Empirical Evidence for Constraint Relaxation in Insight Problem Solving

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Abstract

Using a new developed task environment that allows to control for depth and width of problem space (Match Stick Algebra problems), three experiments were conducted to investigate the role of implicit constraints in insight problem solving. The first experiment showed that constraints caused by prior knowledge of common algebra lead to large differences in solution times, when they were encountered for the first time. No differences were found after the constraints had been relaxed. In the second experiment complimentary moves had to be applied in two different equation structures, one similiar to common algebra, one dissimiliar to common algebra. Consistent with our predictions different problem structures lead to a reversed order of task difficulty for the same moves depending on the activation of prior knowledge from real algebra. In the third experiment it was shown that a re-distribution of activation in a network causes the removing of constraints. Non-detectable priming of the solution lead to significantly more solutions in the experimental group as compared to a control group.

Introduction

Insight can be defined as the act of breaking out of an impasse that has been encountered during problem solving (Ohlsson, 1992). Impasses describe mental states in which no active problem solving occurs. But what is the cause of such impasses and what is their function? One possible cause of impasses are implicit constraints that problem solvers put on the goal state in certain problems. The resolution of impasses (restructuring) then could be caused by constraint relaxation, i.e. a gradual decrease in activation of these constraints.

To avoid the circular definition of insight problems in former studies where insight problems are often defined as problems in which insight occurs (Dominowski & Dallob, 1994), we looked for a task environment where (a) different levels of constraints can be distinguished on the basis of a formal task analysis and b) the depth (i.e., length of solution path) and width (i.e., branching factor) of the problem space can be controlled across individuals.

The domain of Match Stick Algebra (MSA) fulfills these requirements. The goal in MSA problems is always to make an equation true by moving a single stick. For example in IV = III + III or in III = III + III, the solutions are VI = III + III and III = III = III.

The depth of the problem space is constant in these problems because the solution can be obtained in one step (by moving one stick) and no intermediate knowledge state has to be remembered. The width of the problem space is known because the task analysis reveals which moves can be applied to a problem within the given instruction (move one single stick to make the equation true; the resulting equation can only contain Roman numerals between I and XIII and the symbols for +, - and =).

This allows us to vary the number and type of constraints independently from serial problem solving aspects. The obvious source of implicit constraints in this task domain is the subjects′ prior knowledge of common algebra. Because the prior knowledge regarding algebra problems is very systematic, clear hypotheses can be formulated to which extent different problems should activate constraints. This fact can be used to select MSA tasks that vary on a dimension from "not activating constraints" to "activating several constraints". The first experiment was carried out to determine the effect of different types of constraints on task difficulty.

Experiment 1: Impasses and Transfer as a Function of Constraints

In Experiment 1, participants solved problems that were more or less likely to activate constraints (see Table 1).

Constraints should not be active in a problem like "VI = VII + I" because this equation can be made true by moving a match from one numeral to another numeral, which is the same as subtracting 1 from one side of the equation and adding 1 to the other side in common algebra.

Table 1: Tasks used in Experiment 1 and the constraints that must be relaxed to solve them. Solutions are shown in parentheses.

Constraint	Block 1	Block 2
NO	VI = VII + I (VII = VI + I)	II = III + I (III = II + I)
OP	I = II + II (I = III - II)	III = V + III (III = VI - III)
EQ	IV = III - I (IV - III = I)	V = III - II (V - III = II)
SEQ	III = III + III (III = III = III)	IV = IV + IV (IV = IV = IV)

In an equation like "I = II + II", a match has to be moved from the plus sign to a numeral, resulting in "I = III - II". This type of operation has no analog in standard algebra and hence requires a relaxation of what we call the OP-constraint i.e., that operators can not be changed in an equation.

An equation like "IV = III - I" requires relaxation of what we call the EQ-constraint i.e., that the equal sign can not be changed in an equation, because it defines an equation. The solution in this case is to move a match from the equal sign to the minus sign, resulting in "IV - III = I".

Finally the SEQ constraint, i.e. that an equation allows only one equal sign, needs to be relaxed in an equation like "III = III + III". Rotating the vertical match of the plus sign results in "III = III = III", which is the solution in this case.

We expected solution times to increase with each type of constraint because the moves that have to be carried out are less and less similiar to moves in common algebra. The constraint relaxation hypothesis predicts that there should be no differences between these problem types, once the constraints have been relaxed. To test this prediction, we presented a second task for each type of move in a transfer condition.

Participants

Twenty undergraduates from the University of Hamburg participated for course credit. They were all assigned to the same condition, because all factors were varied within subjects.

