Neural Correlates of Mathematical Reasoning: An fMRI Study of Word-Problem Solving

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Abstract

We examined brain activation, as measured by functional magnetic resonance imaging, during mathematical problem solving in six young, healthy participants. Participants solved problems selected from the Necessary Arithmetic Operations Test (NAOT) which is known to correlate with fluid reasoning tasks. In three conditions, participants solved problems requiring (1) one operation (Easy problems), (2) two operations (Hard problems) or (3) simple reading and matching of words (Match problems) in order to control for perceptual, motor and text reading demands of the NAOT problems. Major bilateral frontal activation and minimal posterior activation was observed while subjects solved Easy problems relative to Match problems. Minor bilateral frontal, temporal and lateralized activation of left parietal regions was observed in the Hard problems relative to Easy problems. All of these regions were activated more by Hard than by Match problems. Many of these activations occurred in regions associated with working memory. These results suggest that fluid reasoning is mediated by a composite of working memory systems that include central executive and domain specific numerical and verbal working memory.

Introduction

Mathematical problem solving is a multicomponent cognitive task which requires all facets of working memory, slave systems and central executive. Execution of arithmetic operations is one well-studied component of mathematical problem solving. A number of lesion and brain imaging studies have localized the brain regions critical for arithmetic operations to areas associated with working memory. During execution of basic arithmetic operations, working memory is required as intermediate products, necessary for later operations, must be actively maintained until current processing is completed.

Another component of mathematical problem solving that has not received as much attention is arithmetic reasoning. Arithmetic reasoning is required in more complex problems to determine which arithmetic operations are required to solve a given problem. During execution of arithmetic reasoning, goal management, strategy shifting, and planning are required as evaluations of intermediate products must be made and neccessary subsequent operations must be determined.

A variety of methods including lesion studies and brain imaging studies have been utilized in attempts to localize basic arithmetic operations in the human brain. Acalculia (the inability to perform arithmetic operations) was initially described by Henschen (1919) who localized this disorder to posterior brain regions, mainly the angular gyrus. Similarly, Gerstmann (1930) described a syndrome which included agraphia, acalculia, finger agnosia and right/left disorientation in patients with damage involving the left angular gyrus. More recent studies have shown broader regions of involvement in acalculia. Jackson & Warrington (1986), for instance, examined performance of right and left hemisphere lesion patients on WAIS arithmetic subtests, digit span, and a "Graded Difficulty Arithmetic test." Their results indicated that a left hemisphere group showed greater deficits in the Graded Difficulty Arithmetic test compared to a right hemisphere group and controls (see also Warrington et al., 1986). Similarly, Grafman et al. (1982) subdivided patients into left-anterior, left-posterior, right-anterior and right-posterior lesion groups. Mean quantitative and qualitative error scores from simple arithmetic operations revealed that left-posterior patients were more impaired than other groups. While the above studies appear to localize basic arithmetic operations in the parietal or parietooccipital regions, lesion studies occasionally implicate frontal regions (e.g., Luria, 1973). Brain imaging studies have also indicated involvement of frontal regions in performance of arithmetic operations. Burraud et al. (1995) performed functional MRI (fMRI) on normals while they serially subtracted prime numbers. Their results indicated activation mainly in left dorsolateral prefrontal cortex (BA 46). Their study was, however, limited to investigating prefrontal regions. Roland & Frieberg (1985) utilized PET imaging while subjects serially subtracted by 3s. Their results indicated bilateral prefrontal activation.

Other studies have more finely parsed which brain regions may be mediating different components of these arithmetic operations. Chase et al. (1984), for instance, performed 2dg PET on Alzheimer's Disease patients and normals while they performed WAIS tasks including the arithmetic and digit span subtests. Their results showed that the arithmetic tests activated left angular gyrus while digit span activated anterior frontal areas bilaterally. Petrides et al. (1993) have also shown maintenance components of working memory to be localized in frontal regions (but see Paulesu et al, 1993).

