

# The Influence of Semantic Magnitude Representations on Arithmetic: Theory, Data, and Simulation.

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## Abstract

Arithmetic research reveals longer RTs for large problems (6x8) than small problems (2x3). While several factors have been implicated, they cannot be dissociated in normal arithmetic. Subjects were trained on an artificial operation designed to independently manipulate these variables. Results suggest that semantic operand representations and presentation frequency are involved. A new theory of arithmetic fact retrieval is introduced which suggests that arithmetic facts are stored and retrieved using a magnitude representation of the problem operands. Simulations suggest the theory is able to account for the major arithmetic fact retrieval phenomena.

## Introduction

Arithmetic fact retrieval is one of the most important skills underlying our ability to manipulate numbers. Two central findings have dominated the literature and appear key to our understanding of the cognitive processes which underlie the ability to retrieve simple arithmetic facts (e.g.,  $6 \times 7 = 42$ ) from memory.

### Problem Size Effect

Perhaps the most pervasive finding from simple arithmetic research that is responses to larger problems such as  $9 \times 8$  are on average, slower and more error prone than responses to smaller problems such as  $2 \times 3$  (see Figure 1, Campbell, 1985). This *problem size effect* has been robustly reported in studies of both addition (Ashcraft & Battaglia, 1978), and multiplication (Campbell, 1985; Harley, 1990; Miller, Perlmutter, & Keating, 1984), and has been reported across cultures (Geary, 1996).

Several possible sources of the problem size effect have been proposed, including the size of the operands (Gallistel & Gelman, 1992), the frequency of problem presentation (Ashcraft, 1982), the frequency with which non-retrieval strategies are used (e.g., successive addition; Lefevre, Sadesky, & Bisanz, 1995; Siegler & Shrager, 1984), and the order in which the problems were acquired (Campbell & Clark, 1992). None of these theoretical positions have been widely accepted, in part because each of these factors are highly intercorrelated in normal arithmetic and little independent evidence supporting these positions has been acquired (see Ashcraft, 1992).

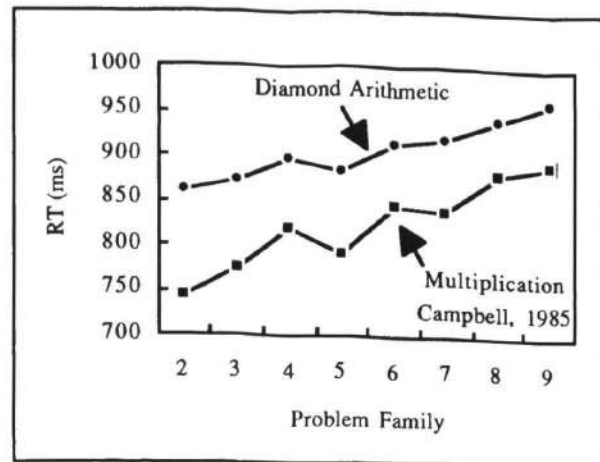


Figure 1: Simple arithmetic reaction times grouped by problem family (e.g.,  $2 \times N/N \times 2$ , etc.)

### Error Types

The most common error that subjects make when producing answers to simple arithmetic problems is to produce the correct answer for another problem which shares one operand with the presented problem. For example, if the problem  $7 \times 6$  was presented, a typical error might be to produce the answer "forty eight". Reports indicate that *operand errors* account for approximately 75-80% of all errors in production tasks (Campbell, 1985; Harley, 1990).

An interesting characteristic of operand errors is that the erroneous responses are usually correct for a problem that shares one operand with the presented problem and is also close in magnitude with respect to the other operand (McCloskey, Harley, & Sokol, 1991). For example, it is more likely that "forty eight" would be an error for the problem  $7 \times 6$  than "twelve". Most errors have been found to have an *operand distance* of two or less; as operand distance increases, the frequency of operand errors systematically decreases. Most theories have suggested that errors are due to confusions during retrieval from a network of stored facts (Ashcraft, 1992; Campbell, 1995), or to errors made when acquiring the facts and using strategies to solve problems such as repeated additions (Lefevre et al., 1995; Siegler, 1988).

Unfortunately, there has been little progress in disambiguating the various possible sources of the problem size effect and error rates because current theoretical

positions make very similar predictions. For example, in comparison to larger problems, small problems (e.g.,  $2 \times 3$ ) are likely to be taught first, retrieved more often, and solved using strategies more accurately.