Procedure

The problems were presented on a computer screen. There were two blocks of MSA tasks. In each block 6 tasks (including 1 for each level of constraints, i.e NO, OP, EQ, SEQ, and 2 additional tasks) were presented. Participants were instructed to hit a button as soon as they knew the correct solution and say it out aloud afterwards. Tasks were presented in random order within blocks. If the time participants spent on one task exceeded five minutes, the trial was interrupted and they were told the solution. The time needed for the solution and the number of solution attempts were recorded.

Results

Figure 1 displays the mean solution times for 4 levels of constraints for the first and the second presentation of each task type. The solution times were analyzed by computing a 4 X 2 ANOVA with the levels of constraints (NO vs. OP vs. EQ vs. SEQ, within) and presentation (First vs. Second, within) as the factors.

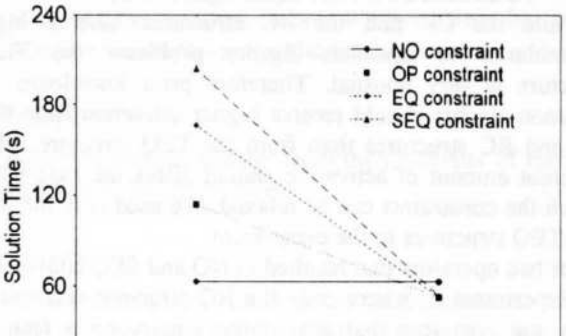

Figure 1: Mean solution times in first and second presentation as a function of constraints (solution times for not solved problems were replaced with maximal solution time = 300 s).

Subjects needed more time for moves that are not allowed in common algebra, resulting in a main effect for levels of constraints, $F(3, 57) = 3.89$, $MS_e = 8636.96$, $p < .05$. There was a large transfer effect between the first and second presentation of a problem type, yielding a significant main effect for presentation, $F(1, 19) = 18.81$, $MS_e = 14094.52$, $p < .001$.

While there were large differences in solution times for different levels of constraints in the first block, there were almost no differences in the second block, resulting in a significant interaction between levels of constraints and presentation, $F(3, 57) = 6.77$, $MS_e = 6308.70$, $p < .001$.

Discussion

Both predictions of the constraint relaxation hypothesis were supported by the results. Problems that require relaxation of more constraints take longer to solve than tasks that require relaxation of fewer constraints. However, once the constraints have been relaxed, these differences disappear.

Experiment 2: Interaction between Structure and Constraints

We conducted Experiment 2 to determine the influence of different task structures on implicit constraints. MSA equations can have three different types of structure:

(a) Numeral +/- Numeral = Numeral (II + III = V), connection is followed by result (CR);

(b) Numeral = Numeral +/- Numeral (V = II + III), result is followed by connection (RC);

(c) Numeral = Numeral = Numeral (VI = VI = VI), a structure with two equal signs (TEQ).

While the CR and the RC structures have a high resemblance to common algebra problems the TEQ structure is very unusual. Therefore prior knowledge of common algebra should receive higher activation from the CR and RC structures than from the TEQ structure. The different amount of activation should affect the ease with which the constraints can be relaxed. We used only the RC and TEQ structures in the experiment.

For two operators that resulted in NO and SEQ constraint in Experiment 1, where only the RC structure was used, there are operators that are complimentary on a feature level in the TEQ structure. A task like VI = VII = V (VI = VI = VI) of the TEQ structure can be solved by moving a stick from one numeral to another numeral as in a task like VI = VII + I of the RC structure. We call this operator "Change of value". It should be easy to apply in both structures, because no implicit constraints are expected to occur.

The situation is quite different for the operator that resulted in the SEQ-constraint in the RC structure (we call it "Change of structure"). As we know from Experiment 1 it was very hard for problem-solvers to obtain the solution to a problem like III = III + III (III = III = III). The complimentary move in the TEQ structure in a task like VI = III = III (VI = III + III) should be much easier to apply because no implicit constraints from common algebra are expected for the TEQ structure. Moreover, changing the equal sign to a plus sign transforms the TEQ structure into the more usual RC structure. Therefore we expect a large difference in task difficulty between RC and TEQ structures for applying that operator.

Table 2: Tasks used in Experiment 2. Complementary moves on the feature level are required to solve problems of the RC and TEQ structures.

