

Effects of externalization on representation and recall of indeterminate problems

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Abstract

Naïve reasoning and problem solving is error-prone. One such pattern is manifested in that people err more often when problems are indeterminate than when problems are determinate. We suggest that an incomplete problem representation could account for the observed pattern of errors. We further contend that in verbal reasoning such incomplete representation stems from a lack of systematic representations of connectives (e.g., *and*, *or*, *if*, etc.), and, therefore, externalization of relations denoted by sentential connectives should improve people's representations of multiple possibilities. These predictions were tested in three reported experiments. Results indicate that determinate problems were easier to represent and recall than indeterminate problems. Furthermore, there was a tendency to represent and recall indeterminate problems as if they were determinate ones by truncating the number of possibilities compatible with the problem. Finally, external aids dramatically improved representation and recall of indeterminate problems. These results are discussed in relation to theories of representation and reasoning.

Introduction

Naïve reasoning and problem solving is error-prone (see Evans & Over, 1996; Johnson-Laird & Byrne, 1991; for reviews) and errors exhibit systematic and predictable patterns. One particular pattern of errors is especially robust: people err more often if problems are indeterminate (i.e., compatible with multiple possibilities) than if problems are determinate (i.e., compatible with a single possibility). For example, the problem is compatible with multiple possibilities if an observed outcome could be caused by several factors, an observed symptom could be indicative of several conditions, or an observed pattern could be generated by several rules.

When problems are indeterminate, people (including expert scientists, politicians, lawyers, and physicians) tend to attribute observed outcomes to one or few factors, overlooking other plausible possibilities. Such a pattern was found in the study of scientific thinking and problem solving (Kuhn, Garcia-Mila, Zohar, & Andersen, 1995; Mynatt, Doherty, & Tweney, 1977), verbal reasoning (Evans & Over, 1996; Johnson-Laird, 1998), and in learning and recall (Sloutsky, Rader, & Morris, 1998).

The most obvious theoretical explanation of the reviewed difficulties is that indeterminate problems require more working memory resources than determinate problems. As a

result, the overall error rate in indeterminate problems is greater than that in determinate ones. Such an interpretation assumes that people attempt to construe veridical problem representations, but they err because they cannot keep all these possibilities in their working memory. Although cognitive overload is a plausible factor, it is likely to result in an increase of unsystematic errors, whereas the observed patterns of errors are quite systematic. Therefore, it seems necessary to carry out a more detailed cognitive analysis and to consider other factors.

It seems that deriving a problem solution requires at least five steps. These include: (1) encoding of a verbal description or perceptual arrangement of the problem; (2) construing a problem representation that includes various elements of the problem and relations among them; (3) search through memory space for a putative solution; (4) manipulation of components of the problem for a putative solution; and (5) mapping a solution onto a verbal response. Therefore, the described difficulties may arise at each of these steps.

We believe that, when other factors are controlled or eliminated via simplifying the task, an incomplete problem representation could account for the observed pattern of errors, although with more demanding tasks other factors may also add to the problem difficulty. The idea stems from prior findings that children do create an incomplete problem representation by truncating the number of possibilities compatible with the problem and often considering just one possibility (Sloutsky, et al., 1998). We call such problem representation the "minimalist" representation and suggest that untrained adults may also tend to construe such problem representations.

One possible mechanism underlying the "minimalist" representation could be a lack of representations of relations among the alternative possibilities. Typically, in the case of verbal descriptions, these relations are denoted by sentential, or logical, connectives, such as *and*, *or*, *or else*, *if*, etc. In the case of *and* the considered possibilities (*A and B*) must co-occur, whereas in the case of *or*, the considered possibilities (*A or B*) may or may not co-occur. However, if people do not have consistent representations of sentential connectives, they would create identical (or similar) problem representations for conjunctive (those connected with *and*) and disjunctive (those connected with *or*) problem statements. In fact, it has been demonstrated (Sloutsky, et al., 1998) that people tend to recall

disjunctions as conjunctions, but not vice versa.