To overcome difficulties associated with studying normal arithmetic, subjects were trained on an artificial arithmetic operation designed to independently manipulate the various potential sources of the problem size effect. While previous artificial arithmetic studies have been performed, none have trained subjects on a full table of problems up to a level of performance comparable to normal arithmetic (e.g., Graham & Campbell, 1992; Zbrodoff, 1995).

## Diamond Arithmetic

### Procedure

Sixty-four problems were constructed using the operands 2 through 9, the operations symbol " $\diamond$ " and answers from 23 to 98. Operand size and answer size were uncorrelated, allowing the effects of operand size and answer size to be studied independently. Further, operands and answers were not systematically related (e.g.,  $2 \diamond 3 = 97$ ;  $2 \diamond 4 = 43$ ), so problems could only be solved by retrieving the appropriate fact from memory (non-retrieval strategies such as counting up could not be used). Subjects were trained over several sessions (12-20) until their response accuracies and latencies approximated normal arithmetic performance (100% correct; average RTs under 1100ms). During training, subjects were presented all problems equally frequently.

In summary, for Diamond Arithmetic operand size and answer size were independently manipulated, and other variables such as frequency, order of acquisition, and non-retrieval strategies were controlled.

### Results

**Reaction Time** Final test results reveal that responses were faster for problems with smaller operands ( $2 \diamond 3 = 47$ ) than those with larger operands ( $9 \diamond 5 = 37$ ; see Figure 1). However, no effect of answer size was found, implying that fact retrieval is largely operand driven.

**Errors** Error types and rates during training were quite similar to those found in normal arithmetic. A majority of errors (74%) were found to be operand errors even when the answers to the problems were quite different (e.g., stimulus:  $6 \diamond 4$ ; response: 98; where  $6 \diamond 4 = 31$  and  $6 \diamond 5 = 98$ ). As in normal multiplication, a operand size effect was also found. The likelihood of an operand error varied relative to the difference between problem operands. For example, when presented with the problem  $6 \diamond 4$ , subjects were more likely to produce the answer corresponding to the problem  $6 \diamond 5$  than  $6 \diamond 9$ .

### Discussion

The results from the Diamond Arithmetic experiment suggest that the problem size effect is related to the magnitude of the operands (Gallistel & Gelman, 1992), and

is not related to the magnitude of the answer (Campbell, 1994).

There are several implications of the Diamond Arithmetic Experiment for theories of arithmetic fact retrieval. First, it appears that non-retrieval strategies (such as multiple additions) are not required to elicit a problem size effect, or standard error types. Despite the absence of typical non-retrieval strategies (e.g., multiple additions), a problem size effect and typical error pattern were found, suggesting that these characteristics of normal arithmetic may not be attributed to non-retrieval strategy use (e.g., Siegler & Shrager, 1984).

There was a problem size effect in Diamond Arithmetic (even when all problems were presented with equal frequency), suggesting that the frequency of problem presentation is not the primary source of the problem size effect, as was suggested by Ashcraft (1992). Further, it also appears that the magnitude of the answer is not provide a significant contribution to reaction time latencies, or error rates, as was suggested by Campbell (1995).

In summary, most major theories of arithmetic fact retrieval have difficulty accounting for the findings of the Diamond Arithmetic experiment. However, it does appear that aspects of the various theories are important components which can be integrated to provide a new theory of arithmetic fact retrieval. For example, the notion that arithmetic facts are retrieved using a representation of the magnitude of the operands (Gallistel & Gelman, 1992) is quite consistent with the findings from the Diamond Arithmetic study. Further, the notion that arithmetic facts (Ashcraft, 1992; Campbell, 1994) are retrieved from a network of stored facts corresponds well with the reported error pattern, in which most answers were errors to other problems which had operands which were close in magnitude to the correct operands. Due to the limitations of current theories of arithmetic fact retrieval, a novel theory is introduced.

### The Semantic Network Retrieval Theory

The semantic network retrieval theory can be considered an amalgamation of three current theoretical positions on arithmetic fact retrieval, adopting the position that arithmetic facts are retrieved from a network of related facts (Ashcraft, 1992; Campbell, 1985), and the position that arithmetic facts are stored and retrieved in a semantic form (McCloskey, 1992) in which numerals are represented in terms of their magnitude (Gallistel & Gelman, 1992).

