The Representation of Relational Information

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Abstract

Most graphic and tabular displays are relational information displays—displays that represent relational information, which is a relation on a set of dimensions. In this paper, we argue that relational information displays are *distributed representations*—representations that are distributed cross the internal mind and the external environment, and display-based tasks are *distributed cognitive tasks*—tasks that require the interwoven processing of internal and external information. The basic components of relational information displays are dimensions. Through a theoretical analysis of dimensional representations, we identified four major factors that affect the representational efficiencies of relational information displays: the distributed representation of scale information, the relation between psychological and physical measurements, the interaction between dimensions, and the visual and spatial properties of dimensions. Based on the representational analysis of relational information displays, we proposed a representational taxonomy of relational information displays. This taxonomy can classify most types of relational information displays. In addition, it can be used as a theoretical framework to study the empirical issues of relational information displays in a systematic way.

Most graphs and tabular displays are *relational information displays*—displays that represent relational information. Relational information is a relation on a set of dimensions. If $\mathbf{A_1}, \mathbf{A_2}, \ldots, \mathbf{A_n}$ are N dimensions and each dimension $\mathbf{A_i}$ is a set with C_μ elements $(a_{i1}, a_{i2}, a_{i3}, \ldots, a_{ic_\mu})$, then a relation **R** on these N dimensions is a subset of the Cartesian product $\mathbf{A} = \mathbf{A_1} \times \mathbf{A_2} \times \ldots \times \mathbf{A_n}$. Each element in the relation set **R** is a n-tuple. Let us consider an example. Table 1 shows the directory information on a Macintosh computer. It has five dimensions: **Name** = (data, final, record, work), **Size** = (120K, 100K, 70K, 55K), **Type** = (Draw, Excel, Word), **Label** = (hot, warm, cold), **Time** = (2:40pm, 3:35pm, 4:30pm, 6:20pm). The relation **Directory** on these five dimensions is a subset of the Cartesian product **Name×Size×Type×Label×Time**:

Directory = ((final, 120K, Word, hot, 2:40pm),
(work, 70K, Word, warm, 6:20pm),
(record, 55K, Draw, warm, 3:35pm),
(data, 100K, Excel, cold, 4:30pm))

Table 1. A Simplified Directory (Folder) Display on a Macintosh Computer

Name	Size	Type	Label	Time
final	120K	Word	hot	2:40pm
work	70K	Word	warm	6:20pm
record	55K	Draw	warm	3:35pm
data	100K	Excel	cold	4:30pm

In addition to the tabular display shown in Table 1, other types of tabular displays and a variety of graphic displays can represent the same relational information **Directory**. Different representations of the same information have different representational efficiencies and can cause different cognitive behaviors. This effect is called *representational effect*, which has been the focus of many studies on graphics (e.g., Bertin, 1983; Carswell & Wickens, 1988; Cleveland, 1985; Schmid, 1983; Tufte, 1990).

In this paper, we study the representational properties of relational information displays from the perspective of distributed representations (Zhang, 1992; Zhang & Norman, 1994). We argue that relational information displays are distributed representations—representations that are distributed cross the internal mind and the external environment, and display-based tasks are distributed cognitive tasks—tasks that require the interwoven processing of internal and external information.

Relational information is represented in the dimensions of a relational information display. Thus, the representation, perception, interaction, and nature (whether visual or spatial) of these dimensions are the critical factors of the representational effect of relational information displays. In the first part of this paper, we analyze these four factors. Then, based on the analysis of dimensional representations, we propose a representational taxonomy of relational information displays.

Dimensional Representations

The Distributed Representation of Scale Information

Every dimension is on a certain type of scale, which is the abstract measurement property of the dimension. Stevens (1946) identified four types of psychological scales: ratio, interval, ordinal, and nominal. Each type has one or more of the following properties (Table 2): category, magnitude, equal interval, and absolute zero. *Category* refers to the property that the instances on a scale can be distinguished from each another. *Magnitude* refers to the property that one instance on a scale can be judged greater than, less than, or equal to another instance on the same scale. *Equal interval* refers to the property that the magnitude of an instance represented by a unit on the scale is the same regardless of where on the scale the unit falls. An *absolute zero* point is a value which indicates that nothing at all of the property being represented exists.