It is known that external representations are capable of improving the process of inference and problem solving (Bauer & Johnson-Laird, 1993; Larkin & Simon, 1987; Zhang, 1997; Zhang & Norman, 1994). It seems plausible that if people do construe defective or "minimalist" representations of possibilities denoted by different sentential connectives, failing to represent multiple possibilities compatible with some of the connectives (e.g., disjunction or conditional), then helping them to externally represent multiple possibilities should reduce errors in representation and recall. On the other hand, if externalization of possibilities corresponding to sentential connectives fail to improve representation and recall of propositions containing these connectives, then it is likely that difficulties in recall stem from other reasons than the "minimalist" representation.

If these theoretical considerations are true, then people should err when problems are indeterminate, corresponding to multiple possibilities, and these errors should exhibit a particular pattern -- people construe "defective" problem representations, truncating the number of alternatives compatible with a problem. Therefore, fully determinate problems, those that correspond to exactly one possibility, should be the easiest ones. Furthermore, in many problems, such as propositions with connectives, the total number of possibilities is fixed ($\sum P_i = 2^n$, where $\sum P_i$ is the total number of possibilities, and n is the number of atomic statements in the connective). Therefore, TRUE and FALSE possibilities in such problems are related inversely. As a result, the proportion of errors when the task is to represent what is TRUE, should be a mirror image of the proportion of errors when the task is to represent what is FALSE. Finally, if participants are assisted only in externalizing connectives, the predicted effects should decrease. The following critical predictions were tested in the three reported experiments:

1. People err more often when problems correspond to multiple possibilities rather than a single possibility: they construe "defective" or "minimalist" problem representations, truncating the number of alternatives compatible with a problem.
2. Because the total number of TRUE and FALSE possibilities are related inversely, proportions of errors when the task is to represent what is TRUE, should be a mirror image of the proportion of errors when the task is to represent what is FALSE.
3. Externalization of relations denoted by sentential connectives should improve people's representations of multiple possibilities.

Experiment 1

The goal of this experiment was to test hypotheses 1-2 via investigating people's representation and recall of propositions with various logical connectives. Participants were presented with content-based propositions having different connectives (e.g., *A and B*; *A or B*; *If A then B*) and cutout cards containing each of the atomic statements and their negations (e.g., *A*, *not-A*; *B*, *not-B*). Note that there

were no cutout cards corresponding to the connectives. The task was to select those cutouts that would (a) communicate that the proposition is true (True) or (b) communicate that the proposition is false (False).

Method

Participants The sample consisted of 26 Ohio State University post-baccalaureate students majoring in education (8 men and 18 women; Mean age = 24.9 years). They received extra credit for participation.

Materials The experiment included five logical forms: conjunctions (*A and B*), inclusive disjunctions (*A or B, or both*), exclusive disjunctions (*A or B, but not both*), conditionals (*If A then B*), and bi-conditionals (*If and only if A then B*). Each logical form appeared three times with different neutral content. The content was rotated across the connectives. Below are the examples of propositions:

Conjunction: *This person drinks orange juice in the morning and watches the history channel.*

Inclusive Disjunction: *This person likes fishing or volunteers in a public school, or both.*

Exclusive Disjunction: *This person either collects stamps or teaches classes on Thursdays, but not both.*

Conditional: *If this person works on weekends, then he supports scientific research.*

Bi-Conditional: *If and only if this person is honest he drives a blue minivan.*

Each proposition was accompanied by several cutout cards. For all logical forms, cutout cards stated atomic propositions in the sentences, negations of atomic propositions, and unrelated filler statements. For each statement, there were two cards that stated each atomic proposition in the sentence, two cards that stated the negation of each atomic sentence, and two unrelated filler items. For example the sentence *If and only if this person is honest he drives a blue minivan* was presented with the following cutout cards: *This person is honest* (two cards), *This person is not honest* (two cards), *This person drives a blue minivan* (two cards), *This person does not drive a blue minivan* (two cards), and two unrelated cards: *This person likes hamburgers*, and *This person plays tennis*. Such an arrangement was created to allow participants to veridically represent the logical forms. For example, to veridically represent conjunctions, they had to select just two cards (*A & B*), whereas to veridically represent inclusive disjunctions, they had to select six cards (*A & not-B*; *not-A & B*, and *A & B*).

