

Ways of Locating Events

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Abstract

This paper argues that the basic modes of spatial cognition can be best identified in terms of argument/participant location, and shows that natural language uses "simple" types of semantic denotations to encode spatial cognition. First we review event-based approaches to spatial location, and point out that spatial expressions should be interpreted not as locating an event/state as a whole but as locating arguments/participants of the sentence/event. Section 2 identifies the ways of locating events/states in terms "argument orientation", which indicates the ways of interpreting locative expressions. We identify four patterns of argument orientation which reveal substantial modes of spatial cognition--spatial properties and relations. Section 3 illustrates various classes of English transitive verbs with which spatial expressions induce argument orientation. We consider four types of locative prepositional phrases and show that the argument orientation pattern of a sentence is not determined by the type of spatial expressions but mostly by the type of the verb, i.e., the event type of the sentence. Section 4 concludes that semantic denotations of locative prepositional phrases are restricted to the "intersecting" functions mapping relations to relations, which are "basic and familiar" semantic objects out of the "heterogeneous" field of functions from relations to relations.

Event and Location

There have been proposals which take "events" as primitive entities and introduce locational properties for events. Davidson (1967, 1970), Fillmore (1968, 1971), Sondheimer (1978), and Parsons (1990) among others. Davidson (1967, 1970) interprets spatial reference adverbs as locating an event in space as illustrated below.

- (1) John walked to the barber's
= $(\exists e) \text{Walking}(\text{John}, e) \ \& \ \text{To}(e, \text{the barber's})$
- (2) John walked across the street to the barber's
= $(\exists e) \text{Walking}(\text{John}, e) \ \& \ \text{Across}(e, \text{the street})$
& $\text{To}(e, \text{the barber's})$

Davidson's semantics, however, does not tell us what the properties of events really mean. That is, what does it mean for an event to have the property "To the barber's" or "Across the street". Thus the meaning representations in (1-2) cannot account for a simple and basic entailment pattern illustrated in (3-4).

- (3) John walked to the barber's
entails 'John was at the barber's'
- (4) John walked across the street to the barber's
entails 'John was on the street'

Fillmore's (1968, 1971) case-theoretic approach is also far from solving this problem: Fillmore's (1968) "Locative" case (Source/Goal/Path introduced later in Fillmore (1971)) was defined as identifying the location or spatial orientation of the "state" or "action" identified by the verb. Again, his interpretation of spatial expressions does not account for the entailment of (3-4).

Sondheimer (1978) gives a strong lexicalist account of the entailment pattern. Sondheimer interprets locative prepositions not as a property of events but as a property of individuals, so he gets the following interpretation:

- (5) John stumbled in the park.
= $(\exists e)(\exists p) \text{Stumbling}(e) \ \& \ \text{O}(e, \text{John}) \ \& \ \text{P}(e, p)$
& $\text{IN}(p, \text{the park})$

The case indicated by "O(bject)" is a neutral case referring to the entity that is affected or discussed. Sondheimer, further, gives meaning postulates to account for (3-4). The meaning postulates are claimed to be determined by the event type. For example, we have a meaning postulate of (7) to interpret the sentence (6)

- (6) On the bus, Mary thought of John
- (7) Meaning postulate:
 $(\forall x) \text{Thinking}(x)$
--> $(\exists y)(\exists q) \text{BEING-AT}(y) \ \& \ \text{O}(y, (ta)A(x,a))$
& $\text{P}(x, q) \ \& \ \text{P}(x, q) \ \& \ (q \in (tp)P(x, p))$

Since the event involved in (6) is "Thinking", Sondheimer claims, the location of the event says something only about the location of the agent. He does not examine various event types, nor gives any generalization over the patterns of locating events. Sondheimer's approach is important in that his lexical treatment reduces the location of event to the location of participants of the event. Similar approaches have been proposed in Geis (1975) and Cresswell (1978): The former is about some stative locational expressions as in (8) and the latter about directional ones as in (9).

