Array Representations for Model-Based Spatial Reasoning

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Abstract

To date, the major focus of research in knowledge representations for artificial intelligence has been on sentential or linguistic formalisms involving logic and rule-based reasoning. There is a growing body of evidence suggesting, however, that much of human problem solving is achieved, not through the application of rules of inference, but rather through the manipulation of mental models. Such a model is represented by a system with a similar relational structure to the reality it represents. Moreover, spatial reasoning with models involves the inspection and transformation of representations in ways that are analogous to visually inspecting and physically transforming entities in the world. Since a crucial component of knowledge acquisition is to capture an expert's mental state and reasoning strategies, it is important to shift some of the attention of AI research to the study of representation techniques that correspond to the mental models used by humans. The paper begins with a cognitive perspective on model-based reasoning. A knowledge representation scheme for spatial reasoning with models is then presented. In this scheme, which has evolved from research in computational imagery, spatial models are represented as symbolic arrays where dimensions of the array correspond to transitive order relations among entities.

Introduction

Contemplate the planning problem of rearranging the furniture in your living room. One approach to solving this problem is to physically move the furniture about the room to evaluate the alternative arrangements. A less backwrenching approach is to mentally visualize and analyze the various possibilities. Cognitive psychologists have acknowledged that mental models are fundamental to human problem solving, particularly for their predictive and explanatory power in understanding human interactions with the environment and with others (Stevens & Gentner 1983). These models correspond to internalized representations that can be mentally inspected and transformed. Mental models can also be applied metaphorically in problem solving; a heuristic cited by orators is to consider a speech as a voyage through a building where objects along the way act as cues to the next topic. Although some mental models may be specialized and require training to develop (e.g. models for reasoning about the physical behavior of complex mechanical devices), others are more accessible and correspond to everyday problem solving (e.g. a mental map for planning a route from your bedroom to the refrigerator).

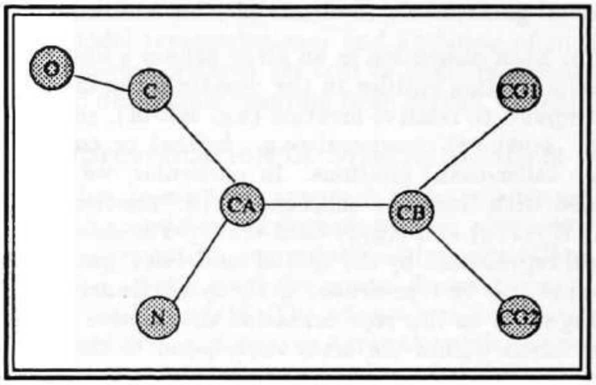


Figure 1: Model of a molecular structure

Johnson-Laird (1983) describes several types of mental models. The first, and most fundamental, is a *relational* model, which is a static frame consisting of tokens that represent entities in the world and a set of relations that define the physical relationships among entities. A *spatial model* is a relational model in which the relations of interest are spatial in nature; tokens are located within a symbolic multi-dimensional space. For example, the graphical depiction in Figure 1 could be considered as a spatial model of a molecular structure: each entity (atomic part) within the structure is represented as a symbolic token (node in the graph) and structural relations among entities (relative location and bonding) are represented using spatial dimensions and links. This model is neither complete or totally accurate; knowledge about the bond lengths and angles, and the relative size of atoms is not captured. However, it explicitly depicts information that can be used for reasoning about molecular structure and interactions, while discarding irrelevant details.

Just as mental models are pervasive to human problem solving, computational models for spatial reasoning provide a foundation for problem solving in AI. This paper is concerned with the development of a computational methodology for spatial reasoning with models.

A knowledge representation scheme is presented in which symbolic array data structures are used, in conjunction with imagery inspection and transformation operations, to reason about the spatial properties of a do-