### Basic Structure of the Semantic Network Retrieval Theory

The semantic network retrieval theory (SNRT) assumes that arithmetic facts are retrieved from an associative network of facts using a semantic representation of the problem. The fact retrieval system is composed of three groups of nodes representing: the current arithmetic problem, the stored arithmetic facts, and the answer output. The problem input includes a representation of the magnitude of the first operand, the magnitude of the second operand, and the arithmetic operation (not presented in

Figure 2). The answer output nodes represent the magnitude of the tens component of the answer, and the magnitude of the ones component of the answer. Separate tens and ones representations are postulated due to findings from error priming (Campbell, 1996) and acquired dyscalculia (McCloskey et al., 1991).

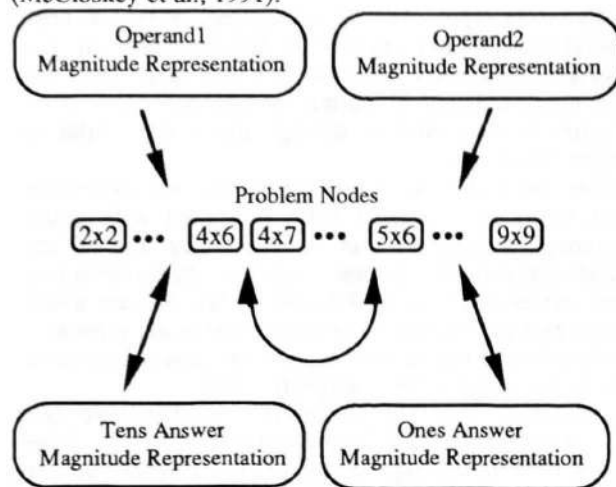


Figure 2: Semantic Network Retrieval Theory

Each arithmetic fact is represented by an individual problem node which has excitatory connections with its problem representation (e.g., the representation of  $3 \times 4$ ) and its corresponding answer (tens:1; ones:2). Problem nodes also have inhibitory connections to problem and answer nodes which are not involved in representing its answer (e.g., the  $3 \times 4$  problem node has inhibitory connections to tens nodes which are not involved in representing ten). Connections between problem nodes and answer nodes are bidirectional, allowing the activation of the answer to influence the activation of individual problem nodes. However, the connections between the presented problem and the problem nodes are unidirectional: the presented problem representations remain fixed through the fact retrieval process. It is assumed that problem nodes are also connected to other problem nodes in an inhibitory manner.

**Simulation Framework** The basic components of the semantic network simulation include 3 groups of nodes: 500 input nodes representing the presented problem, 64 problem nodes representing the stored multiplication problems, and 500 output nodes used to represent the answers to a problem.

The 500 input nodes are divided into two equal groups: those that represent the magnitude of the first operand, and those representing the magnitude of the second operand. Only one arithmetic operation was simulated and therefore no arithmetic operation units were included in the simulation. The 500 output units also are divided into two equal groups of 250 nodes. The first group of nodes represent the magnitude of the tens component of the answer, and the second group represent the magnitude of ones component of the answer.

Each problem node represents a unique arithmetic fact by the nature of its connection weights with the operand and

answer nodes. Problem nodes have positive connections to operand nodes which compose the representation of its operands. For example, the  $8 \times 5$  problem node has positive connections to the nodes in the first operand representation which represent 8, and to the nodes in the second operand which represent 5. All remaining operand nodes have negative connections with the  $8 \times 5$  problem node.

Problem nodes are also connected to answer units. Each problem node has positive connections to answer nodes which correspond to its answer. For example, the  $4 \times 6$  problem node has positive connections to all of the units in the tens answer representation which correspond to the magnitude 2, and to all of the units in the ones answer representation which correspond to the magnitude 4. All remaining answer nodes which are not involved in representing that problem node's answer have negative connections with the problem node. These connections are bidirectional: answer units receive activation from the problem units, and the problem units receive activation from the answer units. The weights between problem and answer nodes are weaker than those between operand and problem nodes. This difference allows operand activation to be the primary influence on problem node activation.

## Representations of Magnitude

In order to introduce the manner in which numerical magnitude is represented, evidence from related arithmetic tasks is introduced which provides insight into the nature of the underlying magnitude representations.