The lesion studies and imaging studies cited thus far lead to the conclusion that both anterior and posterior brain regions contribute to processing of basic arithmetic operations. The results of these studies are consistent with the notion that processing elements of arithmetic operations are mediated by posterior brain regions while active maintenance of intermediate products is mediated by dorsolateral frontal regions (viz., Goldman-Rakic, 1987). This facet of working memory essential for performing complex cognitive tasks, comprehension, learning and reasoning has been referred to by Baddeley (1986) as the *Central Executive,* in contrast to the more peripheral visual and phonological *Slave Systems.* The relation of individual differences in central executive working memory to differences in reasoning abilities has been captured in a computational model of human performance on well-studied reasoning tasks such as the Raven Progressive Matrices (Carpenter, Just & Shell, 1990).

Snow et al. (1984) have suggested that complex reasoning tasks like the Raven Progressive Matrices reflect executive processes necessary to structure behavior and analyze problems. They have proposed a "radex" model for fluid reasoning. In this multidimensional scaling solution, tests near the periphery (e.g., digit span, addition, multiplication, subtraction) cluster by content and may reflect slave systems primarily used for the basic operations of transformation and calculation in particular processing domains. Complex tasks such as the Raven Progressive Matrices, Sternberg Verbal Analogies and the Necessary Arithmetic Operations Task (NAOT) cluster tightly together near the center despite differences in content and are believed to reflect executive processes important for reasoning and common to all these tasks.

While previous studies have examined basic arithmetic operations, we sought to understand the basic neural substrates involved in arithmetic reasoning. Since the NAOT appears near the center of the radex model and correlates highly with other fluid reasoning tasks, we constructed our mathematical reasoning task using these word problems. Solving word problems involves both arithmetic reasoning and basic arithmetic operations. The NAOT, however, is composed of word problems which require the subject to determine the mathematical operations necessary to solve a given problem rather than the actual execution of these operations. This test allows us to examine arithmetic reasoning processes while minimizing involvement of basic arithmetic operations.

The computational processes of solving word problems have been modeled by Nathan, Kintsch, and Young (1992). Word-problem comprehension is conceptualized to involve textual representation (textbase), a situation model (a conceptual understanding of the problem to be solved) and the problem model (a mathematical formalization of the problem to be solved).

Development of the situation model and translation into a problem model may be thought of as executive and control processes (i.e., mathematical reasoning) and may be tapped by tests that appear near the center of the radex model such as the NAOT. Execution of the problem model, calculation and transformation of information, may be thought of as basic operations and may be tapped by tests that appear near the periphery of the radex model.

Kintsch and Greeno (1985) tease apart the distinctions of the problem model and the situation model by distinguishing the representations of these two models - (representation of events in mind- the situation model; representation of formal relations in mind - the problem model). To comprehend a problem, the subject must make a correspondence between the formal equations and the subjects own informal understanding of the situation described. According to Greeno (1989), difficult word problems require that inferences be made relating the two disconnected systems of situational model and problem model for the succesful completion of the problem, while easier word problems can be solved with manipulation of formal mathematical expressions without reasoning about related real-word events.

The aim of the present study was to understand the basic neural substrates involved in arithmetic reasoning. In our study we had subjects carry out difficult and easy word problems drawn from the NAOT during fMRI scanning. Earlier research by Prabhakaran et al. (in press) utilizing a fluid reasoning task, the Raven Progressive Matrices, has isolated brain regions involved in fluid reasoning. In that study we found activation of bilateral frontal and left-lateralized activation in parietal, temporal and occipital regions. Since the Raven Task and the NAOT are highly correlated (Snow et al., 1984), we expected to observe regions of activation involved in goal managment, strategy shifting and induction of abstract relations, similar to that seen during the Raven Task. Since the NAOT requires subjects to reason from verbal descriptions of problems while the Raven Task does not require word problem comprehension, we expected to see activation of regions in frontal and temporal regions not observed during the Raven Task.

Method

Participants

The participants were graduate students from Stanford University. All of them were right-handed (3 men and 4 women) and between the age of 23 and 30 (M = 26). Each participant provided a written consent which was approved by the Institutional Review Board at Stanford University.