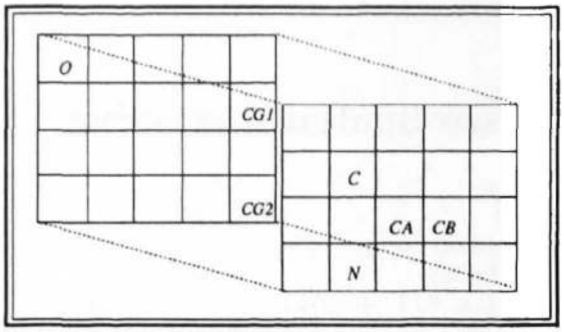


Figure 2: Array model of a molecular structure

main. Each dimension in an array defines a linear-order relation among entities in the domain. The order may correspond to relative location (e.g. left-of), geographic (e.g. north-of), temporal (e.g. before) or conceptual (e.g. taller-than) relations. In particular, we are concerned with transitive relations — i.e. relations r such that if $r(x, y)$ and $r(y, z)$ then $r(x, z)$. For example, the world represented by the spatial model in Figure 1 could alternatively be represented as the symbolic array model in Figure 2. In this representation the relative locations of symbols within the array correspond to the relative locations of atoms within the molecular structure. The relations *left-of, right-of, above,* etc. in the array map to the corresponding relations in the world. Topological relations, such as bonded-to, contained-in, touching, etc., can also be represented in an array. In the above example, adjacency in the array corresponds to bonding between atoms in the molecular structure. Reasoning at varying levels of abstraction is achieved in the scheme by allowing array representations to be defined using recursive data structures; symbols in the array can themselves denote models at a more detailed level of abstraction.

The formalism presented in this paper has evolved from research in the area of computational imagery (Glasgow & Papadias 1992; Glasgow 1993b), which involves the study of AI knowledge representation and inferencing techniques that correspond to the representations and processes for mental imagery. In computational imagery, a mathematical theory of arrays provides a basis for representing and reasoning about visual (e.g. shape) and spatial (e.g. relative location) properties of entities in the world. Although results of cognitive studies offered initial motivation for the representations and functionality of the formalism, the ultimate concerns of research in computational imagery are expressive power, inferential adequacy and efficiency; whenever possible, the limitations of the human information processing system are overcome.

To provide a cognitive perspective of the research area, the paper begins with a discussion of mental models and their use in spatial reasoning and problem solving. A representation scheme for spatial reasoning using symbolic arrays is presented in Section 3. The paper concludes with a a summary of the described research and its contributions.

Mental Models

Results of experimental studies in cognitive psychology suggest that much of human problem solving is not achieved through rule-based reasoning, but rather through the manipulation of mental models. That is, humans often reason by constructing and transforming a class of representations that are structurally similar to the reality they depict. The primary purpose of this paper is to present a computational framework for reasoning about spatial models of the world. Although we do not suggest that the proposed representation scheme is necessarily a model of cognition, an understanding of the underlying principles and behavior of mental models is important to the development of AI systems for spatial reasoning and problem solving. In particular, it is useful to discriminate between mental models and other forms of mental representation.

Principles of Mental Models

Although there does not appear to be an agreed on account for what constitutes a mental model, Johnson-Laird (1983) has proposed some weak constraints on these representations:

- Mental models, and the machinery for constructing and interpreting them, are computable and finite.
- A description of a single state of affairs is represented by a single mental model. Indeterminacies are directly represented only if their use does not result in exponential growth in the number of models.
- The structure of a mental model is isomorphic to the structure of the state of affairs it represents; a model is constructed from tokens corresponding to the entities in the domain.

Johnson-Laird asserts that models are akin to how people perceive the world, yet may be incomplete or simplified. Moreover, mental models are specific, and can be used to represent relations concerning space or time. Inferences are formed, not through the application of formal rules, but through the construction and inspection of alternative models that are used to validate or refute a putative conclusion.

Problem solving with spatial models is often associated with the reasoning abilities of mental imagery. A large body of experimental data has been generated and theories proposed concerning the representations involved in imagery. These theories fall into three categories: 1) theories that suggest that image representations are analogue or picture-like, 2) theories that liken image representations to linguistic descriptions, and 3) those that suggest that there may exist multiple image representations, corresponding to different task demands. Johnson-Laird proposes, as a resolution to the imagery debate, that there exist three kinds of representation involved in imagery: a propositional descriptive representation, a mental model and a visual image. What distinguishes a mental model from other forms of representation is the degree of specificity, which can be measured by the amount of information that is made explicit by the representation.

Reasoning with Mental Models

One purpose of a mental model is to simulate and thus predict and/or plan for the behavior of a system. Humans are adept at reasoning about space, yet it is not well understood how this is accomplished. Forbus (1983) suggests that it is not through logical theorem proving or through algebraic calculations, but through diagrammatic reasoning, that we achieve this competence. He states that the spatial structure of a diagram allows us to use our perceptual apparatus to inspect and interpret models in a way that is analogous to how we inspect and interpret entities in the world. He further conjectures that people can reason with less detailed representations than diagrams — representations which symbolically describe places and relationships among these places.