Table 2. Properties of Psychological Scales

	ratio	interval	ordinal	nominal
category	yes	yes	yes	yes
magnitude	yes	yes	yes	no
equal internal	yes	yes	no	no
absolute zero	yes	no	no	no
example	length	time	softness	shape

From Table 2 we can see that the four types of scales have an order of representational power: ratio > interval > ordinal > nominal. A higher scale (e.g., ratio) possesses more information than a lower scale (e.g., nominal). The scale information of a dimension can be distributed across internal and external representations (Figure 1). When a higher dimension is represented by a lower dimension (e.g., a ratio dimension "length" represented by a nominal dimension "shape"; see Figure 1A), the extra information of the higher dimension either has to be represented internally, or not represented at all, because it is not embedded in the physical properties of the lower dimension. When a lower dimension is represented by a higher dimension (e.g., a nominal dimension "types of fruits" represented by a ratio dimension "length"; see Figure 1B), the extra information of the higher dimension may cause misperception on the lower dimension (Mackinlay, 1986; Norman, 1993). Thus, in order for a representation to be efficient and accurate, the scale types of the represented and the representing dimensions should match (e.g., a ratio dimension "length" represented by a ratio dimension "length"; see Figure 1C).

Figure 2 shows examples of the mapping between represented and representing dimensions of relational information displays. The represented dimensions are the dimensions of the relation **Directory** (see Table 1): size (ratio), time (interval), label (ordinal), and type (nominal). The representing dimensions are the physical dimensions used to represent the represented dimensions: length (ratio), orientation (interval), density (ordinal, see next section for explanations), and shape (nominal). In the four displays on the diagonal in Figure 2 (A, F, K, P), the scale types of the represented dimensions match the scale types of the representing dimensions. In these displays, the scale information of the represented dimensions is represented effectively and accurately. In the six displays above the diagonal, the representing dimensions have more information than the represented dimensions. The extra information may cause misperception on the represented dimensions. For example, in Figure 2C, a "warm" file may be perceived as twice as active as a "cold" file. This is a misperception, because "hot", "warm", and "cold" only indicate the relative activities of the files: they have no ratio and interval information. In the six displays below the diagonal, the representing dimensions have less information than the represented dimensions. In these displays, the extra

