extracted from the world (external memory) and worldly (or imagined) data involving spatially adjacent elements can be extracted more easily (spatial memory load). The learning of rules for operating on these data (rule learning, rule application) then determines problem difficulty. Real world knowledge serves as an error term, accounting for differences not conveniently or completely explained by the other factors. Ecological explanations of isomorph differences (Zhang, 1990, Lewis & Toth, 1992) switch the emphasis to real world knowledge, and partition "rules" into those which reside in the head, and those which reside "in the world" (are cognitively inseparable from the representation). An example of what Zhang refers to as external constraints would be the Tower of Hanoi "rule" prohibiting moving a disk from the bottom of a stack. This constraint is *external* in the sense that it exists for any person interacting with this type of physical situation independent of explicit rules, goals, or problem setting. Another

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example would be the rules of chess prohibiting players from changing pieces' size, shape, or color. These external constraints, referred to as *nomic attunements* by Barwise and Perry (1983) are notable in that they go universally unnoticed yet provide the lion's share of constraint in most of the situations we encounter. In this scheme, a mental model is simply a cognitive situation constrained by nomic attunements. The ecological account attributes isomorph differences to rule learning and rule application just as Kotovsky et al do, but only for such rules as needed to supplement the mental model. The present experiment grew out of a study testing these ecological hypotheses. In our experiments subjects solve problems by manipulating tableaus on a workstation rather than using paper and pencil under "think aloud" instructions. We were surprised to find that our subjects had substantially less difference between solution times for easy and hard problems than reported elsewhere (Hayes and Simon, 1977, Kotovsky, Hayes, & Simon, 1985). This experiment supplements these data with two additional control conditions designed to eliminate the possibility that the observed speed-up for difficult problems was due to either a methodological artifact or differences in subject populations.

Tower of Hanoi and its Isomorphs

The classic description of the Tower of Hanoi and other problems (Newell & Simon 1972) characterizes a problem as a pair of states (initial and goal), a set of operators for transforming states, and a set of rules governing the application of these operators. The problem solving task is depicted as a problem space in which states are the vertices and edges are (legally applied) operators. Problem solving is modeled as a search of this graph to identify some sequence of operators (edges) which transforms the initial state into the goal state. The model predicts that subjects will find problems with larger spaces to search, more distant goals, or more choices to make, more difficult. It ignores, however, cognitive difficulties which may arise in traversing even relatively simple spaces. In the three disk Tower-of-Hanoi problem disks of three sizes are moved between three pegs. A problem rule specifies that a larger disk cannot be moved on top of a smaller one. Subjects are presented with the task of reaching a goal state from an initial state by determining a sequence of moves. In the Monster-Globe (change) problem disks are replaced by monsters and pegs by globes. Monsters are allowed to change the sizes of their globes according to rules of etiquette which require that if two monsters have the same sized globe only the larger monster is allowed to change (only the disk at the top of a stack can be moved) and a smaller monster cannot change its globe to a size held by a larger monster (a larger disk cannot be placed on a smaller one). Subjects find Monster-Globe problems much more difficult. Hayes and Simon (1977), for example, report differences in average solution times of less than two minutes for the three disk Tower of Hanoi problem, and half an hour for the corresponding Monster-Globe (change) problem. The Tower of Hanoi problem involves a potentially larger problem space because it distinguishes the ordering of disks on pegs. This results in an unconstrainted space of 60 states and 468 transitions. The unconstrained MG space, by contrast, has only 27 states and 81 transitions. The Tower of Hanoi situation however simplifies its space in two ways. Human attunements to situations involving the movement of physical objects prevents us from imagining that disks might dematerialize from their pegs to reappear at arbitrary locations on other pegs. This nomic constraint simplifies the space to 108 transitions and 60 states. Because the presence of a larger disk on top of a smaller disk is a fact which can be determined by inspecting the involved peg, compliance of an action with the second rule can also be determined without additional processing. These are effects associated with rule application and spatial memory load in the the Kotovsky et al taxonomy. This reduces the problem to a search of a space of 50 states and 75 transitions in which each of the 36 prohibited moves can be immediately ruled out by inspection at an illegal path length of 1. The Monster-Globe problem space has a 27 states and 81 transitions. Although it has 23 fewer states and only 6 more transitions it is much more difficult. We attribute this difficulty to the absence of a viable mental model. We assume that situation-types involving tangible objects have available some very general attunements which apply to physical situations including such things as:

- object constancy- physical objects and their properties remain constant
- movement- an object can be moved from one spatial location to another and by nomic attunement, in its new state it will be located at the second spatial location and not be located at the first spatial location.
- extension- two objects cannot occupy the same space at the same time Problems such as

Monster-Globe (change) cannot be supported by mental models because they violate object constancy, a basic attunement which plays a primary role in theories of psychology ranging from cognitive development to perception. The movement of disks or other objects, by contrast, is supported by a mental model because their situations anchor object properties (object constancy) and obey our attunements to constraints affecting movement. As a consequence, we can imagine moving a disk from one peg to another and the states which result without effort but cannot do the same for situations which violate these attunements. We hypothesize that for difficult problems such as Monster Globe (change) the easily manipulated tableau supplies an external mechanism for updating states. Because subjects are no longer required to imagine "unimaginable" events, the problem space can be searched more rapidly.

