The Effect of Heuristic Induction on transfer in Mathematical Problem Solving

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The purpose of this study was to explore the effect of Heuristic learning on transfer in mathematical problem solving. Especially, we have focused on following two questions: (1)whether explicit heuristic training is effective, and (2)whether transfer is promoted if students succeed in making good abstraction themselves that referred to the heuristic. The later question is original one. There is few evidence that the appropriate abstraction promotes transfer in high-level mathematical problem solving.

Method

Subjects. Subjects were 51 undergraduates.

Materials. Subjects were given three examples and three test problems. Every problem dealt with straight lines and they can be solved by using the heuristic: represent the straight lines by using parameters. The sample of the problems is presented in the Appendix. All of the subjects were not able to solve Example 1.

Procedure. Subjects were randomly assigned to two conditions, that is, *Instruction* condition and *non-instruction* condition.

In the training session, they were required to solve three example problems. The correct solutions of the examples were shown to subjects after trying to solve each example. In the instruction condition, the explanation about above mentioned heuristic was presented with each correct solution. In the non-instruction condition, the heuristic was not explained at all.

Subjects ware asked to summarize what they had learned from this problem, the solutions, and the mistakes, after each solution was presented. In both condition, subjects were assigned to two groups based on the content of the summary. If a subject referred to the heuristic mentioned above, he or she was assigned to the *good* students group. The subjects who had not referred to the heuristic were treated as *poor* students.

The test session was done one week later. Subjects were required to solve three test problems.

Results and Discussion

Each problem was marked on a scale of 10. In both sessions the scores on the three problems were totaled. Table 1 shows the scores in the test session.

The effect of explicit heuristic training

The percentage of good students was 76 % in the instruction condition and 65 % in the non-instruction condition.

Table 1 The means of subjects' scores in the test session

	instruction		non-instruction	
	poor	good	poor	good
N	6	19	9	17
Mean	2.17	6.11	2.78	9.47
SD	2.56	6.14	4.06	8.68

There was no difference in the percentages between the two conditions. Moreover, the difference of mean scores in test session between two conditions (instruction and non-instruction) was not significant. Thus, we can say that explicit heuristic training was not effective in this experiment.

The effect of Heuristic Induction

Although the explicit heuristic training was not effective, there is an evidence that supports the usefulness of the heuristic. The good students have shown good transfer in the test session. The means of the good students' scores were significantly different from those of the poor students'(t=2.25, df=20.7, p=.04 in the instruction condition; t=2.68, df=23.8, p=.01 in the non-instruction condition). Note that there were no differences between the two students' groups in the training session. The results of this experiment show that transfer in the high-level mathematical problem solving is facilitated if students do good abstraction.

In the non-instruction condition, the good students have induced the heuristic without any instructions about it. The abstraction seemed to be done by self-explanations (Chi *et al.*,1989). The results also shows that the good self-explanations facilitates transfer in the high-level mathematics.

We will have to explore why the explicit heuristic training was not effective in contrast to the good performance of the good students.

Appendix

Example 1: A straight line L: $\frac{x+6}{2} = y - 2 = \frac{z-6}{-1}$ and a point A(0,-1,-3) is given. Find the coordinate of H: foot of the perpendicular from A to L.

References

Chi, M.T.H., Bassok, M., Lewis, M., Reimann, P., & Glaser, R. (1989). Self-explanations: How students study and use examples in learning to solve problems. *Cognitive Science*, 13, 145–182.