An Empirical Investigation Of Law Encoding Diagrams For Instruction

Peter C-H. Cheng
ESRC Centre for Research in Development, Instruction and Training
Department of Psychology, University of Nottingham
University Park, Nottingham NG7 2RD, United Kingdom
pcc@psyc.nott.ac.uk

Abstract

Law Encoding Diagrams, LEDs, are knowledge representations that correctly encode systems of one or more laws using the geometric and/or the topological structure of diagrams. In an instructional role, LEDs aim to focus learning on the formal relations defined by the correct laws, whilst using diagrammatic representations to aid comprehension. LEDs can be viewed as intermediate representations that aim to bridge the conceptual gulf between abstract laws and the behaviour of phenomena. It is anticipated LEDs will be adopted as key models in the foundation of expertise. This paper describes an investigation in which LEDs for momentum and energy conservation were used for instruction. The LEDs were implemented in a computer based discovery learning environment and the subjects given only minimal instruction on their use in problem solving. However, half the subjects used the LEDs for successful post-test solutions of different classes of problem and exhibited strategies that were expert-like, in marked contrast to their novice-like pre-test performance.

Introduction

Law Encoding Diagrams, LEDs, are knowledge representations that correctly encode the underlying relations of a law, or a system of simultaneous laws, using the geometric and/or the topological structure of diagrams (Cheng, 1993, 1994a). The basic idea of this novel concept is to exploit the benefits that diagrams confer on the making of inferences, whilst ensuring the representations formally capture the relations defined by the laws. There are potential computational advantages of using diagrammatic and externalised representations of information in problem solving (Larkin & Simon, 1987; Larkin, 1989). By formally encoding the laws in the structure of diagrams, legal manipulations of LEDs will always result in correct inferences, consistent with the underlying laws. Different uses of LEDs are possible. For example, Cheng and Simon (1992, and in press) describe the use of LEDs in the induction of scientific laws. Cheng (1993, 1994a, and forthcoming) discusses the use of LEDs for instruction in science, engineering and mathematics. Other uses of LEDs are also being explored.

This paper deals with LEDs in instruction by investigating how problem solving performance is affected by the experience of LEDs in a computer based discovery learning environment. This is the first formal empirical evaluation of LEDs. The instructional domain was the collision of elastic bodies. The investigation provides evidence that LEDs are likely to be effective for instruction. The following section presents the two LEDs used in the investigation and also describes their implementation in the ReMIS-CL computer based discovery learning environment. The next section considers possible types of reasoning with LEDs and their role in instruction. The procedure of the investigation is then outlined and the outcomes described. There is, finally, a discussion of how the findings provide encouraging support for the effectiveness of LEDs in instruction, followed by some general considerations in the conclusion.

LEDs for Elastic Collisions

A Law Encoding Diagram, LED, is a representation that correctly encodes the underlying relations of a law, or a system of simultaneous laws, in the structure of a diagram by the means of geometric and topological constraints. For example, a LED for a law conventionally stated as an algebraic formula may use diagrammatic elements, such as lines and shapes, to stand for variables. The relations defined by the formula would be captured by geometric and topological relations, such as congruence, adjacency and symmetry. The difference between LEDs and other diagrammatic representations, such as Cartesian graphs, will become clearer as the properties of LEDs are discussed below. LEDs for a particular domain are considered here.

Perfectly elastic collisions are important in physics, because both momentum and energy conservation are involved. The class of collisions dealt with here are impacts between two bodies (balls) travelling in a straight line. The collisions are governed by the laws of momentum conservation and energy conservation (see Cheng, forthcoming, for descriptions of conventional problem solving strategies in this domain).

Two LEDs that encode the laws for these elastic collisions are: the *one-dimensional property diagram* and the *velocity-velocity graph*. The one-dimensional property diagram (1DP diagram) uses lines to represent magnitudes of velocities and masses of the bodies. Three examples are shown in Figure 1, each depicts a single collision. The lines *U1* and *U2* are the velocities of the two balls before impact; in Figure 1a they happen to be approaching from different directions but with equal speeds. *V1* and *V2* are the velocities of the bodies after collision; in Figure 1a the two bodies are moving apart with equal speeds. Masses lines,

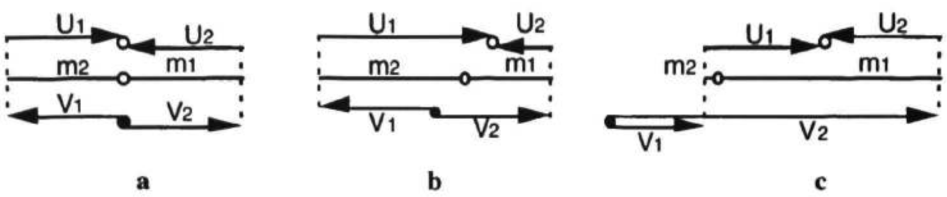

Figure 1: One-dimensional property diagrams.