The experiment had a mixed design with the Truth condition as a between-subject factor and Logical form as a within-subject factor. The Truth condition had two levels: (a) communicate that a proposition is true (True) and (b) communicate that a proposition is false (False).

Procedure Participants were tested individually in a quiet room. The experiment consisted of four phases: warm-up, selection, distraction, and cued recall. During the warm-up phase participants read instructions and completed two practice trials: one in which they represented true meaning of a sentence and another in which they represent false meaning of a sentence.

During the selection phase the participants were presented

with one sentence at a time, and, to control for encoding, were asked to repeat the sentence. If encoding was incorrect the sentence was reread and participants repeated it again. This procedure continued until the sentence was repeated correctly. After that, the experimenter laid down 11 cards in front of the participant. One card had the whole sentence printed on it, whereas ten cards had atomic propositions (2 propositions * 2 cards = 4 cards), negations of these propositions (2 negated propositions * 2 cards = 4 cards), and filler items printed on them. Then, depending on what condition they were in, participants were asked to select those cards that made the sentence true or false. For each sentence all selected cards corresponding to each proposition were placed into separate envelopes for future cued recall.

The selection phase was followed by the distraction phase, during which the participants solved simple numerical problems. This phase continued for seven minutes.

During the cued recall phase, the participants were presented with the cards that they selected and asked to recall the entire original sentence. Note that the cards stated only atomic statements and filler items, but they did not state connectives. Participants recalled one sentence at a time.

If participants recalled an entire sentence, including the connective, the answer was scored as correct. If participants recalled a sentence correctly, but substituted one connective with another, the answer was scored as a substitution error. If they recalled an affirmative proposition as a negation or a negation as an affirmative statement, if they forgot a proposition in a compound statement, or if they included one that originally was not there, the answer was scored as incorrect.

Results and Discussion

In this section, we consider how participants represented and recalled propositions. The correct (truth table) representations for the True and False conditions are presented in Table 1. The table presents the five logical forms used in the experiment, and possible card arrangements representing correct and incorrect choices by logical forms and truth conditions. A representation was coded as correct in a given condition, if a participant selected all and only card arrangements marked by the plus sign.

Proportions of correct representations are depicted in Figure 1. These data clearly indicate that participants tended to represent correctly conjunctions in the True condition (Percent Correct = 100%, Chance = 6.25%, 95% Confidence Interval = 91.4% to 100%) and disjunctions in the False condition (Percent Correct = 100%, Chance = 6.25%, 95% Confidence Interval = 78.5% to 100%). In other words, they tended to represent correctly only those forms that were compatible with exactly one possibility. In the True condition, all other logical forms generated low correct response rates that did not surpass 5%. Similarly,

low correct response rates were observed in the False condition, except for the conditional, which was still not significantly different from chance (Percent Correct = 37%, Chance = 6.25%, 95% Confidence Interval = 40% to 81%).

Because the proportion of correct responses was surprisingly low, we deemed it necessary to analyze patterns of responses. In the True condition, these patterns fell into three categories: (1) correct representations (selection of cards corresponding to all true possibilities), (2) "conversion-to-conjunction" (a tendency consider only one true possibility, $A \& B$), and (3) "other" errors (those that did not fall in the first two categories). In the False condition, we also identified three response categories: (1) correct representations (selection of cards corresponding to all false possibilities), (2) "conjunction of negations," (a tendency to consider only one false possibility, $\neg A \& \neg B$), and (3) "other" errors (those that did not fall in the first two categories).