Evidence from magnitude comparison judgments suggest that numerosities (e.g., 4, 9) are not represented only as distinct lexical-semantic entities, but are also represented in an analog fashion along a 'mental number line' (Meck & Church, 1983; Moyer & Landauer, 1967). This magnitude representation appears to play a central role in many kinds of numerical processing. For example, when asked to choose the larger of two numbers (e.g., 2 or 3), adult human subjects are faster to judge the larger of very different magnitudes (e.g., 2 and 9) than with more similar magnitudes (e.g., 4 and 5), suggesting that numbers which are close in magnitude have more similar representations than those with dissimilar magnitudes, as would be the case if the representation that mediated the comparison were magnitudes that obeyed Weber's law.

In addition, if the difference between numerals is held constant magnitude judgments are faster for smaller pairs (e.g., 3 and 4) than for larger pairs (e.g., 8 and 9), implying that it is more difficult to differentiate between the representations of larger numerosities. It has often been argued that these results imply that numbers are represented by the same kinds of mental magnitudes that represent, for example, the durations of intervals, and that the judgment of which number is bigger is mentally equivalent to the judgment of which of two intervals is longer, or which of two weights is heavier, etc. (e.g., Dehaene, Dupoux, & Mehler, 1990; Gallistel & Gelman, 1992; Moyer & Landauer, 1967).

Given the robust findings from non-arithmetic numerical tasks, it is proposed that arithmetic facts are stored and retrieved using a representation of numerical magnitude.

This paper introduces one possible instantiation of a magnitude representation. However, the fundamental claim presented here is consistent with the assumptions of Dehaene and Gelman and Gallistel that numerical magnitude is represented such that: (a) the closer two numerals are in terms of nodes of their magnitude the more similar their representations are, and (b) the distinctions between adjacent magnitude representations become smaller the larger the quantity represented.

**Simulating Numerical Magnitude** Figure 4 presents the manner in which numerical magnitudes were instantiated in the simulation. The representations of the numerals 0 through 9 are presented. Each column in Figure 15 represents a unique pattern of activation for the same 250 nodes. Each node has an activation value of 0 if thin and unshaded, and .01 if wide and shaded. The total activation across nodes for each numeral representation sums to 1.

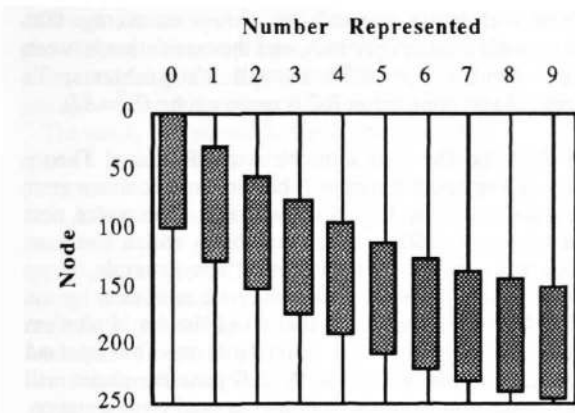


Figure 4: Simulated representation of numerical magnitude. Broad representations correspond to node values of 1, all other nodes have an activation of 0.

As can be seen from Figure 4, numerals with similar magnitudes share features with one another. The closer the magnitudes of the numerals, the more similar their representations are. For example, while the representations of 1 and 6 have only a small proportion nodes with activation in common, the representations of 1 and 2 share most features with one another.

The representation of numerals in Figure 4 also attempts to capture the notion that larger numerals share more features with one another than smaller numerals do. Differences between the representations of the numerals become smaller as the magnitude of the numerals being represented increases. For example, the representations of 1 and 2 share approximately two-thirds of their representations, whereas the representations of 8 and 9 share almost 90% of their representations in common.

### Arithmetic Fact Retrieval

Activation is hypothesized to spread from the operand and operation nodes to the problem nodes, and then to answer nodes. Activation spreads both to the correct problem node, and other problem nodes with operands which are close in

magnitude to the correct operand nodes. For example, consider how activation will spread through the network when the problem  $4 \times 6$  is presented. Problem nodes which share the first operand with the presented problem (e.g.,  $4 \times 5$ ,  $4 \times 6$ ,  $4 \times 8$ ,  $4 + 6$ ) will all receive strong activation from the first operand representation. Other problem nodes with operands close in magnitude to the first operand (e.g.,  $3 \times 6$ ,  $5 \times 7$ ) will receive partial activation from the first operand. Those with a very different first operand (e.g.,  $9 \times 6$ ) will receive inhibition from the first operand representation. These differences are all dependent on the amount of overlap in operand representations between the presented problem, and the operands of each problem node.