Move	RC-Structure	TEQ-Structure
Change of value	VI = VII + I (VII = VI + I)	VI = VII = V (VI = VI = VI)
Change of structure	III = III + III (III = III = III)	VI = III = III (VI = III + III)

Predictions for the transfer between the RC and TEQ structures can also be made from the notion of implicit constraints. While prior experience with problems of the TEQ structure should remove all constraints that normally occur in the RC structure, prior experience with problems of the RC structure should not result in decreased task difficulty for problems of the TEQ structure. This is so because in this structure we do not expect implicit constraints to occur.

Additionally we expect negative effects of prior working on the TEQ structure for Change of value in the RC structure because the prior experience with problems of the TEQ structure should lead to a mental set that favors operators that would normally not be available due to implicit constraints in the RC structure. This should make it harder to carry out the move that is normally the easiest to apply.

Subjects

Twenty-two undergraduates from the university of Hamburg participated for course credit.

Procedure

The procedure was the same as in Experiment 1 except two changes. One group of participants worked on a block of six problems (including one that required Change of value and one that required Change of Structure and four other problems) of the RC structure first and then on a block of six problems (including one that required Change of value and one that required Change of Structure and four other problems) of the TEQ structure. The other group worked on the blocks in reversed order. The time limit was raised to 8 minutes, to allow more solutions to be completed within the time limit.

Results

Figure 2 displays the means for Change of value and Change of structure problems in the RC and TEQ structures for different Order of presentation. The solution times were analyzed by computing a 2 X 2 X 2 ANOVA with the factors Order of presentation (RC-TEQ vs. TEQ-RC, between), Type of structure (RC vs. TEQ, within) and Type of move (Change of value vs. Change of structure, within).

The only significant main effect was for Type of structure, $F(1, 20) = 6.44$, $MS_e = 10180.46$, $p < .05$. The overall difficulty of tasks of the TEQ structure is lower. The significant 2-way interaction between Order of presentation and Type of structure, $F(1, 20) = 6.44$, $MS_e = 12912.31$, $p < .01$, shows that there is transfer from the TEQ structure to the RC structure only. For participants who solved TEQ problems first, Changes of structure were easier to obtain in both structures. For subjects who solved RC problems first, it was easier to Change values in both structures. This is reflected in a significant 2-way-interaction between Order of presentation and Type of move, $F(1, 20) = 11.46$, $MS_e = 12912.31$, $p < .01$.

Further, there is a significant 3-way interaction between Order of presentation, Type of structure and Type of move, $F(1, 20) = 7.19$, $MS_e = 8726.19$, $p < .025$. Long lasting impasses occured only in tasks of the RC structure where problem-solvers had to apply the Change of Structure operator.

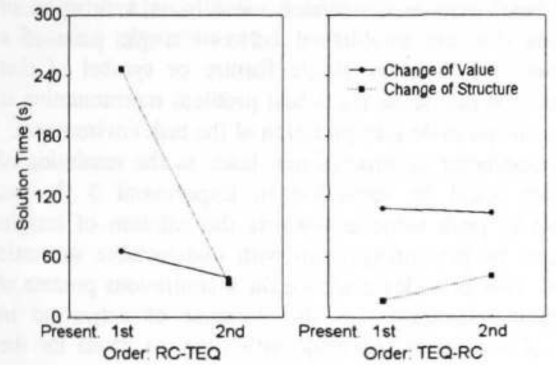

Figure 2: 3-way-interaction between Order of presentation, Type of structure and Type of move

Discussion

The predictions of the constraint relaxation hypothesis were supported by the results. As in Experiment 1 it was very hard for problem-solvers to carry out a move that transformed a common algebraic structure (RC) into an unusual structure (TEQ) because of implicit constraints. The complimentary move that transforms the unusual TEQ-structure into the more common RC-structure was easy to obtain because no implicit constraints occured in this structure. Moreover, prior experience with the TEQ-structure removed all constraints that normally occur in the RC-structure and makes the Change of value move harder to apply.

Experiment 3: Priming the correct solution

Constraint relaxation can be understood as re-distribution of activation over a network. This view implies that adding activation to nodes that are part of the solution should reduce the effect of constraints. An increase of activation of single nodes can be achieved by semantic priming even when the primes are short enough to rule out conscious processing. For the problems that lead to the longest impasses (EQ and SEQ in the Numeral = Numeral +/- Numeral structure) we implemented such a priming procedure. It started after 2 minutes (the time at which least solutions were observed in Experiment 1) if subjects had not solved the problem before. We predicted that such a procedure would prevent or shorten impasses.

Participants:

Thirty-five undergraduates from the university of Hamburg participated for course credit.