In the True condition, the dominant pattern of responses was the "conversion-to-conjunction" (more than 90% of all responses), whereas in the False condition, the dominant pattern of response was the "conjunction of negations" (around 70% of all responses). To establish significance of the prevalence these patterns of responses, within each Truth condition and for each logical form, the number of responses conforming to the patterns was cross-tabulated with the total number of "other" and correct responses and subjected to 2 by 2 chi-square analyses. In the True condition, the prevalence of conversion-to-conjunctions was significant for the bi-conditional, $\chi^2(1, 96) = 80.67, p < .0001$, for the conditional, $\chi^2(1, 96) = 96, p < .0001$, for the exclusive disjunction, $\chi^2(1, 96) = 66.7, p < .0001$, and for the inclusive disjunction, $\chi^2(1, 96) = 80.67, p < .0001$. In the False condition, the "conjunction-of-negations" was prevalent for the conjunction, $\chi^2(1, 60) = 45, p < .0001$, and for the exclusive disjunction, $\chi^2(1, 60) = 9.6, p < .003$. For the conditional and the bi-conditional this pattern, although not reaching significance, accounted for more than a half of all responses.

Recall rates across the Truth conditions are presented on Figure 2. Note that while there were significant differences in correct representations of true and false possibilities of conjunctions and of disjunctions, recall rates for these logical forms did not differ across the Truth conditions ($F(1, 24) = 1.7, p = 0.2$, for conjunctions and $F(1, 24) = 1.6, p = .22$, for disjunctions). Overall, conjunctions were more likely to be recalled than all other logical forms, $F(1, 25) = 74.8, p < .0001$. Recall rates of the other logical forms did not differ significantly from each other.

Pattern of responses was similar to that in the representation phase. In both conditions, the conjunction generated significantly more correct responses than incorrect responses. In the True condition, there were 85% correct responses, $\chi^2(2, 144) = 90.2, z = 6.3, p < .0001$. In the False condition, there were 77% correct responses, $\chi^2(2, 90) = 41.7, z = 4.1, p < .0001$.

Table 1: Truth table representations by the logical forms.

Logical forms	Truth conditions and possible card arrangements							
	True condition				False condition			
	$A \& B$	$\neg A \& B$	$A \& \neg B$	$\neg A \& \neg B$	$A \& B$	$\neg A \& B$	$A \& \neg B$	$\neg A \& \neg B$
Conjunction (AND)	+	-	-	-		+	+	+
Inclusive Disjunction (XOR)	+	+	+				-	+
Exclusive Disjunction (OR)		+	+	-	+		-	+
Conditional (IF)	+	+	-	+	-	-	+	-
Bi-Conditional (IFF)	+	-	-	+	-	+	+	-

Note: "+" indicates a card arrangement that correctly represents a choice in a given condition, whereas "-" indicates an incorrectly selected card arrangement.

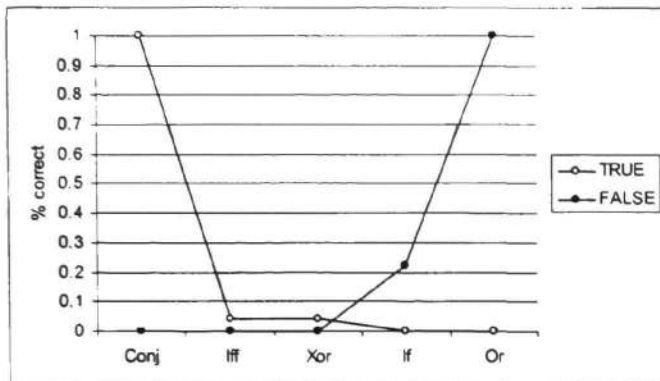


Figure 1: Percent of correct representations in the "TRUE" vs. "FALSE" conditions.