- (8) John stumbled *in the park*.
= 'John stumbled while he was in the park'
- (9) a. Arabella ran *across the yard* from Bill.
b. Arabella stands *across the yard* from Bill.

Recently, Parsons (1990) extends such approach in a form of generalized meaning postulate, which is heavily based on the assumption that "every verb takes a Theme-

argument". The generalized meaning postulates are determined by the type of locative expression. For example, the meaning postulate for prepositional phrases with "onto" is stated as follows: "Any event that is onto something results in a state of being on that thing. The Themes of the event and the state are the same." Parsons claims that this postulate, which is independent of any choice of verb, yields all of the following inferences when applied to the logical forms of the sentences:

- (10) Mary will throw the ball onto the roof.
 --> The ball will be on the roof.
 (11) Mary will push the cow into the barn.
 --> The cow will be in the barn.

Parson's generalization, in short, is that locative expressions always determines the location of the theme-argument. This generalization, however, is both too strong and too weak: As we will see shortly, not every theme-argument is assigned a location by a locative expression, and locative adverbs can determine the locations of non-theme arguments. Further, we will see cases where a locative determines the location(s) of multiple arguments as in (12-13), and we find some constraints on coordination of locative expressions illustrated in (14):

- (12) John escorted Mary into the theater.
 (13) John watched Mary through the window.
 (14) a. *John saw Mary either in the garden or from the rooftop.
 b. John saw Mary in the garden from the rooftop.
 c. ?John saw Mary from the rooftop in the garden.

Argument Orientation of Locative PPs

Now we characterize how natural language uses locative expressions to locate events or states. This paper identifies the ways of locating events in terms of "argument orientation", which indicates the ways of interpreting locative expressions. The patterns of argument orientation reveal substantial modes of spatial cognition -- spatial properties and relations. For example, one of the patterns identifies itself in a sentence such as *John saw Mary in the garden*, where the PP *in the garden* refers to the location of the object argument 'Mary' and so the PP is said to be *object-oriented*. Restricting ourselves to the sentences built from transitive verbs, we illustrate four types of argument orientation. We interpret locative adverbs as predicate modifiers, i.e., functions mapping n-ary relations into n-ary relations for $n > 1$.

Subject-Orientation

The first pattern is *subject-orientation* where locative PPs refer to the location of a subject argument. A general definition of subject-oriented predicate modifiers is given by in (15). Here we write \mathbf{R}^2 for the set of binary relations over a given universe.

- (15) Definition: For all functions $f \in [\mathbf{R}^2 \rightarrow \mathbf{R}^2]$, f is *subject-oriented* (SO) iff for all binary relations $S, T \in \mathbf{R}^2$, if $S_1 = T_1$ then $(f(S))_1 = (f(T))_1$, where $R_1 =_{df} \{ \alpha \mid \exists \beta. \langle \alpha, \beta \rangle \in R \}$, i.e., $R_1 =$ the domain of R .

In other words, subject-oriented functions treat sets of first coordinates of binary relations uniformly. For instance, if at a particular instance 'those who criticized someone' and 'those who mentioned someone' are identical, 'those who criticized someone at the meeting' and 'those who mentioned someone at the meeting' are identical. The locative PPs in (16a,b) are interpreted by subject oriented functions. Thus, (16a,b) entails (17a) but not (17b).

- (16) a. John criticized the teacher at the meeting
 b. John mentioned the teacher at the meeting.
 (17) a. John was at the meeting
 b. the teacher was at the meeting

Object Orientation

The second pattern is *object-orientation* where locative PPs refer to the location or trajectory of an object argument. Thus formally,

- (18) Definition: For all functions $f \in [\mathbf{R}^2 \rightarrow \mathbf{R}^2]$, f is *object-oriented* (OO) iff for all binary relations $S, T \in \mathbf{R}^2$, if $S_2 = T_2$ then $(f(S))_2 = (f(T))_2$, where $R_2 =_{df} \{ \beta \mid \exists \alpha. \langle \alpha, \beta \rangle \in R \}$.