$m1$ and $m2$, are drawn equidistant between the $U1$—$U2$ and $V1$—$V2$ lines. The ratio of the lengths of $m1$ and $m2$ is equal to the ratio of the masses of the two balls; in Figure 1a they are equal.

LEDs utilise the geometric and topological structure of their diagrams to encode laws in the form of diagrammatic constraints. The constraints are, in effect, the rules for the legal manipulation of a LED, and must all be satisfied for the LED to be correct. There are *elementary constraints*, such as the lengths of lines being in proportion to the magnitudes of variables, and more complex *law-encoding constraints*. The 1DP diagram has three of the latter: (i) the tail ends of the arrows for the initial velocities and the points of the corresponding final velocity arrows must be in line vertically, making the total length of the $U1$—$U2$ line equal to that of the $V1$—$V2$ line; (ii) the total length of the mass line equals the length of the velocity lines; and, (iii) the ends of the lines not previously fixed in (i) and (ii), indicated by the small circles, must lie on a straight vertical or diagonal line. The constraints in this LED are relatively simple.

The velocity-velocity graph (V-V graph for short) plots the velocities of the two bodies on separate axes. Two examples are shown in Figure 2; they depict the same collisions as Figure 1a and 1b and the symbols designate the same variables. The arrows in the graph show the velocities before and after collision. The masses are given by the lengths of the labelled lines in the triangle at the bottom left of each V-V graph.

The straight diagonal lines are constant momentum contours and the circle or ellipse are constant energy contours. There are 3 law encoding constraints. (i) The momentum contour line passes through the points for initial and final velocities, as indicated by the small circles, and is parallel to hypotenuse of the mass triangle. (ii) The centre of the energy circle or ellipse is at the origin of the graph and also passes through the points for initial and final velocities. The intersections of the contours give the solutions to the two conservation laws. (iii) The eccentricity of the ellipse is given by the square-root of the ratio masses; $\sqrt{(m_1/m_2)}$. The law encoding constraints of this LED are more complex than those for the 1DP diagram.

The V-V graph and the 1DP diagram are two (of four) LEDs implemented in the ReMIS-CL computer based discovery learning environment (Cheng, 1993). In any LED the magnitudes of variables can be changed by direct on screen manipulation of the diagrammatic elements representing them. The system automatically updates the manipulated LED and any others that are displayed. In the present investigation the 1DP diagram and the V-V graph were always displayed and the subjects were free to manipulate either at any time. ReMIS-CL also includes an animated simulation of the current collision, which is played at the users discretion.

Pedagogy and Problem Solving

One of the foundational ideas of LEDs is that an appropriate focus for learning in mathematics, engineering and science is with correct formal laws. Comprehending the laws provides the learner with possibility of correct, precise and general understanding of the concepts being taught. The diagrammatic nature of LEDs aims to make the laws easier