For the other logical forms, in the True condition the dominant pattern of recall was the "conversion-to-conjunction" (recall of a proposition as if it had a form A and B), whereas in the False condition, the dominant pattern of recall was the conjunction of negations. Significance of the prevalence of conversion-to-conjunction and conjunction of negations responses was established in the same manner as it was established for representations. In the True condition, the bi-conditional conversion-to-conjunctions accounted for 41% of all responses, *NS*, for 54% of exclusive disjunctions, $\chi^2(2, 144) = 15.8, z = 2.5, p < .001$, for 67% of inclusive disjunctions, $\chi^2(2, 144) = 36.8, z = 4.0, p < .0001$, and for 44% of conditionals, *NS*.

In the False condition, in addition to conjunctions, bi-conditionals generated mostly correct responses, 57%, $\chi^2(2, 90) = 11.8, z = 2.2, p < .005$. At the same time, the conjunction of negations accounted for 57% of exclusive disjunctions, $\chi^2(2, 90) = 11.1, z = 2.1, p < .005$, for 63% of inclusive disjunctions, $\chi^2(2, 90) = 18.9, z = 2.8, p < .0001$, and for 40% of conditionals, *NS*.

In short, in the True condition participants tended to represent and recall different logical forms as conjunctions, whereas in the False condition, they tended to represent propositions of all logical forms as the "conjunction of negations," while recalling them (except for the bi-conditional) as conjunctions of true possibilities. Across the conditions, conjunctions were likely to be recalled correctly. These findings support our

hypothesis that people tend to construe the "minimalist" representation, one that is compatible with a single possibility.

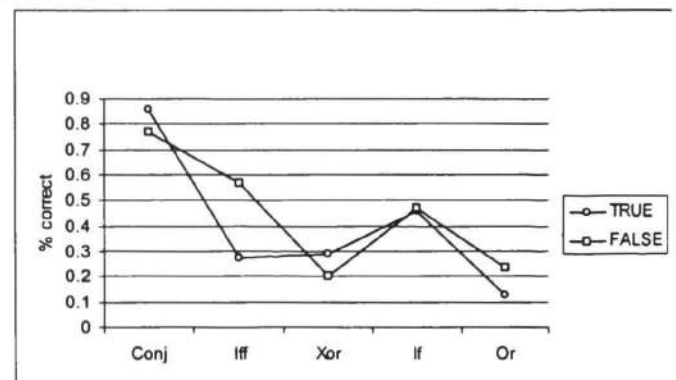


Figure 2: Percent of correct recall in the "TRUE" vs. "FALSE" conditions.

However, the experiment left a number of issues unresolved. The strongest argument against the reported findings could be that participants might have misinterpreted the task. In particular, they may have thought that the task was to select possibilities merely exemplifying that the proposition is true or false rather than selecting all possibilities that are compatible with the true or false state of affairs. It could be also argued that people poorly understand meanings of all logical connectives, except for conjunctions.

To address these arguments, we conducted the Experiments 2 and 3. In Experiment 2, we clarified the instructions, specifically asking participants to select ALL possibilities corresponding to true or false states of affairs respectively. In Experiment 3, we provided them with simple representational aids that allowed them to externalize possibilities denoted by a connective.

Experiment 2

Method

Participants Participants were 109 undergraduate students at the Ohio State University (Mean Age = 22.1 years; 15 men and 94 women). Students were recruited through psychology classes and were given extra credit for participating in the study.

Materials The problem set was identical to that in the first experiment. There were, however a number of important differences. First, each problem was followed by a full ("truth table" type) list of possibilities. For example when a problem consisted of two atomic statements *A* and *B* connected by a sentential connective, a full list of possibilities included the following: *A & B*; *not-A & B*; *A & not B*; and *not-A & not-B*. These possibilities were randomized within each problem and accompanied by two filler items. An example of an item and subsequent choice options is as follows: *This person drinks orange juice in the morning and watches history channel.*

- A. This person drinks orange juice in the morning and does not watch history channel
- B. This person has running shoes and does not smoke cigars
- C. This person does not drink orange juice in the morning and does not watch history channel
- D. This person drinks orange juice in the morning and watches history channel
- E. This person does not drink orange juice in the morning and watches history channel
- F. This person does not have running shoes and smokes cigars

The participants were presented with booklets with experimental tasks. The instruction asked them to encircle ALL choices that correspond to the proposition if it was TRUE (True condition) or if it was FALSE (False condition).