In other words, object-oriented functions treat sets of second coordinates of binary relations uniformly. For instance, if at a particular instance, 'those who were kicked by someone' and 'those who were thrown by someone' are identical, 'those who were kicked by someone into the box' and 'those who were thrown by someone into the box' are identical. The locative PPs in (19) are object oriented, and (19a,b) entail (20).

- (19) a. John threw the ball into the box
 b. John kicked the ball into the box
 (20) The ball moved into the box

Subject and Object Orientation

The third pattern is *subject and object-orientation* where locative PPs refer to the location or trajectory of a subject and an object argument independently. The definition goes:

- (21) Definition: For all functions $f \in [\mathbf{R}^2 \rightarrow \mathbf{R}^2]$, f is *subject and object-oriented* (S+O) iff f is both subject-oriented and object-oriented.
 (22) entails both (23a) and (23b), and (24) entails both (25a) and (25b), so the PPs are oriented to both of the subject and the object.
 (22) John met Mary in the office
 (23) a. John was in the office
 b. Mary was in the office
 (24) John escorted Mary into the room
 (25) a. John moved into the room
 b. Mary moved into the room

(23) and (24) use a single spatial property to locate arguments, so 'being in the office' is used in (23), and 'moving into the room' in (24). But, the sentences in (26) illustrate that a locative PP can refer to the locations of subject and object arguments with a pair of different spatial properties which are complement to each other, i.e., '**be-in-the-control-tower**' and '**¬be-in-the-control-tower**'. Thus both (26a) and (26b) entail (27).

- (26) a. John spied on Mary from the control tower
 b. John saw Mary from the control tower
 (27) John was in the control tower and Mary was not in the control tower

Subject-Object Orientation

The last pattern is *subject-object orientation* where locative PPs refer to a spatial dependency between the subject and the object arguments. For example, (28) entails (29), which imposes a unique spatial relation between the two arguments. The definition is given as (30) below:

- (28) John saw Mary through the window
 (29) John and Mary were on the opposite sides of the window
 (30) Definition: For all functions $f \in [R^2 \rightarrow R^2]$, f is *subject-object-oriented* (S×O) iff for all $S \in R^2$, if $\langle x, y \rangle \in f(S)$ then $\langle x, y \rangle \in R_f$, where R_f is a spatial relation determined by f .

This pattern is different from *subject and object orientation* (S+O) in that the functions in (S×O) cannot be reduced to a boolean compound of a subject oriented function and an object oriented function, while the latter functions in (S+O) can. Symmetric path locatives (PPs headed by *across, through, over, past, and around*) induce this pattern (S×O), and refer to a spatial dependency between subject and object arguments.

We note, without proof, relations among the argument oriented functions: First, if a function is subject and object oriented (S+O), then it is subject oriented (SO) and object oriented (OO). Second, if a function is SO or OO, then it is subject-object oriented (S×O), so $SO \subseteq S \times O$ and $OO \subseteq S \times O$.

Argument oriented functions take a binary relation R as a pair of sets, i.e., $\langle R_1, R_2 \rangle$, where R_1 and R_2 refer to the set of first coordinates and that of the second coordinates, respectively, and restrict one or both of the sets. In other words, a locative PP modifying a binary relation R uniquely determines a function to restrict R_1 or/and R_2 . We will show that such functions are restricting, increasing, and additive, and thus intersecting. Further we will prove that the set of possible denotations of unary/binary locative PPs is isomorphic to the set of properties/relations.

For instance, *into the room* induces the property $P =$ 'move from outside to inside of the room' to restrict R_1 or/and R_2 . Thus *x pushed y into the room* is true iff 'x pushed y' and 'y moved from outside to inside of the room', i.e., **into the room**(push)(y)(x) = (push)(y)(x) \wedge P(y).