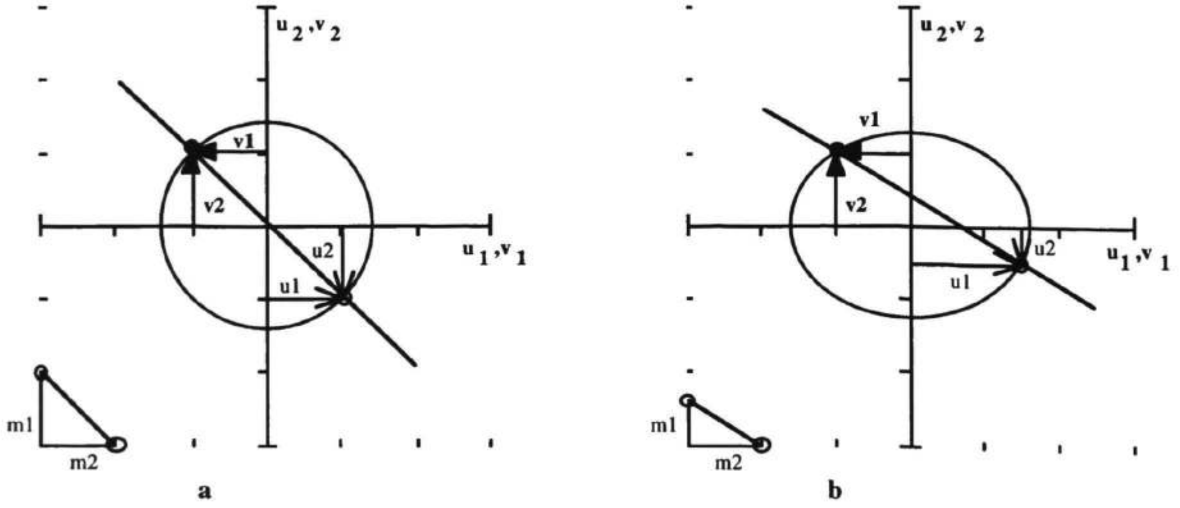

Figure 2: Two Examples of Velocity-Velocity Graphs

to comprehend than under conventional representations. A desirable prospect is for learners to adopt LEDs as their basic conceptions in particular domains, supplanting existing knowledge that is incorrect or incomplete, but supplementing existing formal knowledge with useful (externalised) mental models for problem solving.

One way to view LEDs is in terms of White's (1989, 1993) conceptualisation of the instructional role of *intermediate representations.* Intermediate representations bridge the conceptual gulf between abstract mathematical laws and the behaviour of phenomena. White proposes the use of *intermediate causal models* that represent causal and temporal relations important for explaining phenomena. LEDs differ by emphasising the underlying formal relations of the laws they encode. Thus, learning with LEDs comprises understanding classes of relations and interrelations at different levels of abstraction. There are three classes[1]: (i) relations between LEDs and the abstract formal laws (e.g., the constant energy contour's elliptical shape, in the V-V graph, is a direct consequence of the of the squared velocity terms in the energy conservation law); (ii) relations between LEDs and the behaviour of phenomenon (e.g., the 1DP diagram and V-V graph depict single collisions); and, (iii) relations among different LEDs (e.g., how is the structure of the 1DP diagram reflected in the V-V graph?).

A potential of LEDs is to make learners reason more like experts, by providing them with a range of different models suitable for solving different classes of problems quickly and efficiently. The basis for LEDs in instruction is discussed in more fully in Cheng (1994a). The purpose of the present investigation was to find empirical support for these claims.

What forms of reasoning can be conducted with LEDs? The key point to remember is that they encode laws. So, the forms of reasoning that can be conducted under conventional representations are likely to have a diagrammatic equivalent. (1) Quantitative reasoning can be performed by changing the size of diagrammatic elements representing the magnitudes of variables. For example, when the length of a velocity line is doubled in the 1DP diagram, the magnitude is doubled. (2) One form of qualitative reasoning with LEDs involves gradually changing the size of one diagrammatic element and watching how the rest of the diagram changes. (3) Configurational reasoning is qualitative reasoning about different phenomenon states, distinguished by single variables being positive, negative, or zero, and pairs of variables being equal or unequal. (4) Extreme case reasoning considers what happens when one or more of the variables tends to some limiting value, such as infinity or zero. (5) LEDs permits constraint based reasoning, in which the value of any variable can be determined when the others are known, irrespective of the which variables were independent or dependent in the physical situation. For instance, the ratio of masses can be found using the 1DP diagram given both pairs of velocities. (6) LEDs naturally support reasoning at the conceptual level; for example, the two contours in V-V graph shows that two laws govern the collisions. Important properties of LEDs for more complex problems are their compositionality and extendibility. (7) Composite diagrams can be constructed by joining together LEDs; for instance to explain complex series of collisions (e.g. Newton's Cradle). (8) LEDs can be formally modified to cover extensions of the encoded laws; for example the 1DP diagram and the V-V diagram can be extended to cover inelastic collisions. (9) LEDs can also be used for debugging and trouble shooting. Cheng (1994a) discusses these forms of reasoning more fully and gives detailed examples. The current investigation provides evidence that subjects can quickly learn to solve classes of single collision problems with LEDs.

The Procedure

The investigation was carried out with a total of six undergraduate and postgraduate physics and engineering students. Four were doing PhDs and three had first class honours degrees. They are designated by initials: NT, NG, TI, QC, IP, SP. Each subject did: (i) a pre-test; (ii) a discovery learning session using the ReMIS-CL system, following the pre-test after a short break; and (iii) a post-test following approximately a fortnight later. Each part is described in turn.