Procedure Participants were tested in groups ranging from 20 to 60 participants. The experiment was conducted in a single 20-minute session.

Results and Discussion

In the True condition, the proportion of correct representations was above chance for the conjunction (Chance = 0.167; Confidence Interval = .88 to 1, $p < .001$) and the exclusive disjunction (Chance = 0.03, Confidence Interval = .2 to .37, $p < .01$). Correct performance was at the chance level for the inclusive disjunction (Chance = 0.008, Confidence Interval = 0 to .16) and for the bi-conditional (Chance = 0.03, Confidence Interval = 0 to 0.29). Finally, the proportion of correct performance was below chance for the conditional (Chance = 0.008; Confidence Interval = 0, $p < .05$). In the False condition, the proportion of correct representations was above chance for the exclusive disjunction (Chance = 0.167, Confidence Interval = .5 to .9, $p < .01$), and at the chance level for the other forms. Overall patterns of responses were similar to those in Experiment 1. In the True condition, the majority of errors (more than 75%) were conversion-to-conjunction responses, whereas in the False condition a large number of propositions were represented as the conjunction of negations (about 55%).

Experiment 3

The goal of this experiment was to find a remedy for

"conversions-to-conjunctions" and "conjunction-of-negations." If reported errors stem from the "minimalist" representation of sentential connectives, then providing participants with external tools to represent these connectives, should allow for more complete representations, thus decreasing the proportion of errors.

Method

Participants, Materials, and Procedure The sample was selected from the same population of Ohio State University post-baccalaureate students majoring in education, as the sample for Experiment 1. It consisted of 23 Ohio State University students (5 men and 18 women; Mean age = 23.8 years). They received extra credit for participation. The experiment had the same materials, design, and procedure as Experiment 1, except that participants were given sheets of paper to externally represent sentential connectives.

Results and Discussion

Aggregated effects of externalization on correctness of representation and recall are presented on Figures 3 and 4 respectively.

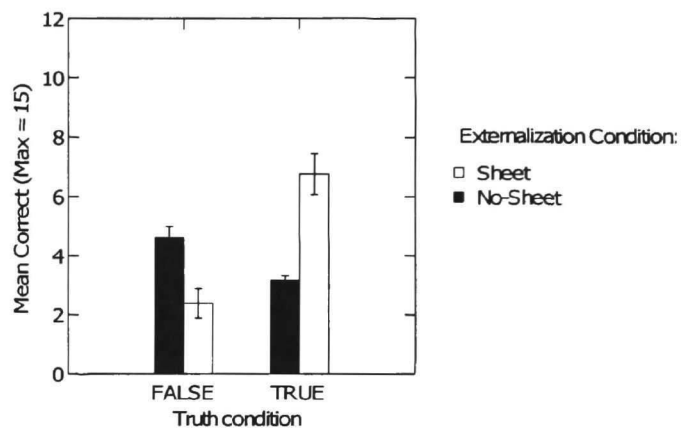


Figure 3: Effects of externalization on representation aggregated across logical forms.