Activation will spread from the second operand in a similar manner. Those problem nodes with second operands which are similar or identical to the presented second operand (e.g.,  $4 \times 6$ ,  $4 + 6$ ) will receive strong activation from the operand representation, and those with a less similar operand will receive less activation (e.g.,  $4 \times 5$ ) or be inhibited (e.g.,  $4 \times 2$ ) depending on the difference between the presented operand and the problem node's operand.

Activation also spreads from the operation nodes (note the simulation does not currently include operation nodes). Problems with the correct operation will receive activation while those in other operations will be inhibited.

Several factors affect the activation levels of the problem nodes. The strongest single factor is the activation from the operand and operation nodes. Problem nodes with operands close in magnitude to the presented problem (in the correct operation) will receive much more activation (and not inhibition) from the input representation than problem nodes with very different operands.

The second factor is inter-problem-node inhibition. Problem nodes mutually inhibit one another so that as activation accumulates over time, one node will generally win out and dominate answer node activation.

Answer node activation is the third influence on problem nodes. As the problem nodes accumulate activation, they in turn activate answer representations. Activated answer nodes feed activation back to problems with answers corresponding to the answer node activation pattern, and inhibit problem nodes with dissimilar answers. For example, if the answer node activation pattern approximated 72, all problem nodes with a answer in the seventies (i.e.,  $8 \times 9$ ) would be activated by the tens representation, and all others would receive less activation, or be inhibited by the tens representation (depending on the difference between their tens representation, and the representation of seventy). Similarly, all problem nodes with ones digit of 2 would be activated by the ones answer representations (e.g.,  $3 \times 4$ ;  $6 \times 7$ ;  $8 \times 9$ ), and others with more dissimilar activation patterns would receive less activation, or receive inhibition.

Finally, it is also assumed that there is some random noise in the system such that incorrect problem nodes will sometimes exceed the activation of the correct problem node activate the wrong answer for the presented problem. This provides an opportunity for errors to occur.

Arithmetic fact retrieval involves accumulation of activation in the retrieval system until the pattern of activation across the tens and ones answer nodes each arrives

at the activation pattern for one of the ten magnitude representations (e.g., the tens nodes adopt an activation pattern representing thirty, and the ones nodes adopt an activation pattern representing two).

On a typical retrieval attempt, several problem nodes initially become active. Problem nodes with operands which are closest in magnitude to the presented problem (including the correct problem node) will accumulate more activation than other nodes, creating sufficiently strong inhibition to reduce the activation of other problem nodes to zero. Over time this process of mutual inhibition typically results in a single winning problem node. Once the winning node sufficiently inhibits other problem nodes, the winning node will be the predominate influence on the answer nodes and will activate its answer node activation pattern. When one problem node is dominant, the activation pattern within the answer nodes will no longer be an amalgamation of several answers, and instead will form one of the ten activation patterns, allowing an answer to be retrieved. The answer retrieved has a separate tens and ones magnitude value which can then be converted into the appropriate form for production.

### Account of Arithmetic Phenomena

**Problem Size Effect** The Semantic Network Retrieval Theory accounts for the fact that larger problems (e.g.,  $7 \times 8$ ) have longer RTs than smaller problems (e.g.,  $4 \times 6$ ) by hypothesizing that larger problems have more inhibition from competing problem nodes than smaller problems do. Because larger operand representations are more similar to one another (e.g., the representations of 8 and 9 are more similar to one another than the representations of 3 and 4), larger operands will more strongly activate numerically close problem nodes. For example,  $7 \times 8$  will activate its strongest competitors (e.g.,  $7 \times 9$ ,  $8 \times 8$ ) more strongly than  $4 \times 6$  will activate its competitors (e.g.,  $4 \times 7$ ,  $5 \times 6$ ). Because problems with larger operands (e.g.,  $7 \times 8$ ) activate incorrect problem nodes (e.g.,  $7 \times 9$ ) more strongly than problems with smaller operands (e.g.,  $4 \times 6$ ) activate their competitors, the larger problems will receive more inter-problem-node inhibition than will problems with smaller operands. Because both problems with small and large operands receive the same amount of activation from operand nodes, the correct problem node for a problem with larger operands will take longer to accumulate activation than problems with smaller operands.