Argument Orientations with Various Transitive Verbs

This section illustrates various classes of transitive verbs with which locative PPs induce argument orientations. Here we consider four types of locative PPs:

- (31) a. Stative Locatives: PPs with *in, on, under, above, in front of, behind*
 b. Asymmetric Path Locatives: PPs with *into, out of, onto, off, up, down*
 c. Symmetric Path Locatives: PPs with *across, through, over, past, around*
 d. Source Locatives: PPs with *from*

The four types show different syntactic and semantic characteristics (Nam 1985). In the following, we give examples of combination of two place predicates and locative PPs. To the right of each example, we noted the argument orientation pattern of the locative PP in it. For simplicity, the following abbreviation is used: O(object-orientation), S(subject-orientation), S+O(subject and object orientation), S×O(subject-object orientation).

- (32) Motion-Causative verbs: *draw, drag, pull, push, throw, hit, knock, run, walk, jump*
 Verbs of 'Sending/Carrying': *mail, convey, deliver, pass, return, carry, take, bring*
 a. John drew the box in/into the room [O]
 b. Kim pushed Mary off the bed [O]
 c. Sue threw the ball across the field [O]
 d. Sue passed the book across the table [O]
 e. Tom took the kids from their school [O]

- (33) Verbs of Placement: *place, arrange, install, position, set, situate, put*
 Verbs of 'Hunting': *dig, hunt, mine, shop, watch*
 a. John installed the machine in the office [O]
 b. Kim dug a fork into/out of the pie [O]
 c. Sarah watched the man across the street [O]
 d. *Sarah put the book from the bag.

As (33d) shows, verbs of 'placement' or 'hunting' do not go with a source type locative like *from the bag*. (33c) is ambiguous that the reference point for the PP can be interpreted deictically (e.g., 'from here') or as given by the location of Sarah (i.e., 'from Sarah'). Both of the readings induce object-orientation of *across the street*.

- (34) Verbs of 'Combining/Attaching': *mix, whip, tape*
 Verbs of 'Housing': *house, contain, fit, hold, seat, sleep, store, serve*

- a. John mixed water and flour on the plate [O]
 b. They sleep four people in each room [O]
 c. The captain housed the soldiers into the big hotels [O]

- (35) Verbs of 'Perception': *see, touch, feel, hear, sense, observe, examine, discover, watch*

Verbs of 'Communication': *call, wire, cable*

Verbs of 'Contact': *touch, pat*

- a. John saw Mary in the garden [O]
 b. *John saw Mary into the garden [O]
 c. John touched Mary across the table [S×O]
 d. John watched Mary from the rooftop [S+O]

The verbs of perception exhibit three different types of argument orientation: (35a) is an example of object orientation; (35c) subject-object orientation so the PP *across the table* refers to a spatial dependency between John and Mary, i.e., 'John and Mary are on the opposite sides of the table'. (35d) is another example of multiple orientation but a different one from (35c): (35d) entails 'John was on/at the rooftop' and 'Mary was not on/at the rooftop', so the PP *from the rooftop* involves the locations of John and Mary but independently. This type of argument orientation (S+O) is illustrated by other transitive verbs below.

(36) Verbs of 'Co-movement': *escort, accompany, chase, follow, tail, lead, guide*

- a. John escorted Mary in/into the museum [S+O]
- b. The dog chased the cat across the garden [S+O]
- c. The teacher led the kids from the playground [S+O]

(37) Verbs of 'Social Interaction': *meet, date, hug, marry, fight, visit, quarrel*

- a. John met Mary at the meeting [S+O]
- b. *John visited Mary into her office

The lexical meaning of each verb in (36-37) naturally implies the subject and the object are located in the same place and locative PPs refer to it. Finally, we illustrate verbs which only induce subject orientation of locative PPs.

