Transferring and Modifying Terms in Equations

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Abstract

Labeling and elaboration manipulations were used in examples to affect the likelihood of students learning to represent workers' rates and times in algebra word problems dealing with work. Learners studying examples with labels for rates and times were more likely to transfer and correctly modify the representations compared to learners who did not see the labels. An elaborative statement describing the possible representations for the different terms in the work equation did not reliably affect performance. These results extend prior work (Catrambone, 1994, 1995) on subgoal learning by demonstrating that representations, not just sets of steps, can be successfully transferred and modified through a manipulation (labeling) that has been shown to aid subgoal learning.

Introduction

A good deal of research has examined the transfer success people have after studying training materials such as those containing step-by-step instructions (Kieras & Bovair, 1984; Smith & Goodman, 1984), examples (e.g., Ross, 1987, 1989), or both (Fong, Krantz, & Nisbett, 1986). Although there have been some exceptions (e.g., Fong et al., 1986; Zhu & Simon, 1987), the usual finding from such research is that people can carry out new procedures or solve new problems that are quite similar to those on which they were trained, but have difficulty when the novel cases involve more than minor changes from what they had previously studied.

This transfer difficulty seems to stem from a tendency by many learners to memorize a solution procedure from examples that consists of a linear series of steps rather than a more meaningful organization. A linear series of steps provides a learner with little guidance for modifying the solution procedure for problems that can not be solved just like the examples. One potentially useful organization for a solution procedure would be a set of goals and subgoals with methods for achieving them (e.g., Anzai & Simon, 1979; Card, Moran, & Newell, 1983; Catrambone & Holyoak, 1990; Newell & Simon, 1972; Singley & Anderson, 1989).

Problems within a domain typically share the same set of subgoals, although the methods for achieving the subgoals might vary from problem to problem. For instance, in the materials used in the present study, the subgoals to represent each worker's rate and time are in each example and test problem, yet the representations for work and time vary (e.g., a constant, a variable).

Prior work with subgoal learning has demonstrated that if a student learns the subgoal structure for solving problems in a domain, then he or she is more likely to adapt old procedures for novel problems, where novel problems are those involving the same subgoals as the examples but requiring new or modified methods (sets of steps) to achieve the subgoals (Catrambone, 1994, 1995). The present study extends the subgoal-learning work by examining the likelihood of learners transferring and modifying *representations* for conceptual entities in equations as a function of whether they studied training materials emphasizing the subgoals achieved by those representations.

Two manipulations were used to convey subgoals in the present study: 1) the use of examples that contained or did not contain descriptive labels for the terms in the equation, and 2) the use of introductory elaboration prior to the examples that described the possible representations for the different terms in the work equation.

Related Work

The justification for the labeling manipulation is based on a series of studies (Catrambone, 1994, 1995, in press) that develop the subgoal-learning model. In brief, this model proposes that:

1) A label leads learners to group a set of steps;
2) After grouping the steps, learners are likely to try to self-explain why those steps go together;
3) The result of the self-explanation process is the formation of the goal that represents the purpose of that set of steps.

The present study exploits the labeling methodology in order to extend the scope of the model.

Earlier studies involving algebra word problems found that learners were relatively unlikely to successfully modify old representations for terms in equations (Reed, Dempster, & Ettinger, 1985). Rather, they tended to rely on a syntactic approach, that is, learners frequently tried to map old equations from examples to new problems at a symbol by symbol level rather than in terms of the conceptual entities that groups of symbols represented (see also Ross, 1987, 1989). In addition, Reed, and Bolstad

(1991) found that providing learners with rules for solving algebra word problems did not have a large effect on performance.

While some studies have shown that learners can benefit from rule-based instruction for solving problems (e.g., Fong et al., 1986), in general learners seem to prefer and frequently derive more from examples. For instance, Chi, Bassok, Lewis, Reimann, and Glaser (1989) found that after studying a text on mechanics, good and poor students (as defined by a subsequent problem solving test) seemed to possess similar declarative knowledge. However, after studying worked examples, good students were more likely to acquire knowledge about, among other things, the conditions for applying actions/operators and the consequences of those actions. In the framework of the present study, they were better at determining the subgoals being achieved by those actions in the examples.