**Simulation Results** One way of measuring the reaction times of the simulation is to use the number of cycles the simulation takes to reach the criterion activation level. If we assume that processing time in the simulation may approximate the process of updating activation levels in the actual system, then the number of cycles can be considered analogous to the time a subject might take to respond, and the 'pseudo RTs' from the simulation may be compared to reaction times from normal subjects.

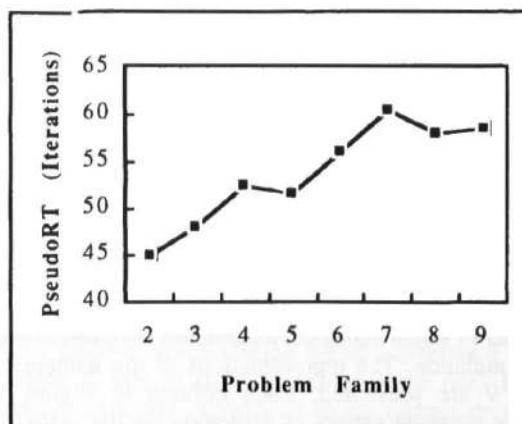


Figure 3: Iterations until solution for simulation of Semantic Network Retrieval Theory

As can be seen in Figure 3, the simulation of arithmetic fact retrieval does reveal a typical problem size effect. Problems with larger operands are slower on average than problems with smaller operands, and the correlation between problem family (i.e. RTs for all 2's problems, 3's problems...) and simulation RT is quite strong ( $R^2=.82$ ).

**Error Effects** The Semantic Network Retrieval Theory predicts that operand errors will be the most common error type because the most highly activated problem nodes, next to the correct problem node, are those which share an operand with the correct problem. For example, if we consider which problem nodes may be activated by the problem  $4 \times 6$ , it is predicted that all of the  $4 \times N$  problem nodes will receive full activation from the first operand representation, and all of the  $N \times 6$  problem nodes will receive activation from the second operand representation. Of the  $4 \times N$  problem nodes, the  $4 \times 6$  problem node should receive the most activation from the operand representations, since this problem node corresponds to both operands of the presented problem. In addition, the  $4 \times 5$  and  $4 \times 7$  problem nodes will receive full activation from the 4 operand representation, and partial activation from the 6 operand representation, since some of the semantic features of the 6 are shared by both the 5 and 7.

	Operand Distance					
	$\leq -3$	-2	-1	+1	+2	$\geq +3$
SNRT Simulation	24	62	151	156	67	26
Normal Multiplication	30	68	182	143	65	21

Table 1: Operand errors separated according to operand distance.

**Simulation Results** The types and frequencies of errors the simulation produced were compared to the errors seen in normal arithmetic. Across one hundred runs of the 64 arithmetic problems the simulation produced 578 errors, for an overall error rate of 9%. As seen in Table 1, the error patterns for the SNRT simulation appear quite consistent

with the findings from normal arithmetic. The most common type of error was the operand error.

The simulation reveals a pattern comparable to the one found in normal arithmetic. Operand errors with smaller 'operand distances' were found to be more frequent than those with larger distances. In fact, approximately 90% of the operand errors had operand distances which were less than or equal to  $\pm 2$ . This is consistent with normal arithmetic, in which the vast majority of operand errors are within an operand distance of 2.

### Summary

The results from this study suggest that arithmetic facts are retrieved from a network of stored arithmetic facts in which numbers are represented in terms of their magnitudes.

The Diamond Arithmetic experiment reveal that representations of numerical magnitude play an important role in arithmetic fact retrieval. In contrast to several different theoretical positions, the standard arithmetic phenomena were replicated despite the elimination of differences in problem presentation frequency, answer size, and non-retrieval strategies (such as multiple additions).

The newly introduced Semantic Network Retrieval Theory appears to adequately account for a number of arithmetic phenomena. The simulation produced a problem size effect both in terms of solution time and error rates. The error rates and types produced by the SNRT simulation also appear to be generally consistent with the findings from normal multiplication. The simulation had an overall error rate of 9% (normal speeded error rates range from 5-15%). As found in normal arithmetic, most errors were operand errors, and replicated the operand distance effect.

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