The pre-test was designed to assess the subjects' conceptual knowledge about the collisions and to examine their problem solving methods. The first part of the pre-test were questions about relevant aspects of the subjects' education and asked what laws or relations governed the collisions. The second, and main part, of the pre-test consisted of problems about particular collisions. Before attempting the problems the subjects were given a description of perfectly elastic collisions, including the statement that energy is conserved. There was a total of 14 problems, which fall into three groups (P1 to P3). (P1) These were six qualitative questions about the outcomes of single collisions under different conditions, designed to examine subjects' knowledge of different configurations of collisions. (P2) These were two problems dealing with extreme cases, by presenting series of collisions in which one variable was increased over successive collisions. (P3) These were six quantitative questions in which the values of different pairs of variables had to be calculated. In different problems, initial velocities, final velocities and masses were the unknowns. This paper will concentrate on problem classes P1 and P3, only.

The questions were printed sheets, on which the subjects also wrote their answers and did their working out. They were asked to think aloud and video recordings were made throughout.

The instructional session on LEDs was based around the ReMIS-CL discovery learning environment, and was divided in to three parts. (i) The subjects first learned how to use the program by following printed instructions describing how to run the simulation and to manipulate the

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[1] No pedagogical precedence is implied by the order.

LEDs. The instructions contained superficial descriptions of the law-encoding constraints of each LED. No instruction was given on problem solving strategies with LEDs, a significant point. (ii) There then followed a free-investigation period of up to 15 minutes, during which the subjects were free to try things out on the system. (iii) The final and longest part of the session had the subjects use the program to answer nine ordered questions about elastic collisions. The questions were more open-ended than the pre-test problems, asking about relations rather than outcomes of particular collisions. The aim was to encourage the subjects to discover relations among the variables, other than the conservation laws. The program logged all the inputs made by the subjects and video recordings were made whilst the subjects thought aloud. Each session typically lasted 90 minutes.

The post-tests were similar to the pre-test, but had minor changes. The subjects were initially asked what they thought they had previously learned and whether they could reproduce the LEDs. The subjects were then shown a picture of the LEDs, which they kept for the remainder of the test. Finally, the subjects worked on collisions problems equivalent to those of the pre-test but with superficial alterations. The subjects were free to choose what ever means they wished to solve the problems.

LEDs In Problem Solving

The subject performance in the three parts of the experiment will be described in turn. The subjects naturally fall into two groups, of three, according to various measures of their behaviour and performance in the computer based trial and the post-test. The groups will be called *LED-users* (QC, SP, IP) and *conventional methods users*. (NT, NG, TI). Reference to the groups will be made during the consideration of the three parts of the experiment.

In the pre-test, all the subjects knew the correct definitions of momentum and energy, but two (NT and IP) did not initially recall that energy conservation was required for perfectly elastic collisions. Two main approaches were used by all the subjects for problem solving in the pre-test, which will be called the *informal* and the *formula* based methods. The formula based method involved written solutions using the correct the algebraic forms of the conservation laws. Subjects typically attempted to derive new formulas for the unknown variables from the given equations. They were mostly successful in problems that required only one of the two laws to be manipulated for the solution. However, when both laws had to be considered, to find final velocities given the initial velocities and the masses, all the subjects failed to successfully complete the necessary algebraic manipulations (even those with first class honours degrees). Using the formula based method, some did two pages of algebraic manipulations and calculations, to no avail. As a result, some subjects resorted to crude trial and error substitution of values into the equations, in vain attempts at making them balance. The informal method is a broad category of the other forms of reasoning that subjects employed, which included use of intuitions and experiences of collisions and vague unsystematic theoretical considerations. The two methods were easily distinguished. The formula based methods were mostly used on the quantitative problems and the informal methods mainly used for the qualitative problems. In summary, pre-test problem solving strategies were only partially effective and can be described as relatively novice-like.

In the first part of the computer based trial, subjects had no difficulty learning how the system worked by following the written instructions. The subjects were effective at solving qualitative and quantitative problems with the system. All the subjects were able to successfully obtain answers to quantitative questions, which they had failed to find in the pre-test (with one exception, caused by a trivial error). The subjects discovered simple quantitative relations, which they did not previously know; for example, when the masses of the balls are equal, the final velocity of each ball equals the velocity of the other ball prior to impact. There was a general preference for the 1DP diagram. During the free-investigation and problem solving periods, the mean proportion of operations performed with the 1DP diagram active, were 68% and 87%, respectively.