Theories of inference based on mental models have suggested that the processing of syllogisms can be achieved by the inspection of symbolic spatial models (Huttenlocher 1968). In such theories, a model is constructed in which the symbols denoting entities in the domain are mapped along an axis corresponding to comparative dimensions such as taller-than/shorter-than or older-than/younger-than. For example, the description *"John is taller than Mary and Mary is taller than Jane"* is representable as the array | *John* | *Mary* | *Jane* |, where the *left-of* relation in the array is interpreted as *taller-than*. Using this model, questions such as *"Is John taller than Jane?"* can be answered using inspection operations analogous to those used in visual inspection. Although it is possible to construct a logical description and rules of inference for syllogistic reasoning, experimental results suggest that mental models that incorporate array representations increase the efficiency and accuracy of problem solving involving transitive inferences (Shaver, Pierson, & Lang 1974; Kosslyn 1983).

Experiments carried out by Taylor and Tversky (1992) presented subjects with both route and survey descriptions of spatial domains. Their findings included the fact that the subjects constructed mental models that were sufficiently abstract to allow inferences from alternative perspectives. They suggest that the advantage of such a representation lies in its flexibility, since it supports exploration of a world from unique points of view as well as adaptation resulting from change in the environment. Other studies by Tversky (1981,1991) provide evidence that spatial mental models might be distorted by an alignment with existing landmarks or frames of reference. For example, when college students were asked to choose a correct map of America from two possibilities, the majority chose the incorrect version which was altered so that South America appeared directly below North America.

Johnson-Laird (1993) cites three fundamental differences between reasoning with mental models and reasoning with logical representations:

- Model-based reasoning is semantic: it relies on the construction and inspection of alternative models, where each model represents a unique state of affairs. Logic-based reasoning is syntactic: conclusions are formed by applying rules of inference to syntactic forms in order to derive new forms.
- In mental models, symbolic tokens correspond to individual entities; models do not contain variables. Much of logical reasoning is based on the instantiation of generalized terms containing variables.
- Whereas logical forms mirror the structure of discourse, mental models are structured to mirror the relations among entities.

Furthermore, Johnson-Laird suggests that the principles involved in mental models have serious advantages for computational reasoning. In particular they allow for the integration of deductive and nonmonotonic reasoning. Derivations occur by simple model checking (inspection of model representations) and updating of models can be achieved without the cost of undoing previously computed deductions resulting from default reasoning.

Representation of Spatial Models

This section presents an approach to knowledge representation for model-based reasoning where, similar to the spatial component for computational imagery (Glasgow & Papadias 1992), symbolic arrays depict the entities and relations in a world. The scheme was developed using a formal theory of arrays. Array theory is the mathematics of rectangularly arranged, nested data objects (More 1981). An array consists of zero or more items held at positions along multiple axes, where rectangular arrangement is the concept of objects having spatial positions relative to one another in the collection. Similar to set theory, array theory is concerned with the concepts of aggregation, nesting and membership. An array can be considered as a multi-dimensional extension of the list data structure used in Lisp (Jenkins & Glasgow 1989). The representation of spatial models involves a special class of arrays — those whose symbols and structure denote entities and their relative locations in the domain of interest. In order to specify the spatial relations, a symbol may occupy one or more cells of an array. For example, the description *"The ball and the lamp are on the table and the lamp is to the right of the ball"* could be represented as the array:[1]

ball	*lamp*
table	*table*

Visual information such as shape, relative distance and relative size is often discarded in a model. However, if desired, distance and shape attributes can be preserved in the array representation. Figure 3(a) illustrates an island map similar to the one used by Kosslyn and colleagues (Kosslyn 1980) to study how humans store and inspect mental maps. Much of the information derivable through the visual inspection of the map image can also be inferred from the symbolic array representation in Figure 3(b). Geographic directions are determined in this representation by comparing the relative locations of entities in the array, e.g., the hut is *south-of* the lake

[1] In the notation presented, adjacent cells containing the same symbol are merged into a singe cell.

and *west-of* the beach. As well, it can be determined that the tree is *near* the lake and that the beach is *closer* to the hut than it is to the lake. Relative size and shape information can be preserved in a representation by increasing the granularity of the array. For example, the shape of the island map is computable from the array representation depicted in Figure 3(c).

A large collection of total, primitive functions, chosen to express fundamental properties of arrays, are described for array theory. These functions, which subsume most of the operations of APL and Lisp, have been implemented in the programming language Nial (Jenkins, Glasgow, & McCrosky 1986). Array theory provides a high-level language that can be used for expressing and proving properties of spatial models. It is currently being employed to specify the primitive operations for constructing, transforming and inspecting array representations. Section 3.2 describes the primitive array functions for model-based reasoning that have been implemented in Nial.

In the remainder of this section, we define an approach to knowledge representation for spatial reasoning. The scheme consists of *array representations*, which model the entities and relations in the world, and a set of *primitive array functions* for generating, inspecting and transforming representations.