(38) Verbs of 'Judgement': *criticize, compliment, honor, thank, insult, ridicule*

Psych-verbs: *adore, idolize, miss, worship, despise*

Intensional verbs: *search, look for, seek, mention*

- a. John criticized Mary at the meeting [S]
- b. John was looking for a knife in the kitchen [S]
- c. John mentioned Mary at the meeting [S]

Table-1: Argument Orientation Patterns of Locative PPs with Transitive Verbs

StativeLoc.	Asym.PathLoc.	Sym.PathLoc.	SourceLoc.
<i>Motion-Causatives, Verbs of 'Sending/Carrying'</i>			
O	O	O	O
<i>Verbs of Placement, Verbs of 'Hunting'</i>			
O	O	O	*
<i>Verbs of 'Combining/Attaching', Verbs of 'Housing'</i>			
O	O	*	*
<i>Verbs of 'Perception', Verbs of 'Communication', Verbs of 'Contact'</i>			
O	*	S×O	S+O
<i>Verbs of 'Co-movement'</i>			
S+O	S+O	S+O	S+O
<i>Verbs of 'Social Interaction'</i>			
S+O	*	*	*
<i>Verbs of 'Judgement', Psych-verbs, Intensional verbs</i>			
S	*	*	*

(Table-1) above summarizes the facts we have seen in this section. The stars (*) in the table indicate the relevant combinations are not acceptable. We note the following facts from the table: (i) If a non-stative locative combines with a transitive verb, it is always oriented to the object argument. That is, it can be either O, S+O, or S×O; (ii) if a transitive verb can combine with a non-stative locative, then stative locatives are object-oriented with the transitive verb, i.e., either O or S+O; (iii) only symmetric path locatives can be S×O, i.e., other locatives are all reducible; (iv) verbs of 'judgement', psych-verbs, and intensional verbs only induce subject orientation (S) with stative locatives, which suggests that object orientation is more prevailing than subject orientation.

Constraints on the Semantic Denotations of Locative PPs

This section characterizes three denotational constraints on the interpretation of locative PPs in English. We claim: English locative PPs are interpreted as denoting (i) *restricting* functions, (ii) *monotone increasing* functions, and (iii) *additive* functions. Due to the constraint of (iv) *argument-orientation* discussed in the preceding sections, these bring us a highly restrictive class of functions the locative PPs can denote, namely *intersecting* functions. In the following, we illustrate that locative PPs should satisfy the constraints. The four constraints (i-iv) are nearly independent from each other, so none of the constraints implies any one of the others but (iii) *additivity* implies (ii) *monotone increasing*.

First, locative PPs denote *restricting* functions (the definition repeated in (39)). Thus (40a) entails (40b), i.e., **in-the-garden**(see) is a subset of the binary relation **see**.

(39) Definition: Let B be a boolean algebra, and let $f \in [B \rightarrow B]$ be arbitrary.

Then f is *restricting* iff for each $x \in B$, $f(x) \leq x$.

- (40) a. John saw Mary in the garden
- b. John saw Mary

Second, locative PPs denote *monotone increasing* functions defined as in (41). (a)-sentences in (42-44) entail (b)-sentences, respectively. These illustrate locative PPs denote increasing functions mapping binary relations to binary relations.

(41) Definition: Let B be a boolean algebra, and let $f \in [B \rightarrow B]$ be arbitrary.

Then f is *monotone increasing* iff for all $x, y \in B$, if $x \leq y$, then $f(x) \leq f(y)$.

- (42) a. John roughly pushed Mary into the bus
- b. John pushed Mary into the bus
- (43) a. John returned the book from LA to San Jose
- b. John sent the book from LA to San Jose
- (44) a. John saw and touched Mary through the window
- b. John saw Mary through the window and John touched Mary through the window

Now we show locative PPs denote *additive* functions. Thus (a)-sentences of (46-47) entail (b)-sentences, respectively, and vice versa.

- (45) Definition: Let B be a boolean algebra, and let $f \in [B \rightarrow B]$ be arbitrary.
Then f is *additive* iff for all $x, y \in B$,
 $f(x \vee y) = f(x) \vee f(y)$.
- (46) a. John kicked or pushed the ball into the room
b. John kicked the ball into the room or John pushed the ball into the room
- (47) a. John walked or jogged in the park
b. John walked in the park or John jogged in the park

The fourth constraint is *argument orientation* of locative PPs. The following just repeats the definitions of *subject orientation* and *object orientation* given in the previous section. We claim that locative PPs denote argument oriented functions.