The first distinguishing feature of the LED-users in contrast to the conventional-methods group is the LED-users' greater inclination to explore. Four measures were good indicators of this tendency. (i) The LED-users spent more time on the system during the free-investigation period, in absolute terms and as a ratio of the total time on the computer based trial. (ii) They performed more operations during the free-investigation period. During the instruction and free-investigation periods, they examined (iii) a greater variety of types, or configurations, of collisions and (iv) considered a greater range of the ratios of the masses. During the problem solving period, the LED-users and the conventional methods groups did not differ on these measures.

The first part of the post-test asked the subjects to reproduce the LEDs. The sketches that subjects made were scored on a lists features important to the diagrammatic constraints of the two LEDs. The LED-users group reproduced between 50 and 56% of the features, whereas the conventional methods group reproduced between 19 and 25%. The conventional methods-group members were all unable to draw any relevant parts of the 1DP diagram. The particular LED chosen for problem solving, by each LED-user, was the one for which they had recalled the most features; SP and QC preferred the 1DP diagram and IP preferred the V-V graph.

The post-test showed small, not statistically significant, improvements in the qualitative problem scores over the, already, reasonable pre-test scores. Similarly, quantitative problem scores were slightly higher than the pre-test scores. This overall increase can be mostly attributed to SP and QC. They drew correct 1DP diagrams for all six quantitative problems (P3). The conventional methods users showed little improvement, and used formula or informal approaches in the post-test. IP did poorly in most attempts at using the V-V graph, as will be discussed below.

LEDs were found in 50 to 79% of the solutions of LED-

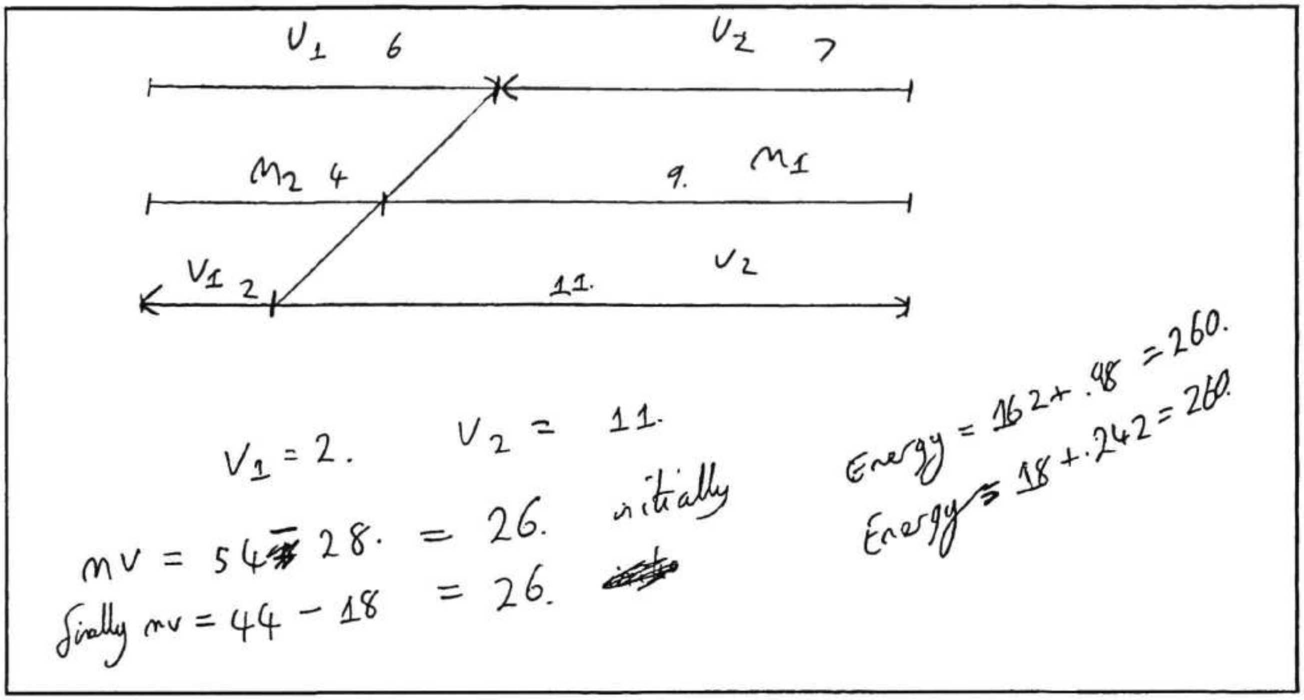

Figure 3: SP's Solution to a quantitative problem.