Overview of Study

The present study has two main purposes. One is to examine whether the benefits of subgoal learning that have been previously found for modifying sets of steps for novel problems (Catrambone, 1994, 1995) will also apply to transferring and modifying the terms used to represent conceptual entities in equations. While some prior work has found that learners can transfer old components into new structures (Elio, 1986), that work focused on reasonably well-practiced procedures rather than transfer after just a small amount of exposure to the training materials. The second purpose is to compare the relative effectiveness of labels (in examples) versus elaborations or rules for representing terms in algebra word problems. Prior work has suggested that examples are more effective than rules in producing knowledge that helps learners to solve novel problems (e.g., Reed & Bolstad, 1991).

Consider the algebra example in Figure 1 in which one has to determine how long it would take someone to do a job given that certain information about their work rate and time and another person's work rate and time are given. This problem involves using an equation for determining work that requires representing each worker's work rate and time: $(\text{Rate}_1 \times \text{Time}_1) + (\text{Rate}_2 \times \text{Time}_2) = 1$.

Learners are good at memorizing how to solve problems isomorphic to the one in Figure 1. In this problem, both workers' rates are represented as constants. The time spent working by worker 1 is represented as a variable and worker 2's time is represented as a function of that variable. However, learners may not encode the example solution in terms of determining a representation for each rate and time and then inserting these representations into the equation, but rather have a more superficial understanding of the solution procedure that involves matching the form used in the example, finding similar values in the problem statement, and inserting them into the equation. As a result, if a new problem requires a different representation of the rates and times, these learners might be unable to solve the problem.

For instance, the first problem in Figure 2 requires that worker 2's rate be represented as a variable. In addition, instead of having the workers' times be represented as a variable and a function of that variable (as they were in the example in Figure 1), the times are now represented as a constant and a function of that constant. Nevertheless, the new representations can be inserted into the same equation as the one used for the example in Figure 1. Similarly, the second problem in Figure 2 requires that one worker's rate be represented as a variable and the other worker's rate be represented as a function of that variable. Their times are both represented as constants. These representations are different than those used in the examples.

Mary can rebuild a carburetor in 3 hours and Mike can rebuild one in 4 hours. How long would it take Mary to rebuild a carburetor if she and Mike work together, but Mike works for 1/2 hour more than Mary?

Solution

$\frac{1}{3}$ = Mary's rate

t = time Mary spent rebuilding carburetor

$\frac{1}{4}$ = Mike's rate

$t + \frac{1}{2}$ = time Mike spent rebuilding carburetor

$$\left(\frac{1}{3} * t\right) + \left(\frac{1}{4} * \left(t + \frac{1}{2}\right)\right) = 1$$

$$\frac{7}{12} * t = 1 - \frac{1}{8}$$

$t = \frac{7}{8} * \frac{12}{7} = \frac{3}{2}$ hours = time Mary spent rebuilding carburetor

Figure 1: Training examples.

1. Mr. Jones can refinish a dresser in 5 hours. After working for 2 hours he is joined by Mrs. Jones. Together they finish the job in 1 hour. How much of the job could Mrs. Jones do in 1 hour when working alone?

Solution (not seen by participants)

$(\frac{1}{5} * (2+1)) + (MrsJ * 1) = 1$

$\frac{3}{5} + MrsJ = 1$

$w = \frac{2}{5}$ = Mrs. Jones' rate; so, in 1 hour Mrs. Jones could do $\frac{2}{5}$ of job

2. Barbara and Connie can finish a job in 6 hours when they work together. Barbara works twice as fast as Connie. How much of the job could Connie do in 1 hour when working alone?

Solution (not seen by participants)

$(2c * 6) + (c * 6) = 1$

$12c + 6c = 1$

$18c = 1$

$c = \frac{1}{18}$ = Connie's rate; so, in 1 hour Connie could do $\frac{1}{18}$ of job

3. Joe can stack a shelf of groceries in 3 hours. Sheila can stack a shelf of groceries twice as fast as Joe. If Joe works for 1 hour alone stacking a shelf and then Sheila starts to help him, how long will Sheila be working with Joe until the shelf is stacked?

Solution (not seen by participants)

$\frac{1}{3} * (t + 1) + (2*\frac{1}{3}) * (t) = 1$

$\frac{1}{3}t + \frac{1}{3} + \frac{2}{3}t = 1$

$t + \frac{1}{3} = 1$

$t = \frac{2}{3}$; so, Sheila will be working with Joe for $\frac{2}{3}$ of an hour

Figure 2: Sample test problems.