Method

Displays

The Tower of Hanoi and Monster-Globe (change) problems were implemented as interactive tableaus in the X window system. Subjects interacted with problem tableaus by manipulating problem objects directly. The interface did not itself constrain the problem. In the TOH tableau, for instance, the subject could perform any action which resulted in a single change in the problem state even if this involved an "illegality" such as moving a disk from the bottom of a stack. Problem tableaus were driven by a control program which tracked states and errors, terminating the program only after a legal sequence of moves from the initial state to the goal state had been observed, and the "finish" button pushed. Buttons for viewing problem rules, and task instructions were also available. The display for the Monster Globe (change) problem is shown in Figure 1.

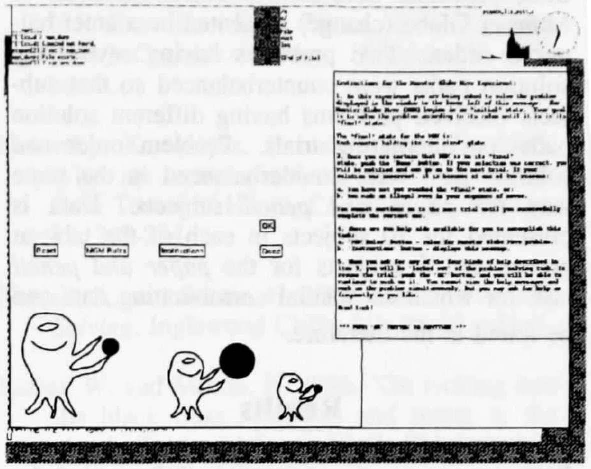

Figure 1. Problem tableau

Experimental Conditions

In computer tableau conditions subjects were presented with the problem's cover story and allowed to interact with the tableau until the problem was solved. In the *reset to last legal* state condition a subject's tableau was reset to the last legal state reached prior to the first error when the "finish" button was pressed in the goal state. If the goal state had not been reached when the "finish" button was pressed a message to that effect was displayed. Because this policy might have allowed subjects to "solve" the problem by ratcheting between legal states, rather than completing an integrated sequence of actions, a second *reset to initial* state condition was run. In the reset to initial condition tableaus were reset to their initial rather than last legal state. Under these circumstances it is assured that each subject successfully reaching completion has performed the full sequence of actions needed to solve the problem. Subjects in the third, *paper and pencil* condition, received "think-aloud" instructions. Completion times were determined from tape recordings of their verbal protocols and validity of final solution path was verified by an experimenter present in the room. Counts and classifications of moves were based jointly on the verbal protocols and accompanying sketches obtained from each subject in this condition.

Procedure

Subjects were 26 University of Pittsburgh undergraduate students who participated for pay. Subjects in the *reset to last legal* condition were part of a control group of an experiment conducted in the Spring of 1992 and reported in Lewis and Toth (1992). Subjects in the *reset to initial* and *paper and pencil* conditions participated in January of this year. Subjects in the computer tableau conditions were presented with up to five problems in an experimental session which was terminated by either completion of the problem set or expiration of an hour. The first two of these problems were always Tower of Hanoi or Monster Globe (change) presented in counter balanced order. Two problems having seven step solution paths were counterbalanced so that subjects received problems having different solution paths on alternating trials. Problem order and solution path were counterbalanced in the same way for *paper and pencil* subjects. Data is presented for 10 subjects in each of the tableau groups and 6 subjects for the *paper and pencil* task for which substantial corroborating data can be found in the literature.

Results

Differences were found among solution times for problems (p < .001) and for experimental conditions (p < .02). No difference was found between solution times for the two tableau conditions, although it is apparent from Figure 2 that differences present are in the direction predicted by the ratcheting hypothesis. Differences were also found in the number of moves preceding solution for problem (p < .005) and for experimental conditions (p < .06). Again, no difference was found between tableau conditions although as Figure 3 shows subjects who reset to the initial state made somewhat more moves. Figure 4 shows the distribution of solution times for tableau subjects solving the Monster-Glove change problem. Although there is a strong mode at 20-26 minutes, the distribution is highly skewed with most subjects achieving relatively rapid solutions (< 15 min) and two requiring almost the entire session to solve the problem.

