Window on the mind? What Eye Movements Reveal about Geometrical Reasoning

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Geometrical concepts involve an intimate mixture of perception and cognition. Most geometry problems are either posed in the form of a diagram, or require construction of a diagram for a successful solution. Yet, unlike the case for reading and arithmetic, a serial algorithm for scanning or constructing diagrams is not taught to students of geometry. Specifically, when learning to read, students are taught to proceed from word to word, from left to right. When learning column arithmetic, they are taught to proceed from right to left through the columns, and from top to bottom within each column. When taught geometry, on the other hand, students are not taught how to look at the diagrams — they must learn an efficient eye movement pattern on their own. Empirical studies of eye movement patterns during reading and arithmetic show that individuals performing these tasks successfully indeed follow an orderly eye movement pattern with very few deviations (Epelboim, Booth & Steinman, 1994; Suppes, 1990). The nature of eye movements performed in order to solve geometry problems is not known. These eye movements and what they can tell us about the underlying mental operations are the topic of our study.

We asked subjects to solve simple geometry problems, each presented to them in the form of a diagram on a computer screen. Their eye movements were recorded, with exceptional precision and accuracy, using the The Maryland Revolving-Field monitor. Three subjects participated. Two of the subjects were skilled at solving geometry problems. They had graduate training in Physics, and encountered problems similar to those used in the experiment in their professional life. The third subject had last solved geometry problems in high school, over 50 years prior to the experiment. He reported that he had "no clue" as to what to do on most of the problems. The subjects were not allowed to write or sketch anything, but the problems were simple enough to solve mentally. The subjects were asked to reason aloud, and their speech was recorded.

Figure 1 shows typical eye movements of one of the skilled subjects. Panel (a) shows the problem as it appeared to the subject. Panels (b-f) show the subject's fixations as he was solving the problem. Each symbol is one fixation. Each panel shows 25 fixations. Each fixation is represented by a circle, a square, or a rhombus, which contains this fixation's sequential number. Circles show brief fixations ($\leq$ 300 msec). Squares show fixations that lasted longer than 300 msec but less than 800 msec. Rhombi show long fixations that lasted over 800 msec. The subject's reasoning aloud for this problem was as follows (numbers in parenthesis indicate the sequential numbers of fixations that occurred while the phrase was being spoken):

ABCD is a parallelogram.	(1-11)
Ok, so where is the unknown angle?	(12-20)
the unknown angle is AEC	(21-31)
... then since ...	(32-68)
oh, ... it's trivial	(69-80)
the lines AD and BC are parallel	(81-92)
and therefore the angle AEC	(93-105)
is alternate interior to the angle DAE	(106-110)
which is labeled 60 degrees	(110-120)
The answer is 60 degrees	(121-122)

The reader is invited to examine Figure 1 and the protocol above and propose a sequence of mental operations that fit these data.

Quantitative analyses and modeling of the eye movement data and spoken protocols of the 3 subjects are now in progress. The results of these analyses will be presented at the symposium.

References

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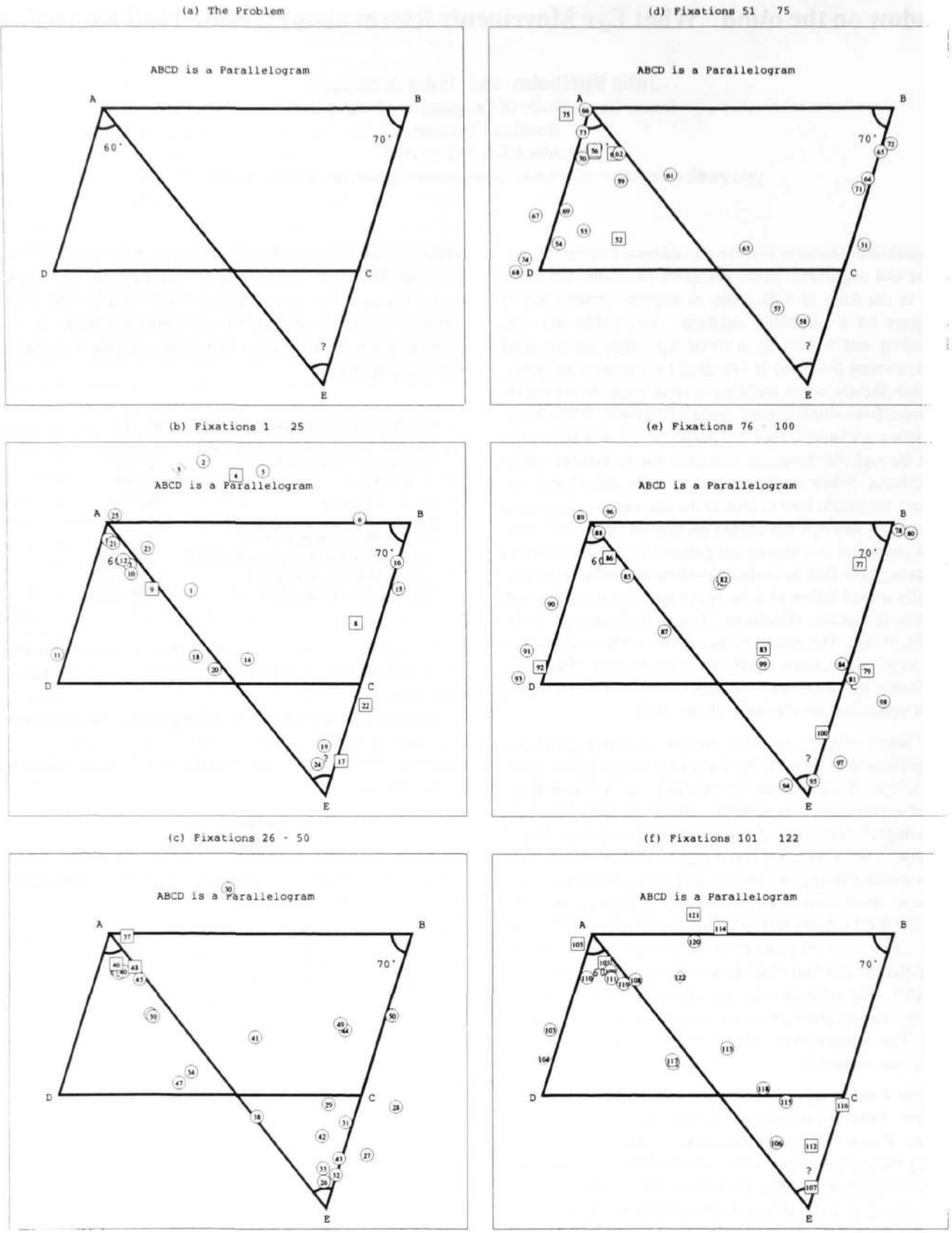

Figure 1: See text.