The following equation can often be used to solve these problems:

$$(Rate_1 \times Time_1) + (Rate_2 \times Time_2) = \text{Tasks Completed}$$

where $(Rate_1 \times Time_1)$ is the amount of work completed by the first worker, $(Rate_2 \times Time_2)$ is the amount of work completed by the second worker, and Tasks Completed is the total work completed by both workers. The Rate of a worker can be represented as a constant, a function of a constant, a variable, or a function of a variable. Similarly, the Time a worker works can be represented as a constant, a function of a constant, a variable, or a function of a variable. The particular representation used depends, of course, on the givens in the problem and the question that is being asked by the problem.

Figure 3: Supplemental text seen by elaboration groups.

Learning was assessed by how successfully learners could transfer or modify representations for terms in the work equation. Learners studied examples that used a subset of the possible representations for the terms and then they solved one isomorph and three novel problems. Novel problems were defined as those that required new representations for at least one of the terms from the equation. Two transfer situations for novel problems were examined. The first was the transfer of old representations. The second was how successfully learners could create a new representation for a term. Note that for purposes of the present study, a "new" representation for a term means that the representation had not been used for that term (e.g., rate) in an example even if it had been used for a different term (e.g., time). For instance, even though time was represented as a variable in the training examples (such as the one in Figure 1), if a test problem required rate to be represented as a variable, this would be considered a new representation for rate since rate had been represented only as a constant in the training examples.

The subgoals in the present study involve finding the correct representations for workers' rates and times for algebra word problems. The assumption is that a learner could learn a particular superficial syntax for the work equation without learning the subgoals for representing workers' rates and times. Thus, when faced with a problem that involves new representations for rate and time, the learner might have difficulty. However, if the learner has learned the subgoals to represent each worker's rate and time, then the learner might have a better chance of producing the correct representations in the novel problems. That is, the learner will be more likely to correctly use old representations for rates or times in the context of new representations for rates and times and also that he or she will be able to determine new representations for rates and times. The labeling and elaboration manipulations were used to affect the likelihood of learners acquiring the subgoals to represent workers' rates and times.

Experiment

Method

Participants. Participants were 80 students recruited from several Atlanta-area colleges who received course credit or payment for their participation. In order to participate in the experiment, a student could not have taken a college-level calculus course.

Materials and Procedure. Participants studied three isomorphic example word problems dealing with work, including the example in Figure 1. A cover page included the following statement: "On the next two pages you will find three example algebra problems dealing with work. Work problems typically describe a situation in which two people work together to complete a task."

Two factors were manipulated: labels and elaborations. The Label groups studied examples with descriptions for rates and times of each worker (see the first four lines under the word "Solution" for the example in Figure 1). The No Label groups studied examples that did not contain these descriptions (i.e., lines 1-4 were not present). The Elaboration groups received a supplement to the statement on the cover page that listed the different representations that could be used for rate and time (see Figure 3). The No Elaboration groups did not receive this supplement. The two manipulations were crossed creating four groups with 20 participants per group.

After studying the examples participants received four problems to solve. The first was isomorphic to the training examples. The next three involved both new and old ways of representing rate and/or time for each worker (see Figure 2). Participants could not look back at the examples when working on the test problems.

Results

All participants solved the isomorphic test problem correctly.

Performance on the three novel test problems was scored in the following ways. First, each problem was scored as correct or incorrect. Each participant was then assigned a proportion correct score.

Second, participants were scored on whether they correctly represented the rate and time for each worker for each of the three novel problems. Across the three problems there were a total of four opportunities to use an old representation for rate or time (i.e., RATE: represented as a constant; TIME: represented as a variable or a variable plus a constant). There were a total of eight opportunities to use a new representation for rate or time (i.e., RATE: represented as a variable, a variable multiplied by a constant, or a constant multiplied by a constant; TIME: represented by a constant or the sum of constants). Participants were assigned a proportion correct for old representations and a proportion correct for new representations.

There was a significant effect of label, but not elaboration, on the proportion of novel test problems solved correctly--label: $F(1, 76) = 8.17, p = .006, MSE = 0.16$; elaboration: $F(1, 76) = 1.44, p = .23$ (see Table 1). The interaction was not significant.

	Elaboration		No Elaboration	
	Label (n = 20)	No Label (n = 20)	Label (n = 20)	No Label (n = 20)
Proportion of Problems Solved Correctly	.70	.42	.57	.33
Proportion Correct Old Representations	.92	.81	.84	.69
Proportion Correct New Representations	.73	.52	.68	.51

Table 1: Performance on novel test problems.