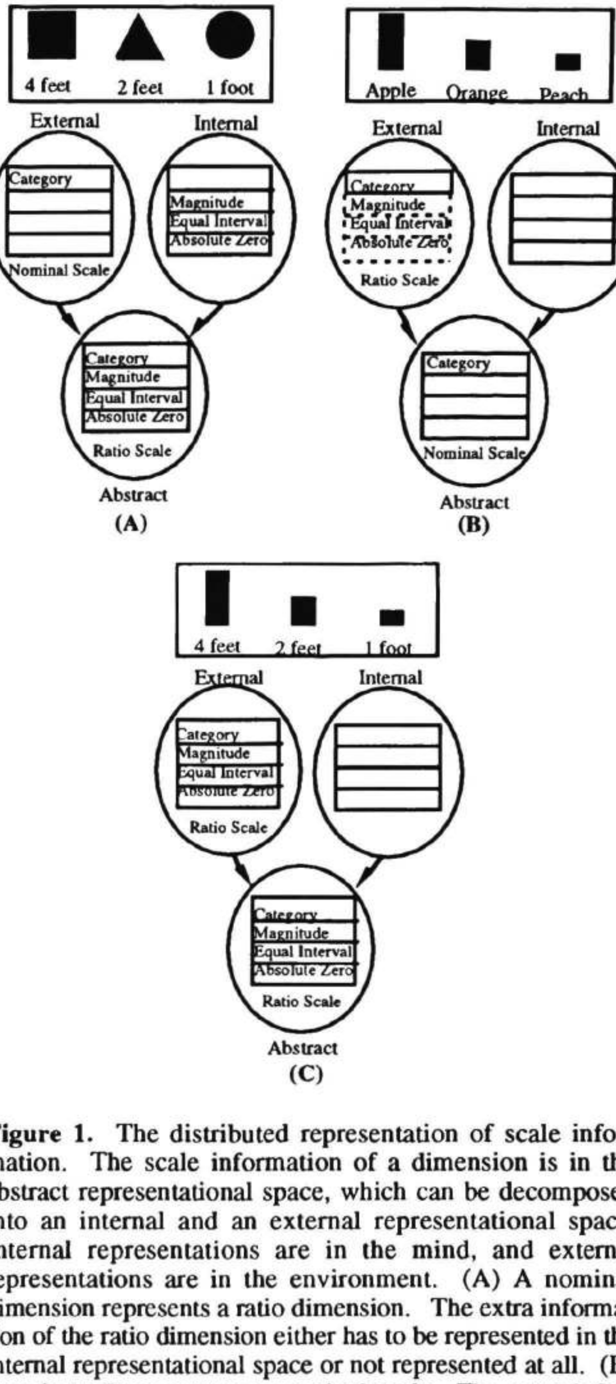

Figure 1. The distributed representation of scale information. The scale information of a dimension is in the abstract representational space, which can be decomposed into an internal and an external representational space. Internal representations are in the mind, and external representations are in the environment. (A) A nominal dimension represents a ratio dimension. The extra information of the ratio dimension either has to be represented in the internal representational space or not represented at all. (B) A ratio scale represents a nominal scale. The extra information of the ratio scale may cause misperception on the nominal scale. (C) A ratio scale represents a ratio scale. This is an efficient and accurate representation.