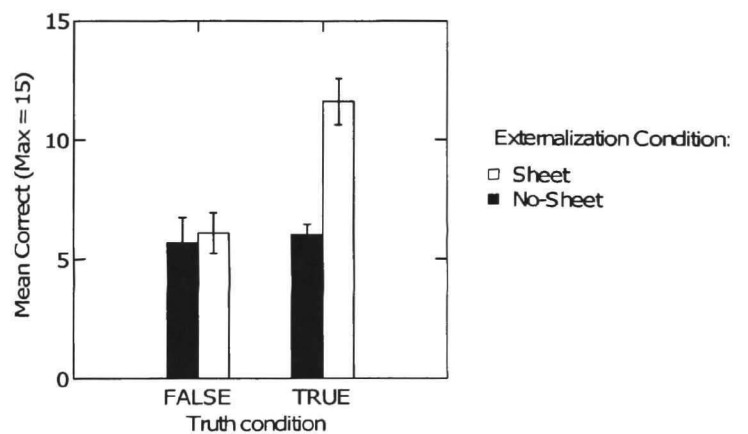


Figure 4. Effects of externalization on recall aggregated across logical forms.

As depicted in Figure 3, externalization had positive effects in the True condition ($M_{\text{external rep}} = 6.8$; $M_{\text{no-external rep}} = 3.2$) and negative effects in the False condition ($M_{\text{external rep}} = 2.4$; $M_{\text{no-external rep}} = 4.6$). A two (Truth condition) by two (Externalization condition) between-subjects ANOVA indicated that there was indeed a significant Truth condition * Externalization interaction, $F(1, 45) = 42$, $p < .0001$. At the same time, the main effect of Externalization was non-significant, $F(1, 45) = 2.4$, $p = .128$.

Data in Figure 4 indicate that in the true condition, recall drastically increased with the introduction of external representation, whereas in the false condition, recall rates remained unchanged. A 2 (Truth condition) by 2 (Externalization condition) between-subjects ANOVA indicated that the main effect of Externalization was significant, with recall rates with external representations ($M = 9.2$, $SD = 4.1$) higher than recall rates without external representations ($M = 5.9$, $SD = 2.3$), $F(1, 45) = 14.6$, $p < .0001$. The main effect of the Truth condition was also significant, $F(1, 45) = 13.7$, $p = .001$ and there was also a significant Truth condition * Externalization interaction, $F(1, 45) = 11$, $p < .003$. While effects of Externalization were pronounced in the True condition ($M_{\text{external rep}} = 11.6$; $M_{\text{no-external rep}} = 6.0$), there were no such effects in the False condition ($M_{\text{external rep}} = 6.1$; $M_{\text{no-external rep}} = 5.7$).

In short, in Experiment 3 it was found that in the True condition, external representations lead to (a) a decrease of conversion-to-conjunction responses; (b) an increase in the number of represented possibilities; (c) an increase in the percent of correct representation; and (d) an increase of correct recall. The results were more mixed for the False condition.

General Discussion

As predicted, people err more often when problems are indeterminate. These errors exhibit a particular pattern: people tend to construe "defective" problem representations, truncating the number of alternatives compatible with a problem. Second, proportions of errors when the task is to represent what is TRUE, were a mirror image of the proportion of errors when the task is to represent what is FALSE. Finally, as predicted, externalization of relations denoted by sentential connectives improved people's representations of multiple possibilities.

These findings indicate that participants tended to minimize the number of represented possibilities, often creating a representation with just one possibility compatible with the problem. We define such representation a "minimalist" representation. One possible mechanism underlying the "minimalist" representation could be a lack of consistent representations of logical connectives. In this case they would create identical (or similar) problem representations for conjunctive and disjunctive problems.

Findings support our contention that reasoning errors often stem from an incomplete or "defective" problem

representation. We took special care to eliminate alternative sources of problem difficulty. First, we constructed sufficiently simple tasks where no solutions were required and where there was no need to search through memory or problem space or to simultaneously manipulate many items in working memory. Furthermore, because the tasks have a very simple linguistic form and encoding was controlled, we could assume that errors are unlikely to stem from an inaccurate mapping. Therefore, the likely remaining candidate is the tendency to create an incomplete problem representation, although when problems are more difficult, other factors may also affect error rates.

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Acknowledgements

This research has been supported by a grant from the James S. McDonnell Foundation awarded to the first author.