On the Representation of Number Concepts

Marco Zorzi[1,2] (ZORZI@UNIV.TRIESTE.IT)
Brian Butterworth[1] (B.BUTTERWORTH@UCL.AC.UK)

[1]Department of Psychology, University College London
Gower St., London WC16BT UK
[2]Department of Psychology, University of Trieste
via dell'Universita` 7, 34123 Trieste (Italy)

In this work we address the issue of how numbers should be plausibly represented with a "neural" code, that is, as activation patterns over a set of processing units in neural network models of mathematical (numerical) cognition. Our approach is to evaluate a series of neural network simulations of simple arithmetic, such as single-digit additions and magnitude comparisons. We can therefore evaluate the impact of various representational schemes on the model's performance, in comparison to that of skilled human participants.

Gallistel and Gelman (1992), among others, proposed that numerical expressions in an arithmetical problem are translated into magnitudes prior to accessing the solution which is also a magnitude that needs translating back into a numerical expression. These magnitudes are points or regions on a continuous psychological dimension. This fits in well with other aspects of numerical cognition, such as magnitude comparison tasks, which appear to behave as though numbers fall on a single, possibly compressed, dimension or line. However, it fails to match both our intuitions and standard analyses of number which characterise integer arithmetic as involving cardinalities, i.e., discrete properties of sets (Giaquinto, 1995).

A second problem concerns the implementation of the hypothesis that numbers are regions on a number line. In one of the few attempts to model arithmetical fact retrieval (McCloskey & Lindeman, 1989), numbers are encoded over an ordered sequence of input nodes, where each node stands for a particular number. Moreover, the two immediate neighbours of the number are activated as well: thus 3 is represented as the activation of the node labelled "3" plus (lesser) activation of "2" and "4". One obvious problem of this approach is that two numbers such as "7" and "3" would not have anything in common, since they activate orthogonal representations (i.e., nodes 6-7-8 for "7" and nodes 2-3-4 for "3"). Indeed, Ashcraft (1995) claims that the "consensus model" of fact retrieval consists of associations of (possibly) orthogonal representations of problems and asnwers. However, our intuitive understanding of integer numbers entails that "7" includes "3": this is the notion of number cardinality.

Recent psychological evidence shows that cardinal magnitude information is automatically activated even when it is irrelevant to the task (e.g., Dehaene, Bossini, & Giraux, 1993). In connectionist terms, an abstract representation of magnitude can be straightforwardly represented as the number of units activated, whereby bigger numbers include smaller numbers; therefore, for N>M, a set with M members can be put in 1-1 correspondence with a proper subset of the set with N members (see Figure 1).

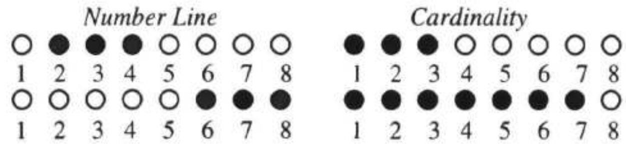

Figure 1: Alternative schemes for representing numbers.

Our study shows that feedforward networks using unrelated, orthogonal representations of number concepts have more difficulties in learning a set of arithmetic facts and offer a poor match to human performance, whereas number representations that are based on cardinal magnitudes are fundamental for successful modelling of the processes involved in simple arithmetic and magnitude comparisons.

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