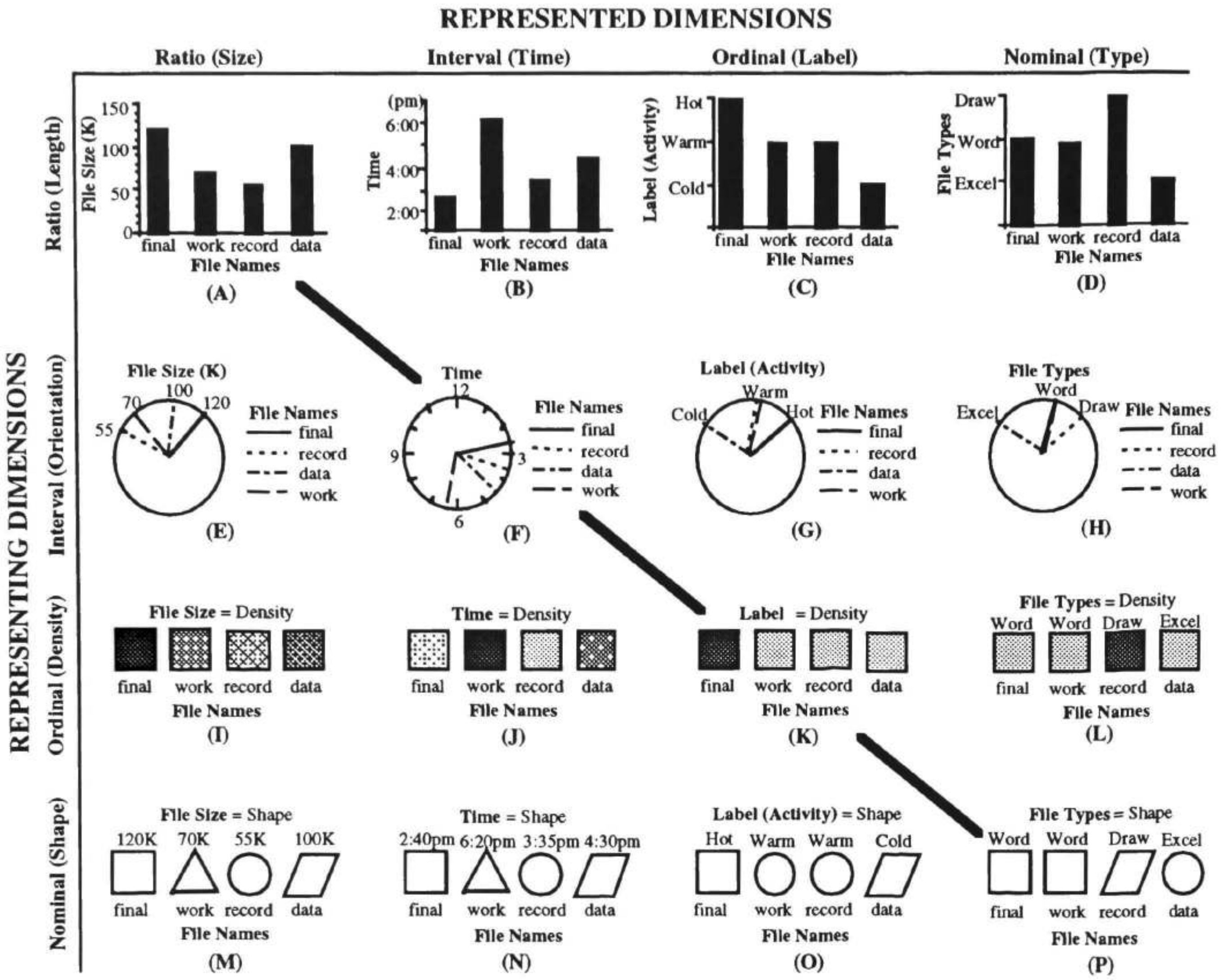

Figure 2. The mapping between represented and representing dimensions. The representing dimensions in the four displays on the diagonal have the same amount information as the represented dimensions. The representing dimensions in the six displays above the diagonal have too much information. The representing dimensions in the six displays below the diagonal have insufficient information.

information of the represented dimensions either has to be represented internally or not represented at all. For example, in Figure 2M, the shapes possess no ratio, interval, and ordinal information of the sizes of the four files. These types of information are represented by the numeric symbols, which have to be read into the mind and get interpreted.

Psychological vs. Physical Measurements

The dimensions in relational information displays can be measured either psychologically (e.g., comparing the area ratio of two circles by visual perception) or physically (e.g., measuring the length of a bar by a ruler). Because the scale type of a dimension is determined by the measurement processes, the same dimension may show different scale properties under psychological and physical measurements.

For ratio dimensions, the relation between psychological and physical measurements follows Stevens' Law (Stevens, 1957): $\psi = \mathbf{kS^n}$, where ψ is the psychological measurement, **S** is the physical measurement, **k** is a constant, and **n** is the power index determined by the properties of the stimulus and the measurement process. Cleveland & McGill (1984) showed that ratio judgments for dimensions with $\mathbf{n} \approx 1$ (e.g., length, $\mathbf{n} \approx 1$) are easy and accurate, while those for dimensions with $\mathbf{n} \neq 1$ (e.g., area, $\mathbf{n} = .6\sim.9$) are difficult and distorted. An informal observation from Figure 2 makes us speculate that interval judgments on these dimensions are also difficult and distorted. For example, it is much harder to judge the difference between two densities than between two lengths. Thus, ratio physical dimensions with $n \neq 1$ (e.g., area, volume, brightness, density, etc.) should not be used to represent ratio and interval information. However, just because of these difficulties and distortions, they can be used to represent ordinal information effectively. For example, density is considered as an ordinal dimension in Figure 2.

The Interaction between Dimensions

The dimensions of a multidimensional stimulus can be either separable or integral (e.g., Garner, 1974). Separable dimensions are those whose component dimensions can be directly and automatically separated and perceived. Integral dimensions can only be perceived in a holistic fashion and can not be separated without a secondary process that is not automatically executed. Relational information displays are multidimensional stimuli. Therefore, the separability of dimensions can affect the perception of relational information. If the dimensions of a relational information display are integral, it is difficult to separate and perceive the information on each dimension. In addition, integral dimensions may produce emergent properties that can cause misperception. For example, in Figure 3, the horizontal distance between "final" and "record" is the same as that between "record" and "data", but the former may be perceived longer than the latter. This is because horizontal and vertical distances between dots, which are integral, produce an emergent property—the absolute distances between dots (Garner & Felfoldy, 1970). For some information integration tasks, however, integral dimensions are better than separable dimensions (e.g., Carswell & Wickens, 1988).

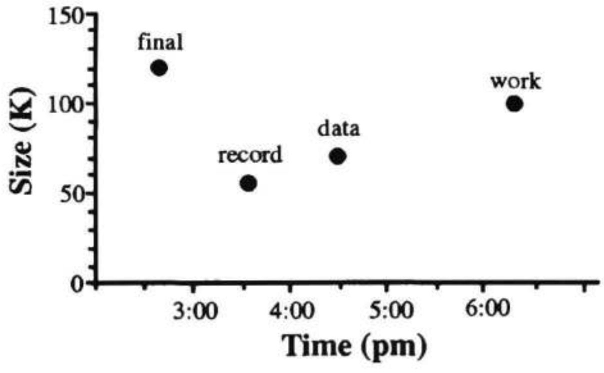

Figure 3. The horizontal and vertical distances between dots are integral. The emergent property (the absolute distances between dots) may cause misperception: the horizontal distance between "final" and "record" is the same as that between "record" and "data", but the former may be perceived longer than the latter.