Procedure

Prior to entering the scanner, subjects were given instructions and shown 3 sample problems to familiarize them with the task. At the outset of each trial, a word problem (drawn from Necessary Arithmetic Operations Test; see Figure 1) was presented on the screen for 30 seconds with 4 answer choices below it. In the last 5 seconds one of the answer choices was highlighted. The answer choices consisted of one or two mathematical operations (e.g., addition, division). The subject squeezed a squeeze ball to indicate whether the highlighted choice was the correct answer to the problem.

Task Design

The NAOT consists of standard word problems in mathematics that utilize basic addition, subtraction, multiplication and division operations to solve the problems. Instead of solving the word problems and finding an answer, the task is to indicate which arithmetic operations would be used to solve the problem. The original task was modified for the scanner.

Three types of problems were created with increasing levels of difficulty. Problem difficulty was operationalized by the number of mathematical operations required in the problem.

Easy problems were chosen from the set of NAOT problems which required only one operation (e.g. multiplication). Since subjects needed to discern only one operation neccessary to solve these problems, construction of a problem model was all that was necessary in order to solve them; minimal reasoning from a related real-word situation was required.

Hard problems required two operations (e.g. multiplication and addition). Since subjects needed to discern which two operations were neccessary to solve the problem, understanding and reasoning from a related real-world situation was necessary to determine the formal operations required to solve the problem.

Match problems consisted of word problems in which the solution was provided in the text of the problem and the subjects were asked to match the answer with the choices provided below. The Match problems were used to control for cognitive, sensory and motor activation (e.g. reading the word problem, visual inspection of stimuli, eye movements, motor response) that were irrelevant to the cognitive factors of interest.

Three sets of problems were presented to subjects in the scanner. The first two sets contained 12 problems or 6 cycles of alternating problem types of Easy and Hard followed by Match (Easy-Match, Hard-Match). The third set contained 12 problems or 6 cycles of alternating problem types of Hard followed by Easy problems (the Hard-Easy condition). The presentation of the three sets of problems were counterbalanced across subjects to eliminate any effects from the order of presentation.

fMRI Methodology

Imaging was performed with a 1.5T whole-body MRI scanner (General Electric Medical Systems Signa, Rev. 5.5). For functional imaging, two 5 inch diameter local receive coils were used for signal amplification. Head movement was minimized using a bite-bar formed with each subject's dental impression. A T2* sensitive gradient echo spiral sequence (Meyer, Hu, Nishimura, & Macovski, 1992), which is relatively insensitive to cardiac pulsatility motion artifacts (Noll, Cohen, Meyer, & Schneider, 1995), was used for functional imaging with parameters of TR = 900 msec , TE = 40 msec, and flip angle = 65 degrees. 4 interleaves were obtained for each image, with a total acquisition time (sampling interval) of 2.88 sec per image. T1-weighted, flow compensated spin-warp anatomy images (TR=500msec; minimum TE) were acquired for all sections that received functional scans, and pixels found to be significantly activated during the functional scan were overlaid on these structural images. Eight 6 mm thick slices were acquired in the horizontal plane of the Talairach and Tournoux atlas (1988) starting from 7.5mm below the anterior commisure (AC) posterior commisure (PC) line, with a 1.5mm inter-slice interval. Stimuli were generated from a computer and back-projected onto a screen located above the subject's neck via a magnet-compatible projector. Visual images were viewed from a mirror mounted above the subject's head. The sequence of the presentations of the stimuli were synchronized with the imaging sequence of the scanner.

Data Analysis

Image analysis was performed off-line by transferring the raw data to a Sun SparcStation. A gridding algorithm was employed to resample the raw data into a cartesian matrix prior to processing with 2d FFT. Once individual images were reconstructed, time series of each pixel was obtained and correlation methods that take advantage of periodically oscillating paradigms were used to analyze functional activation (Friston, Jezzard, and Turner, 1994). As described by Friston et al. (1994), the reference function was computed by convolving a square-wave at the task frequency with a data-derived estimate of the hemodynamic response function. The frequency of the square-wave was computed from the number of task cycles divided by the total time of the experiment. For the experiments, one task cycle consisted of a control block and an experimental block each of equal duration. There were six cycles presented over a 360 sec scan (frequency ~0.0166 Hz). Correlations between the reference function and the pixel response time-series were computed and normalized (see Friston et al., 1994).