An array representation is constructed so that there is a correspondence between the structure of the symbolic array and the structure of the world being modeled. More precisely, a world is representable by an array if there exists a mapping between symbols in the array and entities in the world that preserves the relative location of entities. Array representations provide a basis for deductive reasoning in a spatial domain.

In the proposed formalism, a *world* is defined as a set of entities and a set of spatial relations that are defined on the entities. Our definition assumes a finite set P of predicate symbols that is used to index the relations in the world.

Definition. A **world** w is defined as a pair $< S, R >$ such that:

- S is a finite set of symbols that denote the *entities* of interest in the world.
- R is a finite, P-indexed set of *spatial relations* defined over the set of symbols S for the world. Each n-ary relation in R is defined in terms of a set of n-tuples containing entities in S. The notation w_p is used to denote the relation in R corresponding to predicate symbol $p \in P$.

Similar to a world, an array representation contains a set of spatially organized parts. The spatial relations among parts are determined by a set of boolean functions that are used to inspect an array data structure.

Definition. An **array representation** A is defined as a pair $< \mathcal{A}, F >$ such that

- $\mathcal{A}$ is a multi-dimensional array containing the set of symbols $Sym(\mathcal{A})$. A symbol may occupy more than one location in $\mathcal{A}$, but each location contains at most one symbol.
- F is a P-indexed set of boolean array functions. For each $n+1$-ary function $f_p \in F$ and symbols $s_1, ..., s_n \in Sym(\mathcal{A})$, $f_p(s_1, ..., s_n, \mathcal{A}) \in \{true, false\}$.

The assumption that a location in an array may contain at most one symbol corresponds to the fact that at most one entity can occupy a single location in space. This does not preclude, however, the concept of containment or the fact that a symbol may denote a complex entity consisting of subentities.

An array representation is a *model* for a world if all of the world's relations are representable by a function for the array.

Definition.
Given a world $w = < S, R >$ and an array representation $A = < \mathcal{A}, F >$, an n-ary relation $w_p \in R$ is **represented** in A if and only if $S = Sym(\mathcal{A})$ and for all symbols $s_1, ..., s_n \in S$:

$$f_p(s_1, ..., s_n, \mathcal{A}) = \text{true if and only if } (s_1, ..., s_n) \in w_p$$

for function $f_p \in F$.

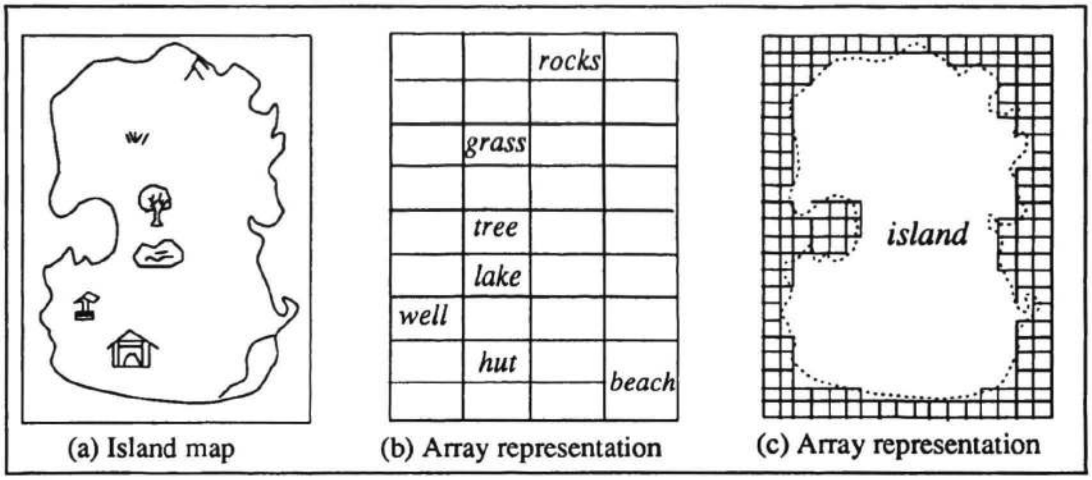
An array representation A is an **array model** for world w if and only if for every relation $w_p \in R$, w_p is represented in A.

If there exists an array model for a world w then we say that w is **representable.**

A representable world is complete in the sense that all spatial relationships among entities are made explicit by the relations in R, and can thus be represented by array inspection functions in F.