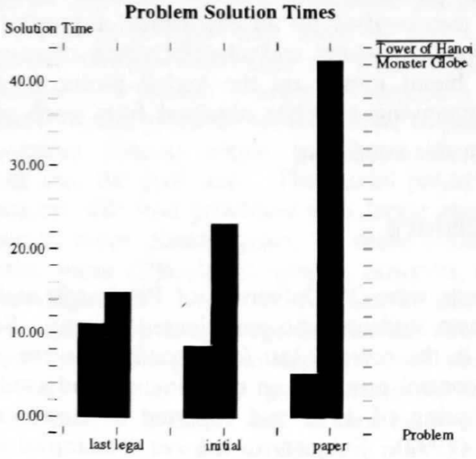

Figure 2. Solution times

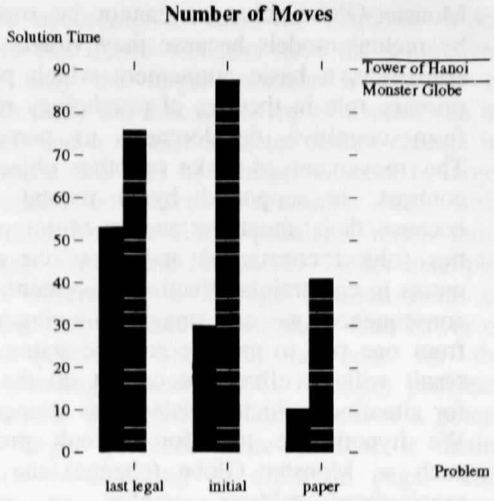

Figure 3. Number of moves

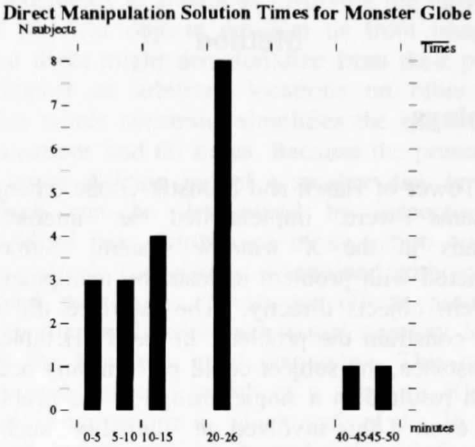

Figure 4. Solution times for tableau

Discussion

On first blush, the average solution time of subjects in the *reset to initial* condition, 23 minutes, appears roughly comparable to the 19 minute average solution time reported by Kotovsky, et al. for Monster-Globe (change) subjects solving the problem using a physical model. Although our sample of six *paper and pencil* subjects is too small to draw strong conclusions, the difference between their solution times and those of Kotovsky et al.'s control group in their experiment (42 vs. 14 min) is large enough to suggest differences in subject populations. Our *paper* *and pencil* subjects' times more closely resemble the 29 minute solutions reported by Hayes and Simon (1977). Within our experiment subjects interacting with the Monster Globe (change) problem through tableaus showed dramatic improvements in performance relative to their cohort deprived of a direct manipulation model of the problem. As shown in Figures 2 and 3 these advantages did not extend to the Tower of Hanoi where longer solution times and a greater number of moves were associated with the use of tableaus. In both problems direct manipulation let to more exploration with more than twice as many moves examined than in the *paper and pencil* condition. This bias toward exploration holds even for the easy Tower of Hanoi problem in which *paper and pencil* subjects were near the optimal seven move sequence with 10.5 moves but tableau subjects averaged over 40. In contrast to Kotovsky et al.'s physical monsters with pneumatic balloons which reduced the number of moves, the relative ease of the tableau's point and click interface served to increase them dramatically. These results lend support to our thesis that interaction with the environment plays an active role in cognition. In Kotovsky et al.'s experiment where operation of valves to inflate and deflate balloons interposed substantial distance between intention, action, and effect, the presence of an external model did little to aid untrained subjects. We argue that this not because the monsters were failing as a memory aid but because lack of memory is not what makes this problem hard. Because the problem's dynamics violate attunements involving object constancy the problem does not support a mental model. As a consequence it is not the state of the problem that is hard to remember but its possible states and their relation to actions. The direct manipulation tableaus help bridge this gap by providing an external model which is easy to manipulate and observe. As a consequence, subjects need not contemplate moves or ruminate about their effects. Instead, they simply click, observe, and retract their moves to explore the space much as they might a model in their imagination.

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