To construct functional activation maps, pixels that satisfied the criterion of $z > 1.96$ (representing a significance at $p < .025$, one-tailed) were selected. This map was then processed with a median filter with a spatial width of 2 to emphasize spatially coherent patterns of activation. The filter was used on the assumption that pixels with spuriously high z values (i.e., false positives due to type I errors) are less likely to occur in clusters than pixels with genuinely high z values, and thus clusters of pixels with high z values are more likely to reflect an active region. The resulting map is overlaid on a T1-weighted structural image.

To obtain composite maps of activation over all subjects, average functional activation maps were created by transforming each section from each subject to a corresponding standardized horizontal section (Talairach & Tournoux, 1988) at the same distance above and below the AC/PC plane (Desmond et al.,1995). This transformation was done individually for all horizontal sections. Following transformation, the average z-value for each pixel in a section was computed across subjects and pixels that reached a statistical threshold of $p < .10$ or lower are displayed on each map.

Results

Behavioral Performance

Participants performed with high accuracy on the Match problems (M = 97.2%), slightly less well on the Easy problems (M = 93.1%), and least well on the Hard problems (M = 77.8%). Performance on each problem type did not differ significantly across scans. Therefore, scores for each problem type were combined across scans and examined in a repeated-measures analysis of variance (ANOVA). Scores differed significantly for the three problem types, $F(2, 5) = 24.114$, $p < .0001$. Participants performed significantly better on Hard than Match problems, $t(5) = 11.76$, $p < .0001$ (one-tailed) and on Hard than Easy problems, $t(5) = 4.61$, $p < .0058$. No significant difference was found between Easy and Match problems, $t(5) = 1.18$, $p < .29$.

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Figure { SEQ Figure * ARABIC }. Activation patterns for Hard-Match , Easy-Match, and Hard-Easy conditions.

fMRI Scans

The Hard/Match scan yielded a number of cortical activations that were all greater for Hard than for Match problems. Major foci of activity occurred bilaterally in the superior, middle and inferior frontal gyri and premotor areas (Brodmann areas 6, 8, 9, 10, 44, 45 and 46). Activation also occurred in superior and inferior parietal areas (areas 7 and 40), more on the left than the right. Early visual areas, precuneus, medial occipital gyri, and lingual gyrus (areas 7, 18 and 19) also showed significant activation. Inferior, middle and superior temporal gyri (areas 21, 22, 37 and 39) were activated. Left parietal areas in supramarginal and angular gyri (area 39) were activated. Minor foci of activity were seen bilaterally in the anterior cingulate (area 32).

The Easy/Match scan yielded activations that were greater for Easy than Match problems. In contrast to the Hard-Match activations, Easy/Match activations were fewer and less pronounced when occurring in the same region. Major foci of activity occurred bilaterally in the middle, inferior frontal gyri and premotor areas (Brodmann areas 6, 8, 9, 10, 44, 45 and 46). Minor foci of activation was observed in posterior regions, bilateral supramarginal/angular gyri and in superior and inferior parietal regions (areas 7, 39 and 40). Inferior and middle temporal gyri (areas 21 and 37) also showed activation.

The Hard/Easy scan yielded activations that were greater for Hard than Easy problems. Major foci of activity occurred bilaterally in the middle, inferior frontal gyri and premotor areas (Brodmann areas 6, 8, 9, 10, 44, 45 and 46). The posterior cortex showed asymmetrically greater activity in left versus right superior parietal, inferior parietal, angular, and supramarginal gyri (areas 7, 39, and 40). Bilateral activation was seen in inferior and middle temporal gyri (areas 37, 21, and 19) and in precuneus, medial occipital and lingual gyri (areas 18, 19, and 37). Minor foci of activity were seen bilaterally in the anterior cingulate (area 32).