To illustrate the concept of an array model, consider the world described by the set $S = \{Britain, Portugal, Spain, ...\}$ of countries in Europe and their corresponding geographical relationships R indexed by the set of predicate symbols $P = \{$*north-of, west-of, east-of, south-of and borders-on*$\}$. We can define an array model $A = < \mathcal{A}, F >$ for w where:

- $\mathcal{A}$ is the array depicted in Figure 4.
- The array functions in F are defined to model the spatial relations in the world. For example, the relation $w_{west\text{-}of}$ is represented by the function $f_{west\text{-}of}$, which is defined so that an application of the form $f_{west\text{-}of}(s_1, s_2, \mathcal{A})$ returns the value true if and only if symbol s_1 occurs in a location that is to the left of the left-most occurrence of symbol s_2 in the array data structure $\mathcal{A}$. Similarly the relation $w_{borders\text{-}on}$ is represented by the function $f_{borders\text{-}on}$, where



(a) Island map (b) Array representation (c) Array representation

Figure 3: Representations of Kosslyn's island map

$f_{borders\text{-}on}(s_1, s_2, \mathcal{A})$ evaluates to true just in the case where symbols s_1 and s_2 are situated in adjacent cells of array $\mathcal{A}$.

Deductive reasoning is achieved in the representation scheme through inspection of array models. This approach relies on the semantics, or the mapping between the representation and the domain of interest. Conclusions are derived by applying array functions that map to the relevant spatial relations in the world. This approach to reasoning can be thought of as a restricted form of model-theoretic deduction, one which is limited to inferences that are made explicit by inspection of array representations.

The notion of an array model presented in this section assumes that a world is representable, that is, all of its relations are made explicit. It is possible to extend the representation scheme to include indeterminate worlds (Glasgow 1994). An indeterminate world is one which has multiple possible interpretations, represented as a set of array models.

Discussion

The work described in this paper extends research in computational imagery by relating the spatial representation for imagery to cognitive studies of mental models, and by formally defining an array model in terms of a mapping from symbols in an array to entities in the world, and from spatial functions in the array model to spatial relations in the world.

A characteristic of the array representation for model-based reasoning is that it brings relevant spatial properties to the forefront. The entities and spatial relations in the world are explicitly denoted as symbols and relations in a multi-dimensional array. This representation provides for a simplified model of the world — one that captures salient spatial features and suppresses unnecessary or irrelevant details. A benefit of the array representation lies in its succinct encoding and its provision for updating and change. It can also be distinguished from traditional logic representations by the fact that it imposes specificity on a representation, yet symbolic arrays are more abstract than the visual representations proposed for reasoning with diagrams.

The array representation for spatial reasoning has measurable computational advantages over proof-theoretic logic systems. In particular, array models can be used to develop *vivid* knowledge bases. Levesque (1986) defines a vivid knowledge base as one that is structured so that there is a one-to-one correspondence between the entities in the world and the symbols in the knowledge base, and for every simple relationship of interest in the world – in our case spatial relationships – there exists corresponding connections among symbols in the knowledge base. Vivid knowledge bases are consistent and complete. Levesque argues that the main benefit of vivid knowledge bases is that they provide for efficient worst case reasoning behavior, since calculating what is logically implicit generally reduces to retrieving what is explicit. Further arguments concerning the computational advantages of the array representation are presented elsewhere (Glasgow 1993b; 1993a).

The proposed model-based approach to reasoning can be motivated and justified by human needs. Simon (1978) has proposed criteria for assessing and selecting representations based on information content and on ease of programming. These criteria are task dependent and partially rely on the ability of the programmer to represent the state of knowledge in the world and the transformations and inferences that may occur. Experimental results in cognitive psychology suggest that humans apply model-based reasoning for problem solving in a variety of domains. Certainly a formalism that captures the representations and processes associated with model-based reasoning would facilitate the implementation of computational reasoning systems in such problem solving domains. Although the scheme was motivated by human needs, it can overcome inherent limitations of the cognitive system. In particular, control strategies can be developed so that no consistent interpretation for a world is overlooked.

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<tr><td></td><td></td><td></td><td></td><td>Norway</td><td>Sweden</td><td>Finland</td></tr>
<tr><td></td><td rowspan="3">Britain</td><td></td><td></td><td>Denmark</td><td></td><td></td></tr>
<tr><td>Ireland</td><td></td><td>Holland</td><td rowspan="2">Germany</td><td>Poland</td><td></td></tr>
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<tr><td></td><td></td><td colspan="2" rowspan="2">France</td><td>Switzerland</td><td>Austria</td><td>Hungary</td></tr>
<tr><td></td><td></td><td rowspan="2">Italy</td><td colspan="2">? Yugoslavia ?</td></tr>
<tr><td>Portugal</td><td colspan="2">Spain</td><td></td><td></td><td>Greece</td></tr>
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Figure 4: Array representation of Europe

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