users and were distributed over every problem category. The majority of LEDs solutions were in the form of sketches or scale drawings. The exceptions were QC's correct solutions to the qualitative problems, in which QC imagined how the 1DP diagram would appear in different collision configurations. All the subjects checked some of their quantitative answers, by substituting values they had obtained from the LEDs into the correct laws, to determine whether the quantities were conserved. For example, consider SP and QC's use of the 1DP diagram to solve a quantitative problem. The problem was to find the final velocities, v_1 and v_2, given the initial velocities were u_1=6 and u_2=-7, and the masses were m_1=9 and m_2=4. None of the subjects solved the problem correctly in the pre-test. In the post-test, SP and QC successfully solved the problem by drawing 1DP diagrams and then double checked the answers with quick calculations. Figure 3 shows SP's solution; at the top is SP's 1DP diagram and below the initial and final quantities of momentum and energy were computed and found to be correctly conserved.

IP used the V-V graph, but was not as successful as QC or SP. In four of the quantitative questions, IP did not use, or failed to use it successfully. These were all problems involving unequal masses that required the application of the law-encoding constraint relating the eccentricity of the energy ellipse to the square-root of the ratio of the masses. Similarly, in one qualitative problem, IP drew a single energy ellipse but considered three different ratios of masses, so found three outcomes when only one is possible. The need to correctly apply this difficult law-encoding constraint, and the fact that the subjects were not told explicitly about the constraints, is a plausible explanation for IP's failures. This interpretation is consistent with the view that the ease of use of LEDs is related to the complexity of the law-encoding constraints required in a particular problem.

The ability of the LED users to reproduce the features implies that they comprehended the diagrammatic constraints of the LEDs. The constraints seem fundamental to problem solving with LEDs, so the correlation between good recall and their subsequent use in problem solving is interesting.

More details of the experiment and the subjects performance can be found in Cheng and Serpell (1994).

Discussion

For at least this one domain some positive conclusions can be drawn about the instructional role of LEDs. LEDs can be an effective basis for qualitative and quantitative problem solving. With no instruction on problem solving strategies and a minimal description of the law-encoding constraints, some learners were able to successfully solve qualitative and quantitative problems. Given a free choice, half the subjects used LEDs for post-test problem solving, abandoning the formula manipulation methods that were dominant in their scientific education. This demonstrates a willingness of the subjects to adopt LEDs as the basis for problem solving, even though the exposure to them was brief.

LED-users can be considered as moving towards more expert-like reasoning away from the crude and ineffective novice-like reasoning in the pre-test. The earlier laborious computations were replaced by drawings of LEDs in the post-test, sometimes followed by simple calculations using the conservation laws to cross check the values found. The LED-users were choosing to use an efficient approach to solve the given problems and then double checking the solutions obtained by another independent means.

The idea of the diagrammatic constraints seems to be a useful theoretical concept for understanding the use of LEDs. The number of structural features or diagrammatic constraints that subjects successfully reproduced was one of the important characteristics of the LED-users group. The complexity of the constraints explains (perhaps determines) the relative ease of problem solving and usefulness of different LEDs. The 1DP diagram is the better of the two LEDs in this respect. It has the simplest constraints and the subjects using it here were more successful. V-V graph may, however, be more useful for high level conceptual reasoning (see Cheng, 1994a).

A possible explanation why the conventional-methods group did not use the LEDs, in the post-test, is that they did not have a firm grasp of the elementary and law-encoding constraints. This is shown by their poor reproductions of the LEDs at the beginning of the post-test. The main distinguishing feature of LED-users and conventional-methods group, in the computer based trail, was the latter's lack of independent exploration with the system during the free-investigation period of the computer based trial. The variety of possible collisions they considered was smaller. They would have seen fewer of the patterns that each LED can legally assume and thus would have had less opportunity to appreciate the diagrammatic constraints underlying those patterns. An implication for instruction with LEDs is that the elementary and law-encoding constraints of the diagrams should be made explicit to the learner, perhaps as a set of rules.

Conclusions

The investigation has provided some encouraging evidence that LEDs are effective for problem solving and that they may be effective for instruction. The research on LEDs for instruction is continuing, with the results of a similar investigation conducted with non science based students being currently analysed.

Although this paper has consider LEDs for elastic collisions, other systems of LED are being developed and some already exist. For example, Cheng (1994b) describes LEDs that have been developed for probability theory and Bayes theorem and Cheng (1994c) considers the role of LEDs in scientific discovery, with examples from the history of science.

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