Discussion

Mathematical reasoning invoked by NAOT performance yielded fMRI activation of an extensive, but specific, network of cortical regions. The Hard/Match comparison revealed activations in bilateral frontal, parietal, temporal and occipital regions with parietal activation being lateralized to the left hemisphere. The Easy/Match comparison revealed predominantly bilateral frontal activations and minimal activity was observed in other regions. The Hard/Easy scan revealed bilateral frontal and temporal activations and left-hemisphere parietal activations. Temporo-occipital activations were diminished in the Hard/Easy scan, in contrast with the Hard/Match scan. This result suggests that temporo-occipital regions were engaged in the Easy/Match scan but at a sub-threshold level.

The psychological model of word-problem solving and comprehension proposed by Nathan, Kintsch, and Young (1992) posits that the understanding and solving of word problems requires three mutually constraining levels of representation that must be constructed by the solver: a) a representation of the textual input itself - the text-base, b) a model of the situation conveyed by the text in everyday terms- the situation model, and c) the formalization of that situation- the problem model. In this view, performance on Hard problems in the current study required Central Executive coordination of situation and problem models. Easy problems were solved with a minimum of coordination between these two levels of representation. Both problems required representation of the text-base. In our study, Hard/Easy scans showed the greatest domininution of activation in temporal and occipital areas, relative to the Hard/Match scan. Other studies have shown that temporal lobes are involved in sentence processing (i.e forming the text base; Partiot et al., 1996; Just et al., 1996). Thus we speculate that these areas are involved in constructing the text-base.

Extensive activation of bilateral frontal regions was observed in Hard/Easy and Hard/Match scans. Other studies have shown that these regions are engaged by tasks requiring thematic coding of stories, generating mental models, temporal ordering of events, goal-management and planning (e.g., Partiot et al., 1996; Baker et al., 1996). We speculate that the frontal regions are involved in generating and coordinating situational and conceptual models in word-problem solving. Greeno (1989) argued that abstract symbol-space representations and transformations (conceptual model), and real-word or situational representations (situation model) coexist as disconnected systems. Our bilateral frontal activation may indicate representation of the two different models in these disconnected systems. Some evidence for this hemispheric distinction comes from a recent hemispheric ECT study by Deglin and Kinsbourne (1996). These authors demonstrated, using a syllogism task, that under conditions of left-hemisphere suppression, the right hemisphere solved syllogisms using prior knowledge of real-world facts. Under conditions of right hemisphere suppression, the left-hemisphere used more formal-logical operations.

Further evidence for this hemispheric dissociation comes from a recent fMRI study of Raven's Progressive Matrices (Prabhakaran et al., in press). These researchers demonstrated that the left-hemisphere was activated more while subjects solved "Analytic" problems that required formal logical operations than "Figural" problems where only visuospatial analysis was required.

Goal management is a fundamental Central Executive process that has also been attributed to frontal lobes. Tasks that emphasize this process such as Tower-type tasks (Tower of London, Tower of Hanoi and Tower of Toronto) and Wisconsin Card Sort have shown activation principally in dorsolateral prefrontal regions. One PET study of the Tower of London task (Baker et al., 1996) showed increased activity in dorsolateral and rostrolateral prefrontal areas with increasing task difficulty. They posited, based on previous imaging and lesion studies, that the rostral prefrontal area is involved in non-routine selection of cognitive strategies while the dorsolateral prefrontal area is involved in goal management or in representing intermediate problem states in visuospatial working memory. Activation of the rostrolateral prefrontal cortex has also been observed in sequence learning paradigms (e.g., Jenkins, Brooks, Nixon, Frackowiak, &

Passingham, 1994). These tasks require planning or strategy-shifting and selection and evaluation of possible alternatives to solving a problem. Additional evidence for prefrontal mediation of goal management processes comes from imaging studies with dual-task procedures and from studies that require simultaneous storage and manipulation of information. (e.g., Jonides et al., in press; D'Esposito et al., 1995).

