Modelling human performance on the travelling salesperson problem

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Abstract

The travelling salesperson problem (TSP) is a classic problem in combinatorial optimisation which is of considerable theoretical and practical importance. Here, we propose and test a computational model of how humans solve TSPs. Model behaviour was highly consistent with human solutions. Human TSP performance depends on boundary formation. A human-emulating model may allow better solutions than conventional algorithms.

Introduction

The Euclidean form of the Travelling Salesperson Problem (TSP) requires finding the shortest path ("tour") through a set of points in the plane and returning to the origin. Although it has attracted a great deal of research, the general problem remains unsolved, in that no practical algorithm has been discovered that guarantees an optimal solution. MacGregor & Ormerod (1996) reported, however, that with a range of TSPs, humans are able to draw solutions that are significantly shorter than those provided by the best known algorithms. Human performance appears to depend upon the ability of visual perception to identify a global figure in the TSP point array. This figure is then utilised to construct a tour (Ormerod & Chronicle, 1999).

The research reported here developed and tested a mathematical model of human performance, with two aims: first, to provide support for the account of human performance suggested by Ormerod & Chronicle (1999), and second to investigate the performance of human-derived heuristics against conventional algorithms.

Method

Three psychologically plausible features were incorporated into the model: (i) a convex hull basis, (ii) a sequential procedure, (iii) capability of producing a variety of solutions. The resultant algorithm is described in general terms below. Note that "closest" can be defined in a number of possible, ways.

Step 1: sketch arcs between adjacent boundary points, select one randomly (the "current arc") and choose a tour direction

Step 2: find the closest interior point to the current arc, and check that no other boundary arcs are closer to that point. If so, the next arc in the direction of travel becomes the current arc and Step 2 is repeated. If not, retain the current arc.

Step 3: replace the current arc with two arcs that include the interior point identified in Step 2. Move to the next arc in the direction of travel, and return to Step 2.

Tour lengths generated by the model were first tested against those produced by MacGregor & Ormerod (1996)'s participants. Second, the optimality of those tours was assessed against known or predicted optimal tour lengths for the 13 problems concerned.

Results

Regression through the origin was used to find the slopes of the lines predicting the experimental data from the model results. In all cases the results supported the goodness-of fit of the model, with r-squared values in excess of 0.99 for all problems.

The best solutions of the model were 1% above optimality across the same 13 test problems. The experiment also employed the highly-structured benchmark 10-node problem described by Dantzig *et al* (1959), and for this problem the best human solutions were 2.7 and 3.0 percent above optimal. By comparison, the present model found the optimal solution.

Discussion

We proposed and tested a model of human performance on TSPs. Because of empirical evidence that people are influenced by the convex hull in generating solutions to TSPs the model was designed to conform in a general way to a convex hull approach. However, it differs from conventional convex hull heuristics by generating solutions from a given starting point and progressing in a specified direction (clockwise or counterclockwise). The results of the model conformed closely to those of the human subjects, both quantitatively and qualitatively. The model also found tours that were nearer to optimal than commonly-used heuristics. Human performance may be capitalised upon in this novel domain for cognitive science.

References

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