Procedure

The procedure was the same as in Experiments 1 and 2 with a time limit of 8 minutes and only one block consisting of 8 different MSA tasks (5 NO constraint tasks, 1 OP-, 1 EQ-, and 1 SEQ-constraint task). In the priming procedure the subjects were instructed to fixate the first position in the task (III in III = III + III) as soon as a sound occurs and to change the fixation to the second position (= in III = III = III) only when the content of the actual position was displayed again after being hidden for 300 msec behind a black mask.

They remained fixated on the second position, until the content of that position was displayed again, after being hidden by a 300 msec black mask. In that manner subjects attended to each part of the equation in succession. In the experimental group at one or two (correct) position(s) the symbol(s) constituting the solution were shown for 17 msec (for example in III = III + III a '=' was shown at the position of the '+'). In the control group only the mask was shown. After having completed this procedure, subjects continued to solve the problem for one minute, then the priming procedure started again. This circle was repeated until subjects solved the problem or the time limit was reached.

Results

Figure 3 shows the cumulative distribution of solution frequencies for the EQ and SEQ problems for different priming intervals in the priming and the control conditions. In the priming condition seven subjects solved one and two subjects solved two of the EQ and SEQ problems after the first and second primes. In the control condition no solutions were observed in this time period.

Using a χ^2-test, we compared the frequency of solutions after the first and second prime with the frequency of solutions that occured after the the third prime (including not solved problems) in both experimental groups. The difference was highly significant, χ^2 (df = 1) = .0004. Subjects in the priming condition solved significantly more problems in the intervals following the first and second primes, although none of the subjects reported having detected the prime during the experiment.

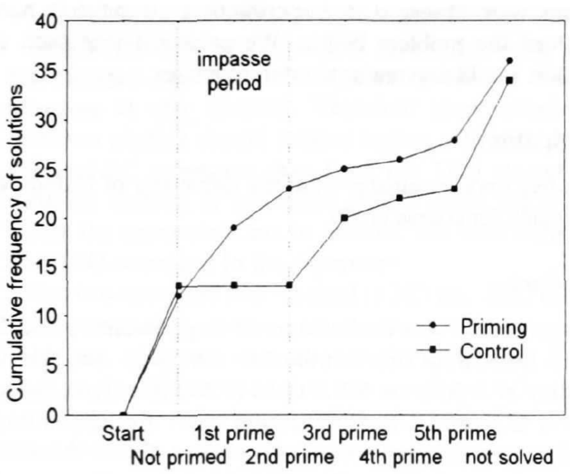

Figure 3: Cumulative frequency of solutions in different priming intervals for the EQ and the SEQ problem in priming and control conditions.

Discussion

The results of experiment 3 provide strong evidence for the notion of re-distribution of activation during insight problem solving. Undetectable semantic primes led to more frequent solutions to insight problems in the impasse phase, where in an control group no subject found a solution to the problem in that phase.

General Discussion

The results of our experiments provide evidence for several facts related to Ohlsson's (1992) framework for insight problem solving, particularly the notion of constraint relaxation. In Experiment 1 it was shown that there are discontinuities in problem solving that can not be attributed to the depth and the width of a problem-solver's problem space (her/his representation of the problem). This can not be explained by theories that hypothesize insight to occur only in problems where a change of problem space becomes necessary, because there are too many possibilities in the original problem space to be tested or certain features of the problem are not included in the problem representation (Kaplan & Simon, 1990). It also rules out any theory that handles insight problem solving from a business-as-usual perspective (Seifert et al., 1994), regarding insight phenomena as an either not important or non-existent cognitive phenomenon (Weisberg, 1981).

Our results support the notion of impasses that describe a mental state in which no problem-solving occurs. They also show that one reason for impasses are implicit constraints that problem-solvers impose on the goal state in insight problems. These constraints are removed as soon as the solution to one problem that activates certain constraints is known. This can be seen from the large transfer to other tasks of the same type.

The results of Experiment 2 show that the structure in which a problem is presented governs the activation of prior knowledge. Different meanings constituted by the structure reversed the pattern of task difficulty. The way in which activation is distributed initially is a function of relations that are established between single parts of a structure, not of every single feature or symbol of that structure. In this sense the actual problem representation is merely one possible interpretation of the task environment.

One restructuring process that leads to the resolution of impasses could be identified in Experiment 3. It was possible to push subjects towards the solution of insight problems by presenting them with undetectable semantic primes. This provides evidence for a continuous process of constraint relaxation, i.e. the decrease of activation in nodes of a semantic network with time, as cause for the subjective experience of sudden restructuring.

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