Visual vs. Spatial Dimensions

Visual (such as size, texture, shape, and color) and spatial (such as position) representations are different anatomically and functionally. Visual information processing follows a pathway from the occipital to the temporal lobe, while spatial information processing follows a pathway from the occipital to the parietal lobe (e.g., Mishkin, Ungerleider, & Macko, 1983). In perceptual tasks, visual and spatial representations are processed differently and position information is required for the integration of visual features (Treisman & Gelade, 1980). There is also evidence showing that spatial representations are superior to visual representations for a variety of cognitive tasks, including the Tower of Hanoi problem (Kotovsky, Hayes, & Simon, 1985; Zhang & Norman, 1994), number representations (Zhang & Norman, 1993), and cockpit instrument displays (Zhang, 1992), just to list a few. Therefore, whether the dimensions in a relational information display are visual or spatial is another important factor of the representational effect of relational information displays.

A Representational Taxonomy of Relational Information Displays

The Hierarchical Structure

Relational information displays can be analyzed at three levels in terms of the properties of their dimensions (Figure 4). At the level of dimensionality, different displays can have different numbers of dimensions, e.g., 2-D, 3-D, 4-D, etc. At the level of scale types, the dimensions of a display can have different scale types: ratio (R), interval (I), ordinal (O), and nominal (N) scales. A n-dimensional display can have $(n+3)(n+2)(n+1)/6$ combinations of scale types[1]. For example, a 2-D display can have $(2+3)(2+2)(2+1)/6 = 10$ combinations: R-R, R-I, R-O, R-N, I-I, I-O, I-N, O-O, O-N, N-N. At the level of dimensional representations, each scale type can be implemented by different physical dimensions. For example (see Figure 4), the scale type R-R can be represented by length-length (Rectangle, Cross, etc.), length-angle (Coxcomb, Polar Plot, etc.), distance-distance (Line Graph, Cartesian Plot, etc.), and so on.

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[1] In the sequence: R••...•I••...•O••...•N••...•, the numbers of •'s after R, I, O, and N are the numbers of dimensions on ratio, interval, ordinal, and nominal scales, respectively. The number of permutations of this sequence with R fixed at the beginning is $(n+3)!$, where n is the total number of dimensions of a n-D display. Because the n •'s and the I, O, and N are interchangeable, their permutations ($n!$ and $3!$, respectively) should be excluded. Thus, the number of possible scale types of a n-D display is $(n + 3)!/n!3! = (n+3)(n+2)(n+1)/6$.