- (48) Definition: For all functions $f \in [R^2 \rightarrow R^2]$ where R^2 is the set of binary relations,
 f is *argument-oriented* iff f is subject oriented or object oriented.

Argument oriented functions take a binary relation R as a pair of sets, i.e., $\langle R_1, R_2 \rangle$, and each function affects R_1 or/and R_2 depending on whether it is SO or/and OO. Thus, if f is object-oriented, f determines a unique function $f^*: P \rightarrow P$ such that $f^*(R_2) = (f(R))_2$ for all $R \in R^2$.

Now to draw out a general claim of intersectivity constraint, we show that the function (f^*) determined by an argument oriented function (f) inherits the properties of f illustrated in (39), (42) and (45): Thus for all object-oriented functions $f \in [R^2 \rightarrow R^2]$, $f^* \in [P \rightarrow P]$ such that $f^*(R_2) = (f(R))_2$, and for all $R \in R^2$, we show:

- (49) If $f \in [R^2 \rightarrow R^2]$ is restricting, $f(R) \leq R$, then $(f(R))_2 = f^*(R_2) \leq R_2$. Thus f^* is restricting, too.
- (50) If $f \in [R^2 \rightarrow R^2]$ is increasing, i.e., if $S \leq T$, $f(S) \leq f(T)$, then if $S_2 \leq T_2$ then $(f(S))_2 \leq (f(T))_2$, i.e., $f^*(S_2) \leq f^*(T_2)$. Thus f^* is increasing.
- (51) If $f \in [R^2 \rightarrow R^2]$ is additive, i.e., $f(S \vee T) = f(S) \vee f(T)$, then
 $(f(S \vee T))_2 = (f(S) \vee f(T))_2 = (f(S))_2 \vee (f(T))_2$, and so
 $f^*((S \vee T)_2) = f^*(S_2 \vee T_2) = f^*(S_2) \vee f^*(T_2)$.
Thus f^* is also additive.

In the same way, we can prove that subject-oriented functions also inherit these properties. That is, for all subject oriented functions $f \in [R^2 \rightarrow R^2]$, there is a unique function $f^* \in [P \rightarrow P]$ such that $f^*(R_1) = (f(R))_1$ for all $R \in R^2$. Let us define the set of such functions as follows:

- (52) $SO^* = \text{df. } \{f^* \in [P \rightarrow P] \mid f^*(R_1) = (f(R))_1 \text{ for all } R \in R^2 \text{ and all subject-oriented functions } f \in [R^2 \rightarrow R^2]\}$
- (53) $OO^* = \text{df. } \{f^* \in [P \rightarrow P] \mid f^*(R_2) = (f(R))_2 \text{ for all } R \in R^2 \text{ and all object-oriented functions } f \in [R^2 \rightarrow R^2]\}$

Then, from the previous discussion in (49-51), we see all functions in SO^* and OO^* are intersecting, increasing, and additive functions. We also note the following:

- (54) Theorem-1: $SO^* = OO^*$
(55) Theorem-2: SO^*/OO^* is identical to $INT[P \rightarrow P]$, the set of intersecting functions in $[P \rightarrow P]$.
(Proof included in Appendix)

Now by the theorem (51) from Keenan and Faltz (1985) we get (57) Theorem-3.

- (56) Theorem (K&F 1985:147): Let B be a boolean algebra, and let the function $u: [B \rightarrow [B \rightarrow B]]$ be defined as follows: given $b \in B$, $u(b)$ is that function from B into B such that for each $c \in B$, $(u(b))(c) = c \wedge b$. Then u is an isomorphism of B onto $INT[B \rightarrow B]$, the set of intersecting functions from B into B .

- (57) Theorem-3: SO^*/OO^* are isomorphic to the set of properties P .