The temporal ordering of events is one operation neccessary for generating a mental model (Partiot et al., 1996). Our tasks required construction of mental models for problem solving. Areas activated during NAOT performance, right dorsolateral prefrontal areas, correspond to activation patterns seen in imaging studies of tasks that involve the ordering of events. (Petrides, Alivisatos, Meyer, & Evans, 1993; Prabhakaran et al., in press)

In addition to regions corresponding to central executive systems, regions corresponding to slave systems are also activated during NAOT performance, further indicating the close link between reasoning and working memory. Regions thought to mediate storage and transformational processes of verbal and numerical information (e.g., letters, digits and semantic information), include left inferior frontal operculum, Broca's area (44, 45 and 47), premotor area (6), and left parietal regions (supramarginal and angular gyrus 39, 40; Paulesu, Frith & Frackowiak, 1993; Petrides et al., 1993; Gabrieli et al., 1996; Raichle et al., 1994; Roland & Frieberg, 1985).

The present study, provides a new and direct view of the neural network underlying a fundamental human ability, the ability to reason mathematically. Hard word problems were associated with regions mediating construction of situational and conceptual mental models, processes that involve goal management, strategy shifting and temporal ordering of events. Similar to other studies of high-level reasoning (e.g., Raven Progressive Matrices) word problem solving was associated with areas mediating working memory central executive and verbal or numerical working memory.

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References

Baddeley, A. (Eds.). (1986). *Working Memory*. Oxford: Oxford University Press.

Baker, S. C., Rogers, R. D., Owen, A. M., Frith, C. D., Dolan, R. J., Frackowiak, R. S. J., & Robbins, T. W. (1996). Neural systems engaged by planning: a PET study of the Tower of London task. Neuropsychologia, 34, 515-526.

Burbaud, P., Degreze, P., Lafon, P., Franconi, J., Bouligand, B., Bioulac, B., Caille, J., & Allard, M. (1995). Lateralization of Prefrontal activation during internal mental calculation: a functional magnetic resonance imaging study. Journal of Neurophysiology, 74, 2194-2200.

Carpenter, P. A., Just, M. A., & Shell, P. (1990). What one intelligence test measures: A theoretical account of the processing in the Raven Progressive Matrices Test. Psychological Review, 97, 404-431.

Chase, T. N., Fedio, P., Foster, N. L., Brooks, R., Di Chiro, G., & Mansi, L. (1984). Wechsler adult intelligence scale performance: Cortical localization by Fluorodeoxyglucose F18-positron emission tomography. Archives of Neurology, 41, 1244-1247.

D'Esposito, M., Detre, J. A., Alsop, D. C., Shin, R. K., Atlas, S., & Grossman, M. (1995). The neural basis of the central executive system of working memory. Nature, 378, 279-281.

Deglin, V. L., & Kinsbourne, M. (1996). Divergent thinking styles of the hemispheres: How syllogisms are solving during transitory hemisphere suppression? Brain and Cognition, 31, 285-307.

Desmond, J. E., Sum, J. M., Wagner, A. D., Demb, J. B., Shear, P. K., Glover, G. H., Gabrieli, J. D. E., & Morell, M. J. (1995). Functional MRI measurement of language lateralization in Wada-tested patients. Brain, 118, 1411-1419.

Friston, K. J., Jezzard, P., & Turner, R. (1994). Analysis of functional MRI time-series. Human Brain Mapping, 1, 153-171.

Gabrieli, J. D. E., Desmond, J. E., Demb, J. B., Wagner, A. D., Stone, M. V., Vaidya, C. J., & Glover, G. H. (1996). Functional magnetic resonance imaging of semantic memory processes in the frontal lobes. Psychological Science, 7, 278-283.

Gerstmann, J. (1930). Zur Symptomatologie der Himlasionen im Ubergangsgebiet der unteren Parietal-und mittleren Occipitalwindung. Nervenarzt, 3, 691-695.

Goldman-Rakic, P. S. (1987). Circuitry of primate prefrontal cortex and regulation of behavior by representational memory. In F.Plum (Eds.), *Handbook of Physiology- The Nervous System V* (pp. 373-417). New York: Oxford University Press.