An analysis of variance was conducted on the proportion of correct representations for rates and times using labels and elaboration as grouping factors and type of representation (old or new) as a within-subjects factor. There was a significant effect of label, $F(1, 76) = 6.40$, $p = .01$, $MSE = 0.16$, but not of elaboration, $F(1, 76) = 1.15$, $p = .29$ (see Table 1). There was also a significant effect of type of representation (old vs new), $F(1, 76) = 43.77$, $p < .0001$, $MSE = 0.04$. There were no significant interactions.

Across the problems, the most common errors that participants made were to inappropriately represent either rate or time in the equation or to write that not enough information was given in the problem.

Discussion

The results from the present experiment are consistent with the hypothesis that students who learned the subgoals of representing workers' rates and times would represent them more successfully on novel problems. This occurred both for old representations in new contexts (i.e., problems that required new representations for at least one term) as well as for new representations. Learners were more successful transferring old representations to novel problems than creating new representations. This is a reasonable finding since the first type of transfer essentially involves the learner recognizing that the old representation is appropriate while the second type of transfer involves the learner creating a representation.

The labeling manipulation affected performance while the elaboration manipulation appeared to be ineffective. This finding is consistent with prior work suggesting that examples play a larger role than explanatory text on the problem solving knowledge students acquire (Chi et al.,1989; LeFevre & Dixon, 1986).

The overall pattern of results is consistent with the claim that when learners are helped to form subgoals for solving problems in a domain, they are more likely to successfully achieve those subgoals in novel problems that require new or modified methods. The twist in the present study is the demonstration that subgoal learning does not benefit just methods that involve a series of steps, but can also benefit a method that is essentially a representation for a conceptual entity in an equation. This finding suggests that the subgoal-learning framework may be applicable to a variety of problem solving situations including those involving changes in representations as well as those involving changes in steps.

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References

Anzai, Y., & Simon, H.A. (1979). The theory of learning by doing. *Psychological Review, 86* (2), 124-140.

Card, S.K., Moran, T.P., & Newell, A. (1983). *The psychology of human-computer interaction.* Hillsdale, NJ: Erlbaum.

Catrambone, R. (1995). Aiding subgoal learning: Effects on transfer. *Journal of Educational Psychology, 87* (1), 5-17.

Catrambone, R. (in press). Generalizing solution procedures learned from examples. *Journal of Experimental Psychology: Learning, Memory, and Cognition.*

Catrambone, R. (1994). Improving examples to improve transfer to novel problems. *Memory & Cognition, 22* (5), 606-615.

Catrambone, R., & Holyoak, K.J. (1990). Learning subgoals and methods for solving probability problems. *Memory & Cognition, 18* (6), 593-603.

Chi, M.T.H., Bassok, M., Lewis, R., Reimann, P., & Glaser, R. (1989). Self-explanations: How students study and use examples in learning to solve problems. *Cognitive Science, 13*, 145-182.

Elio, R. (1986). Representation of similar well-learned cognitive procedures. *Cognitive Science, 10*, 41-73.

Fong, G.T., Krantz, D.H., & Nisbett, R.E. (1986). The effects of statistical training on thinking about everyday problems. *Cognitive Psychology, 18*, 253-292.

Kieras, D.E., & Bovair, S. (1984). The role of a mental model in learning to operate a device. *Cognitive Science, 8*, 255-273.

LeFevre, J., & Dixon, P. (1986). Do written instructions need examples? *Cognition and Instruction, 3*, 1-30.

Newell, A., & Simon, H.A. (1972). *Human problem solving.* Englewood Cliffs, NJ: Prentice-Hall.

Reed, S.K., & Bolstad, C.A. (1991). Use of examples and procedures in problem solving. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 17* (4), 753-766.

Reed, S.K., Dempster, A., & Ettinger, M. (1985). Usefulness of analogous solutions for solving algebra word problems. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 11* (1), 106-125.

Ross, B. (1987). This is like that: The use of earlier problems and the separation of similarity effects. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 13* (4), 629-639.

Ross, B. (1989). Distinguishing types of superficial similarities: Different effects on the access and use of earlier problems. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 15* (3), 456-468.

Singley, M.K., & Anderson, J.R. (1989). *The transfer of cognitive skill.* Cambridge, MA: Harvard University Press.

Smith, E.E., & Goodman, L. (1984). Understanding written instructions: The role of an explanatory schema. *Cognition and Instruction, 1* (4), 359-396.

Zhu, X., & Simon, H.A. (1987). Learning mathematics from examples and by doing. *Cognition and Instruction, 4* (3), 137-166.