

Mental Predicate Logic: An Empirical Examination

Yingrui Yang and Martin D.S. Braine

New York University
6 Washington Place, 8th Floor
New York, N.Y. 10003
yang@xp.psych.nyu.edu

Braine's theory of mental predicate logic (unpublished ms.) will be briefly introduced. A preliminary empirical examination for a major subset of 10 schemas proposed by Braine's model will be reported. The methodology is a modified version of that used by Braine, Reiser, & Rumin (1984). We design a large number (64) of reasoning problems of predicate logic sort-most fairly simple-for which the model predicts the reasoning that subjects will use to solve them. In each problem, one or more facts are given-these serve as premises, followed by a conclusion to be evaluated. Subjects must decide whether the proposed conclusion is true or false, given those facts. They are also to rate each problem's subjective difficulty on a 7-point scale right after evaluating the conclusion given. Within this set of problems, the number of reasoning steps is varied. The mental logic must account (among other things) for which reasoning steps are easy and which are difficult, and it is expected that adding additional steps will make a problem more difficult.

There are three major theses: (a) Because the inferences of the model are claimed to be generally available to people, errors made by subjects in judging the truth value of the conclusion given to each problem will be few overall. (b) The difficulty of a problem will be correlated with the number of inference steps required to solve it, as defined by the reasoning routine associated with the inferences. (c) Problem difficulty will be even more closely correlated with the summed difficulties of those inferences. We conducted a study by using three samples (N=20 for each). With respect to Thesis (a), the results found error rates of lower than 3%, i.e., 97% accuracy, confirming the expectation. With respect to Thesis (b), the mean ratings of problem difficulty were very significantly correlated with the number of inferential steps required by the model to solve a problem ($r=.8$), and the correlations remained significant when extraneous variables, like problem length, were partialled out-again a confirmation of expectation. With respect to Thesis (c), each kind of inference defined in the logic was assigned a difficulty weight estimated from the difficulty ratings of subjects. The difficulty weights are estimated by using the algorithm "Praxis" (Brent, 1973; Gegenfurtner, 1992) to obtain the best least-square fit of predicted problem difficulties to the obtained mean difficulty ratings (taking the predicted difficulty to be the sum of the difficulty weights of inferences involved in solving a problem the model's way). Correlations between predicted and actual difficulties are high (around .93) and remain to .8 when problem length is partialled out. In cross-validation analyses among three samples, the correlations remained at almost the same level.

A certain order of schema difficulty ranks was found consistently among three samples. There are two important applications of this ranking order among schema weights. First, by using this order, we successfully classified the 10 schemas into two groups and made a 2-parameter model. With respect to Theses (b) and (c), by using this 2-parameter model, all the corresponding correlations remained only about .1 lower than that for 10 parameter model. This result offers a simple answer to the question of which inferences are easy and which are more difficult. Second, to form a 2-parameter model by not keeping this ranking order, the results disproved the prediction.

Reference

- Braine, M.D.S., (1995). Steps toward a mental predicate logic.
- Braine, M.D.S., Reiser, B.J., & Rumin, B. (1984). Some empirical justification for a theory of natural propositional logic. In G.H. Bower (Ed.) *The psychology of learning and motivation: Advances in research and theory* (Vol. 18). New York: Academic Press.
- Brent, R.P. (1973). *Algorithms for function minimization without derivatives*. Englewood Cliffs, NJ: Prentice-Hall.
- Gegenfurtner, K.R. (1992). PRAXIS: Brent's algorithm for function inimization. *Behavior Resaearch Method Instruments & Computers*, 24(4) 560-564.