Grafman, J., Passafiume, D., Faglioni, P., & Boller, F. (1982). Calculation disturbances in adults with focal hemispheric damage. Cortex, 18, 37-50.

Greeno, J. G. (1989). Situation models, mental models, and generative knowledge. In D. K. &. K. Kotovsky (Eds.), *Complex information processing: The impact of Herbert A. Simon* Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Henschen, S. E. (1919). Ueber Sprach-, Musik-, und Rechenmechanismen und ihre Lokalisation im Grosshirn. Zeitschrift fur die gesamte Neurologie und Psychiatrie, 52, 273-298.

Jackson, M., & Warrington, E. K. (1986). Arithmetic skills in patients with unilateral cerebral lesions. Cortex, 22, 611-620.

Jenkins, I. H., Brooks, D. J., Nixon, P. D., Frackowiak, R. S. J., & Passingham, R. E. (1994). Motor sequence learning: A study with positron emission tomography. Journal of Neuroscience, 14, 3774-3790.

Just, M. A., Carpenter, P. A., Keller, T. A., Eddy, W. F., & Thulborn, K. R. (1996). Brain activated modulated by sentence comprehension. Science, 274, 114-116.

Kintsch, W., & Greeno, J. G. (1985). Understanding and solving word arithmetic problems. Psychological Review, 92, 109-129.

Lee, A. T., Glover, G. H., & Meyer, C. H. (1995). Discrimination of large venous vessels in time-course spiral blood-oxygen-level-dependent magnetic-resonance functional neuroimaging. Magnetic Resonance in Medicine, 33, 745-754.

Luria, A. R. (1973). *The Working Brain: An Introduction to Neuropsychology*. London: Penguin-Allen Lane.

Meyer, C. H., Hu, B. S., Nishimura, D. G., & Macovski, A. (1992). Fast spiral coronary artery imaging. Magnetic Resonance in Medicine, 28, 202.

Nathan, M. J., Kintsch, W., & Young, E. (1992). A theory of algebra-word-problem comprehension and its implications for the design of learning environments. Cognition and Instruction, 9, 329-389.

Noll, D. C., Cohen, J. D., Meyer, C. H., & Schneider, W. (1995). Spiral k-space MRI of cortical activation. Journal of Magnetic Resonance Imaging, 5, 49-56.

Partiot, A., Grafman, J., Sadato, N., Flitman, S., & Wild, K. (1996). Brain activation during script event processing. Neuroreport, 7, 761-766.

Paulesu, E., Frith, C. D., & Frackowiak, R. S. J. (1993). The neural correlates of the verbal component of working memory. Nature, 362, 342-345.

Petrides, M., Alivisatos, B., Meyer, E., & Evans, A. C. (1993). Functional activation of the human frontal cortex during the performance of verbal working memory tasks. Proc. Natl. Acad. Sci. USA, 90, 878-882.

Prabhakaran, V., Smith, J. A. L., Desmond, J. E., Glover, G. H., & Gabrieli, J. D. E. (in press). Neural substrates of fluid reasoning: An fMRI study of neocortical activation during performance of the Raven's Progressive Matrices Test.

Raichle, M. E., Fiez, J. A., Videen, T. O., MacLeod, A. K., Pardo, J. V., Fox, P. E., & Petersen, S. E. (1994). Practice-related changes in human brain functional anatomy during nonmotor learning. Cerebral Cortex, 4, 8-26.

Roland, P. E., & Frieberg, L. (1985). Localization of cortical areas activated by thinking. Journal of Neurophysiology, 53, 1219-1243.

Snow, R. E., Kyllonen, C. P., & Marshalek, B. (1984). The topography of ability and learning correlations. In R. J. Sternberg (Eds.), *Advances in the Psychology of Human Intelligence* (pp. 47-103). Hillsdale, NJ: Lawrence Erlbaum.

Warrington, E. K., James, M., & Maciejewski, C. (1986). The Wais as a lateralizing and localizing diagnostic instrument: a study of 656 patients with unilateral cerebral lesions. Neuropsychologia, 24, 223-239.