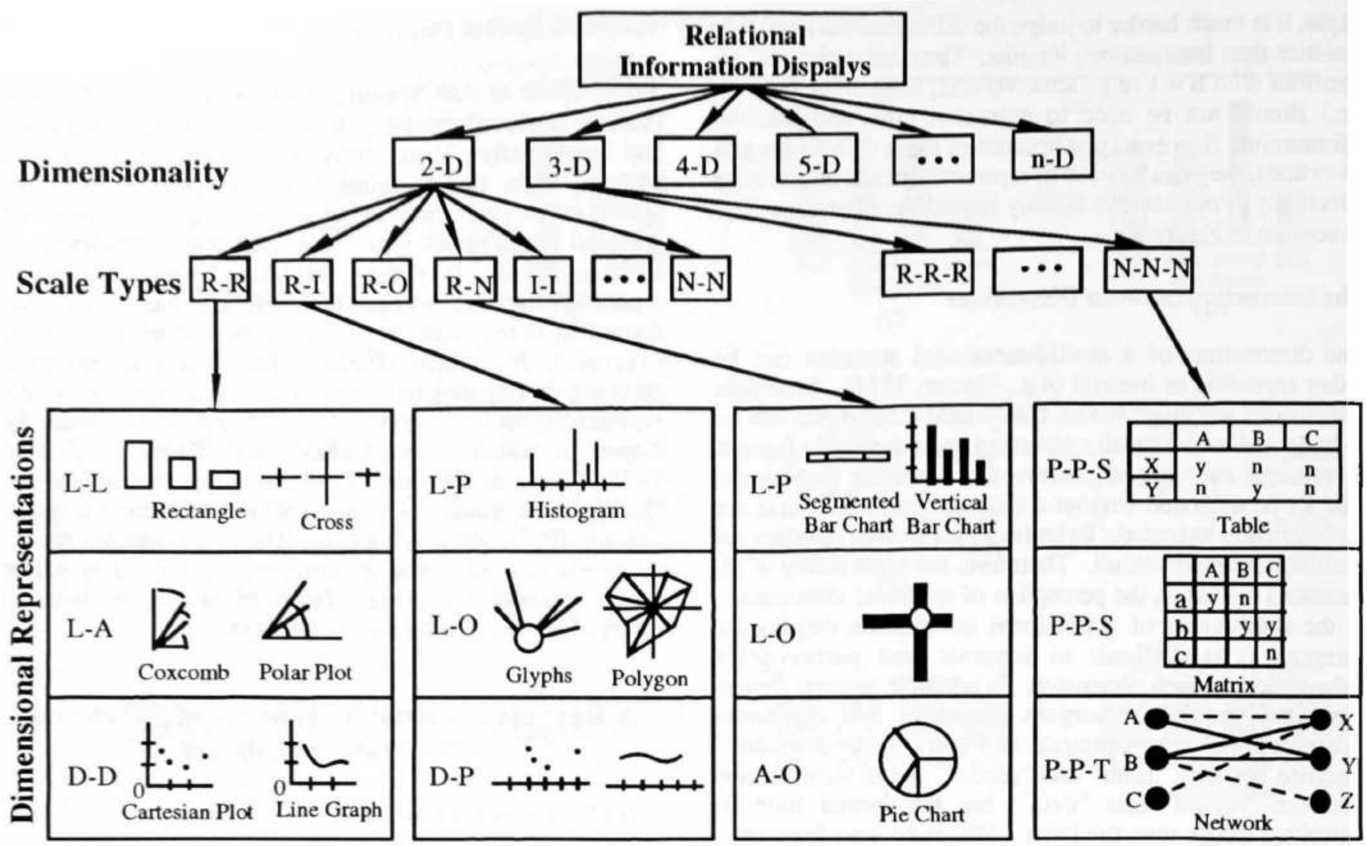

Figure 4. The representational taxonomy of relational information displays. A = Angle, D = Distance, L = Length, O = Orientation, P = Position, S = Shape, T = Texture. See text for details.

Taxonomy

The hierarchical structure in Figure 4 is a representational taxonomy of relational information displays. This taxonomy can classify most types of relational information displays, including graphs, charts, tabular displays, maps, networks, etc. For example, among the displays shown in Figure 4, the pie chart and the two bar charts are in the same category at the level of dimensional representations; the line graph and the pie chart are in the same category at the level of scale types; and all the displays in Figure 4 are in the same category at the level of dimensionality. The lower the level at which two displays are in the same category, the more similar they are. For example, the pie chart and the vertical bar chart are more similar to each other than the pie char and the line graph, because the former two are in the same group at the level of dimensional representations while the latter two are at the level of scale types.

Graphic vs. Tabular Displays

In alphanumeric tabular displays, the dimensions of relational information are represented by alphanumeric symbols and positions of table cells. Alphanumeric symbols are nominal dimensions, which can only represent nominal information externally. Cell positions are ordinal dimensions, which can only represent nominal and ordinal information externally. Thus, neither alphanumeric symbols nor cell positions can represent interval and ratio information externally. In contrast, in graphic displays, not only nominal and ordinal information but also interval and ratio information can be represented externally (e.g., by length, distance, etc.). This is the main reason why graphic displays are better than alphanumeric tabular displays when interval and ratio information needs to be represented. For relational information that only has nominal and ordinal information, graphic and tabular displays do not differ much in their representational efficiencies.

Summary and Conclusion

In this paper we identified four major factors of the representational effect in relational information displays: the distributed representation of scale information, the relation between psychological and physical measurements, the interaction between dimensions, and the visual and spatial properties of dimensions. Our major claim is that relation information displays are distributed representations across the internal mind and the external environment. Based on the representational analysis of relational information displays, we also proposed a representational taxonomy that can classify most types of relational information displays. We suggest that the four major factors and the representational taxonomy can be used as a theoretical framework to study the empirical issues of relational information displays in a systematic way.

The focus of our present study is on the representational properties of relational information displays. Over 100 years of studies on graphics have indicated that there is no universally best display for all tasks: whether a display is efficient for a task depends on not just the representational format of the display but also the structure of the task itself. Thus, in order to understand the representation and comprehension of relational information, we need to consider the relation between representational formats and task structures. This is an interesting issue worth of further studies.

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