(56) and (57) state a main result on the denotational constraints for unary locative PPs. This result also applies to binary locative PPs: Each binary locative PP (Symmetric Path locatives) uniquely determines a function which refers to a spatial dependency between two arguments. And, as we showed for unary locatives, such functions are restricting, increasing, and additive, and the set of their denotations is isomorphic to the set of binary relations (R^2), and so isomorphic to the set of intersecting functions from R^2 into R^2 . Formally, with the definition in (58), we get (59) and (60):

- (58) $S \times O^* = \text{df. } \{f^* \in [R^2 \rightarrow R^2] \mid f^*(S) = S \& R_f \text{ for all } S \in R^2 \text{ and all } f \in S \times O, \text{ and } R_f \text{ is a spatial relation determined by } f\}$

- (59) Theorem-4: $S \times O^*$ is identical to $INT[R^2 \rightarrow R^2]$, the set of intersecting functions in $[R^2 \rightarrow R^2]$.

- (60) Theorem-5: $S \times O^*$ is isomorphic to the set of binary relations R^2 .

Finally, let us consider a sentence with a three place predicate and a binary locative PP:

- (61) John showed the picture to Mary through the window

(61) contains a three place predicate *show* and a symmetric path locative *through the window*. The PP is oriented to the second and third arguments, and the sentence entails 'the picture and Mary were on the opposite sides of the window'. Thus, the PP determines a spatial dependency between the two arguments 'the picture' and 'Mary', which is inherently binary.

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$= f(\alpha) \vee f(\beta)$ f is intersecting
 Thus f is additive.
 (1) and (2) prove that $INT_{P/P} = SO^*/OO^*.$ ###

Appendix

(55) Theorem-2: SO^*/OO^* is identical to $INT_{[P \rightarrow P]}$, the set of intersecting functions in $[P \rightarrow P]$.

Proof:

(1) For all $f \in SO^*/OO^*$, f is restricting, increasing, and additive, we show f is intersecting, i.e., $f(\alpha) = f(\mathbf{1}) \wedge \alpha$, for all $\alpha \in P$.

By the definition of boolean complement, $\alpha \vee \alpha' = \mathbf{1}$, and $\alpha \wedge \alpha' = \mathbf{0}$. Since f is additive, $f(\mathbf{1}) = f(\alpha \vee \alpha') = f(\alpha) \vee f(\alpha')$, where α' is the boolean complement of α .

$$\begin{aligned} \text{Thus, } f(\mathbf{1}) \wedge \alpha &= (f(\alpha) \vee f(\alpha')) \wedge \alpha \\ &= (f(\alpha) \wedge \alpha) \vee (f(\alpha') \wedge \alpha) && \text{Distributive law in } P \\ &= f(\alpha) \vee (f(\alpha') \wedge \alpha) && \text{Since f is restricting,} \\ & && \text{i.e., } f(\alpha) \leq \alpha \\ &= f(\alpha) \end{aligned}$$

The last equation comes from this: Since f is restricting, $f(\alpha) \leq \alpha$, $f(\alpha') \leq \alpha'$, and $\alpha \wedge \alpha' = \mathbf{0}$, so $f(\alpha) \wedge f(\alpha') = \mathbf{0}$, and $f(\alpha') \wedge \alpha = \mathbf{0}$.

Therefore, $f(\alpha) = f(\mathbf{1}) \wedge \alpha$, which proves f is intersecting.

(2) Now we show that intersecting functions are restricting, increasing, and additive: i.e., $INT_{P/P} \subseteq SO^*/OO^*$.

From the above K&F's theorem, each intersecting function is restricting. Let $\alpha \leq \beta$, then $f(\alpha) = f(\mathbf{1}) \wedge \alpha \leq f(\mathbf{1}) \wedge \beta = f(\beta)$. So, f is increasing.

Let $\alpha, \beta \in P$, then

$$\begin{aligned} f(\alpha \vee \beta) &= f(\mathbf{1}) \wedge (\alpha \vee \beta) && \text{f is intersecting} \\ &= (f(\mathbf{1}) \wedge \alpha) \vee (f(\mathbf{1}) \wedge \beta) && \text{Distributive law in } P \end{aligned}$$