Why Some Arithmetic Facts Are Harder to Remember

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A continuing puzzle within the study of simple arithmetic fact retrieval is why some facts take longer to retrieve than others. Although reaction times may reflect representational structure or content—and it is the larger arithmetic facts which take longer to retrieve—our results suggest that this *problem size effect* may be due to more general effects of memory underlying cognitive skill.

It has been variously hypothesized that the problem size effect in adults is due to relative fact frequency (large facts are practiced less often; Ashcraft, 1987), order of acquisition (large problems are learned later and suffer intrusions from early problems; Campbell & Graham, 1985), or initial backup strategy (large problems are more error-prone with counting strategies; Siegler, 1988). Two of these hypotheses assume that initial error rates during learning affect later retrieval times, even after extensive practice.

Two experiments used *alphaplication* facts, in which random letters replaced digits in a non-base-10 arithmetic table, to test the above hypotheses. In one experiment, which tested the *relative frequency* and *strategy* hypotheses, subjects were familiarized with counting and addition in the domain; then half were coached to use a repeated addition strategy while the rest used a mnemonic strategy, and all were given extensive practice (40 trials per item for high frequency items) on selected facts. Despite an effect of strategy on initial error rates, there was no significant effect of strategy on retrieval times for well-practiced items. A second experiment, which tested the *order of acquisition* hypothesis and used no strategy coaching, found greater initial error rates for the first fact subset than the second, but no significant differences for test RT after 40 practice trials per item. Thus in neither experiment can differences in initial error rates be considered a predictor of significant differences in RT.

Of these three hypotheses, support was obtained only for the relative frequency hypothesis. However, since relative frequency and total practice trials were confounded, post hoc analyses were performed; these revealed that the frequency effect was due to amount of practice, not to relative frequency during practice. These results are consistent with Logan and Klapp's (1991) findings that the development of automaticity depends on number of trials per item, not total number of trials, set size, or number of sessions.

These results suggest that once direct retrieval has become the primary or only strategy, response time differences indicate differences in retrieval practice, not fact representational structure or content. Moreover, the effect of early errors on later retrieval speed seems to wash out, at least for the production task. It is an open question whether other less-practiced but skill-related tasks, drawing on the same representational content, would be more vulnerable to early errors. Results from additional tasks in these experiments—fact verification and numeric comparison of pairs in the form "a x b" and "c"— support this speculation for the verification task but not numeric comparison. Zbrodoff and Logan (1990) concluded that arithmetic facts are treated as wholes rather than as one-way links from problem to answer; thus answer priming of incorrect trials may be the reason that verification is vulnerable to early errors while production and number comparison are not.

More generally, our results cast doubt on two kinds of memory theories. Semantic memory theories assume that response times reflect semantic memory structure; however, our results indicate that such a simple measure reflects nothing more than practice. Additional data that we have collected test whether RT-based measures of transfer can detect conceptual structure since same-task RT does not. Our results on the effect of relative frequency versus total practice also seem to contradict the prediction of strength theories. Such a theory would scale trials per problem against total trials to yield a strength proportional to relative problem frequency and fairly insensitive (except for proportion of error trials) to absolute number of trials. Our concurrent computer simulation of different memory theories extends these findings into the theoretical domain.

References

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