Eye Movements during Geometrical Problem Solving

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Diagrams are used extensively in posing and solving geometry problems. The rules for effective strategies for looking at diagrams are not known. It is likely that the eye movement strategies used by good problem-solvers reflect their reasoning about the problem. This suggests that studying the eye movement patterns of geometry experts observed while they solve problems posed with diagrams may reveal new information about cognitive and perceptual steps that they use to arrive at a solution.

Model

Our study of eye movements during solving of geometry problems started with a simple model within which the data was examined. The model consisted of a set of axioms about the nature of eye movements during geometrical reasoning and the relationship between eye movements and cognitive or perceptual operations. This model was based on a similar model proposed previously for eye movements during arithmetic and reading (Suppes, 1990).

Experiment

Three subjects (2 experts and 1 non-expert) were presented with simple geometry problems on the computer screen. Each problem consisted of a diagram in which some angles were labeled with letters and numerical values. Some problems also contained brief text stating initial conditions. For each problem the subject had to find the value of the unknown angle, labeled with a "?". The Maryland Revolving-Field monitor (MRFM) was used to record eye movements. Head movements of subjects were restricted from above using a bicycle helmet attached to the frame of the MRFM. This method of restraining the head allowed the subjects to talk while they were solving problems. Subjects were not allowed to write or sketch anything, but the problems were simple enough to solve mentally. Subjects were asked to reason aloud, and their speech was recorded.

Summary of results

The eye movement data was examined within the framework provided by the model. Many of the model's assumptions were confirmed by the data. There was a strong correspondence between the eye movement pattern and cognitive steps involved in solving each problem. Fixation durations depended, to some extent, on the cognitive or perceptual processing that was taking place at the time. For example, fixation durations were longer when the subjects were performing mental arithmetic than when they were scanning features of the diagram. Variability in durations of fixations was not due solely to the probabilistic nature of the oculomotor processes, although such processes clearly played an important role in determining the eye movement pattern in this task, as they do in other visually-guided tasks. Likewise, saccades were made to features in the diagram that were currently being considered, as indicated by concurrent verbal protocols. Expert subjects combined simple features into more complex, imaginary structures, as was required to solve the problem. They talked about these structures and scanned their imagined locations within diagrams. The non-expert did not construct such structures, and his eye movements were limited to the visible features of the diagram.

Conclusion

We conclude that eye movements provide important additional information about cognitive processes used to solve geometry problems. Future generations of the model will take into account details of individual problems. However, a successful model should be general enough to describe classes of cognitive operations involved in geometrical reasoning in general. and perhaps, with some parametric changes, it could transfer to other complex visually-guided tasks.

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References

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