

Constructing Understanding: the role of animation in interpreting graphical representations

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This paper examines the mathematical conversations of middle school students while they use computer-mediated graphic representations to construct a shared understanding of the concepts of basic probability. The analysis illustrates how mathematical learning and problem solving is "stretched over" the complex cognitive system of individuals, the physical environment, and social interaction.

Thirteen pairs of students were divided into two conditions and given the task of determining if a game of chance was fair to all participants. In the game, which team scored a point was determined by the outcome of three coin flips. In both conditions, the rules of the games were established so that one team won a point if any one of five of the eight possible outcomes occurred and the other team won a point if any of the remaining three outcomes occurred. While this game may seem to be obviously unfair, students often do not realize that there is a sample space, or they do not consider the sample space (all possible outcomes and their inter-relations) relevant to their task. Students often predict that any game based on a random event or a series of random events is a fair game (Enyedy and Vahey, 1996).

Students were provided a simulation of the game which they used to investigate the importance of the sample space. The first condition employed a graphical representation of the sample space—a probability tree. In the second condition, the probability tree was augmented with animation to determine what role dynamic animation can play in helping students grasp abstract relationships. The analysis of the students' protocols focuses on how the visual and dynamic elements of a computer based graphical representation influence students' interpretations of the mathematical concepts.

The data suggests that the students with access to an animated representation had qualitatively different mathematical discussions than those who had access to the static representation. The conversations of the animation group used a narrative structure, suggested by the animation, to organize their talk about the relationship of two complex ideas of probability. The non-animation group described the same relationship using a weaker and less general structure. It is likely that these two types of conversations correlate with local perspectives that focus a students' attention on certain aspects of the probability tree and away from others.

The first and most common perspective, the "outcome perspective," was characterized by attending to the outcomes of the sample space that are represented along the bottom row of the probability tree. Given the nature of the task (a quantitative comparison of expected point totals of the two teams) there is a strong functional reason for the dominance of the outcome view in the student conversations. The bottom row of the probability tree reveals the quantitative structure of the sample space. Students who reasoned from the outcome perspective counted off the different combinations of outcomes for each team and quantitatively compared the proportions.

The second perspective, the "process perspective," is characterized by attending to the structure of the probability tree (especially the internal nodes) that represents the process by which outcomes occur. The narrative structure provides students with a means to immerse themselves in the process, identifying with an agent within the story. By adopting the viewpoint of an agent with a goal, or with someone who benefits by that agent's actions, students reasoned about the process and drew upon situational knowledge of similar contexts.

The animated probability tree provides a means for students to coordinate the situational structure of the sample space (process view) with the quantitative structure of the sample space (outcome view). Hall (1990) has proposed that certain representations allows student to consider both the situation and quantity simultaneously providing redundant constraints on the problem space. This study found that only the students who used the animated probability tree were able to coordinate the outcome and process perspectives and productively connect their intuitions about the sample space to their intuitions about the probability distribution across the sample space.

References

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